(*Video 1: Integration by substitution *)

In[139]:= (* Examples *)
HoldForm[u = g[x]]
HoldForm[Integrate[f[g[x]] × g'[x], x]] == Integrate[f[u], u]

Out[139]= $u = g[x]$

Out[140]= $\int f[g[x]]\, g'[x]\, dx == \int f[u]\, du$

(*Example 1*)

In[126]:= HoldForm[Integrate[20 x³ Sqrt[5 x^4 + 7], x]]

Out[126]= $\int 20\, x^3\, \sqrt{5\, x^4 + 7}\; dx$

In[169]:= g[x_] := 7 + 5 x⁴
f[u_] := Sqrt[u]
g'[x] × f[g[x]]

Out[171]= $20\, x^3\, \sqrt{7 + 5\, x^4}$

In[173]:= Integrate[f[u], u] /. u → g[x]

Out[173]= $\frac{2}{3}\, \left(7 + 5\, x^4\right)^{3/2}$

In[148]:= Integrate[20 x³ Sqrt[5 x^4 + 7], x]

Out[148]= $\frac{2}{3}\, \left(7 + 5\, x^4\right)^{3/2}$

(*Example 2*)

In[130]:= HoldForm[Integrate[(10 x + 3 x²) E^(x^3 + 5 x^2 + 1), x]]

Out[130]= $\int \left(10\, x + 3\, x^2\right)\, e^{x^3 + 5\, x^2 + 1}\; dx$

In[161]:= Integrate[(10 x + 3 x²) E^(x^3 + 5 x^2 + 1), x]

Out[161]= $e^{1 + 5\, x^2 + x^3}$

In[177]:= g[x_] := x³ + 5 x² + 1
f[u_] := E^u

In[182]:= Integrate[E^u, u] /. u → g[x]

Out[182]= $e^{1 + 5\, x^2 + x^3}$

In[160]:= D[e^{x^3 + 5 x^2 + 1}, x]

Out[160]= $e^{1 + 5\, x^2 + x^3}\, \left(10\, x + 3\, x^2\right)$

(*Example 3*)

In[164]:= `HoldForm[Integrate[x (x^2 + 1) ^1500, x]]`

Out[164]= $\int x \left(x^2 + 1\right)^{1500} dx$

In[167]:= `Integrate[x (x^2 + 1) ^1500, x]`

Out[167]= $\dfrac{\left(1 + x^2\right)^{1501}}{3002}$

In[192]:= `g[x_] := x^2 + 1`
`f[u_] := u^1500`

In[195]:= `Integrate[1 / 2 f[u], u] /. u → g[x]`

Out[195]= $\dfrac{\left(1 + x^2\right)^{1501}}{3002}$

In[166]:= $D\left[\dfrac{\left(x^2 + 1\right)^{1501}}{3002}, x\right]$

Out[166]= $x \left(1 + x^2\right)^{1500}$

(*Video 2: Integration by substitution *)

(*Example 4*)

In[203]:= `HoldForm[Integrate[E^(2 x) Cos[E^(2 x)], x]]`

Out[203]= $\int e^{2x} \text{Cos}\left[e^{2x}\right] dx$

In[209]:= `D[E^(2 x), x]`

Out[209]= $2 e^{2x}$

In[213]:= `g[x_] := E^(2 x)`
`f[u_] := Cos[u] / 2`
`Integrate[f[u], u] /. u → g[x]`

Out[215]= $\dfrac{1}{2} \text{Sin}\left[e^{2x}\right]$

In[216]:= `Integrate[E^(2 x) Cos[E^(2 x)], x]`

Out[216]= $\dfrac{1}{2} \text{Sin}\left[e^{2x}\right]$

(*Example 5*)

In[204]:= `HoldForm[Integrate[(Cos[x]^3 + 2 Cos[x] - 1) Sin[x], x]]`

Out[204]= $\int \left(Cos[x]^3 + 2\, Cos[x] - 1 \right)\, Sin[x]\, dx$

In[228]:= `g[x_] := Cos[x]`
`f[u_] := - (u^3 + 2 u - 1)`
`Integrate[f[u], u] /. u → g[x]`

Out[230]= $Cos[x] - Cos[x]^2 - \dfrac{Cos[x]^4}{4}$

In[231]:= `Integrate[(Cos[x]^3 + 2 Cos[x] - 1) Sin[x], x]`

Out[231]= $Cos[x] - Cos[x]^2 - \dfrac{Cos[x]^4}{4}$

`(*Example 6*)`

In[232]:= `HoldForm[Integrate[1 / (x Log[x]), x]]`

Out[232]= $\int \dfrac{1}{x\, Log[x]}\, dx$

`g[x_] := Log[x];`
`f[u_] := 1 / u;`
`Integrate[f[u], u] /. u → g[x]`

Out[241]= $Log[Log[x]]$

In[242]:= `Integrate[1 / (x Log[x]), x]`

Out[242]= $Log[Log[x]]$

In[243]:= `D[Log[Log[x]], x]`

Out[243]= $\dfrac{1}{x\, Log[x]}$