```
(*Video 1: Fundamental Theorem of Calculus *)

In[21]:= (*Antiderivative of x^2, also known as indefinite integral of x^2 *)
HoldForm[Integrate[x^2, x]] == Integrate[x^2, x]
```

Out[21]= $\int x^2\,dx == \frac{x^3}{3}$

```
In[11]:= (*Definite integral of x^2, from x=a to x=b*)
HoldForm[Integrate[x^2, {x, a, b}]]
```

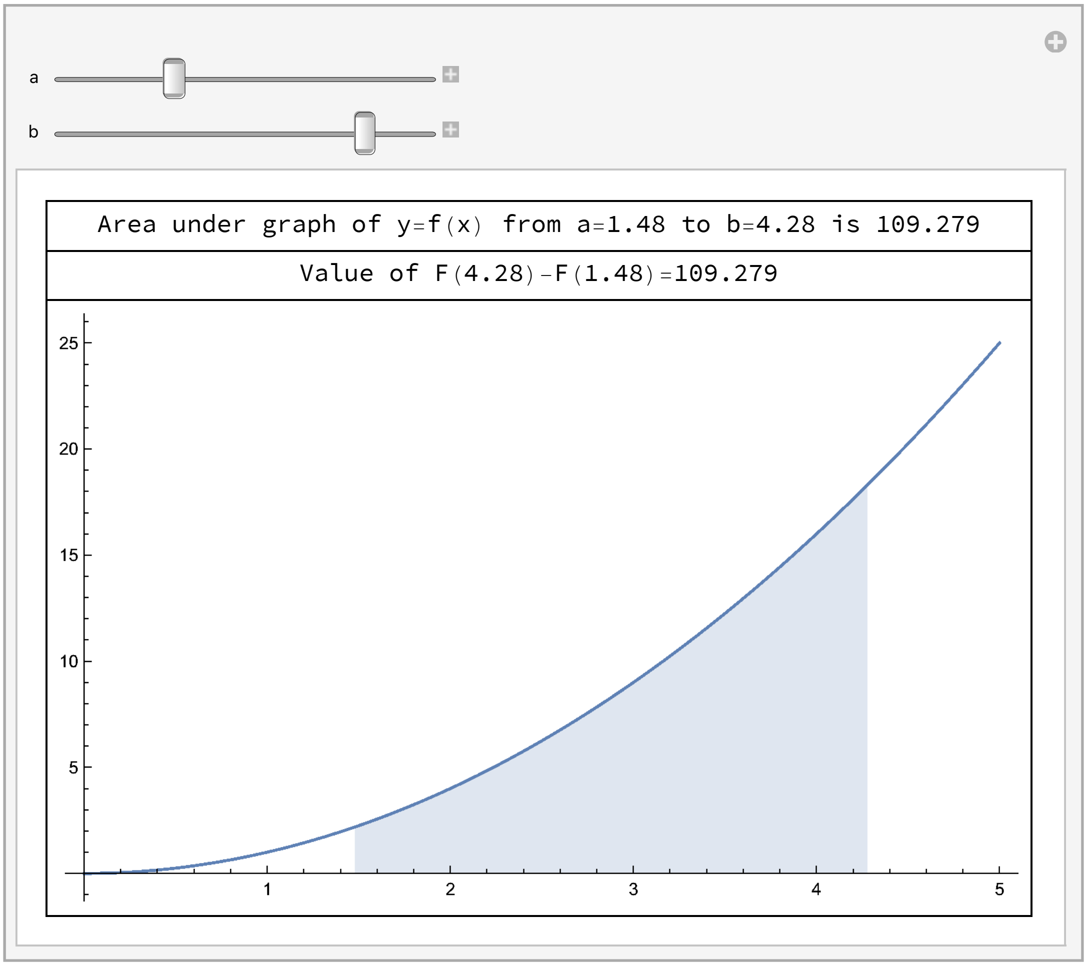
Out[11]= $\int_a^b x^2\,dx$

```
In[22]:= F[t_] := t^3 / 3

(*The above definite integral, by definition,
is the area under the graph of y=x^2 that lies between x=a and x=b*)
```

In[23]:=
```
Manipulate[
 Grid[{{Row[{"Area under graph of y=f(x) from a=", a, " to b=", b, " is ",
      F[b] - F[a]}]}, {Row[{"Value of F(", b, ")-F(", a, ")=", F[b] - F[a]}]},
   {Show[Plot[x^2, {x, 0, 5}, ImageSize → 500], Plot[x^2, {x, a, b},
      Filling → Bottom, PlotRange → {{0, 5}, {0, 25}}, ImageSize → 500]]}},
  Spacings → {1, 1}, Frame → All], {{a, 2}, 0.01, 5}, {{b, 4}, 0.02, 5}]
```

Out[23]=




```
(*Fundamental Theorem of Calculus:*)
```

$$\int_a^b f[t]\,dt = F[b] - F[a]$$

```
(*where F[x] is an antiderivative of f[x], i.e., F'[x]=f[x]. *)
(*Note: on the left we have a definite integral,
on the right we have the difference of values of a indefinite integral*)

(* Let's see that F[x] + C leads to the same result,
that is, any antiderivative can be used:
*)
```

In[26]:=
```
Integrate[x^2, x]
```

Out[26]= $\frac{x^3}{3}$

In[31]:=
```
otherF[x_] := F[x] + C
otherF[x]
```

Out[32]= $C + \frac{x^3}{3}$

In[33]:=
```
Integrate[x^2, {x, 1, 2}]
```

Out[33]= $\frac{7}{3}$

In[42]:=
```
F[2]
F[1]
```

Out[42]= $\frac{8}{3}$

Out[43]= $\frac{1}{3}$

```
F[2] - F[1]
 (*Computing the above definite integral using the FTC and antiderivative F*)
```

Out[37]= $\frac{7}{3}$

In[40]:=
```
otherF[2]
otherF[1]
```

Out[40]= $\frac{8}{3} + C$

Out[41]= $\frac{1}{3} + C$

In[38]:=
```
otherF[2] - otherF[1] (*Computing the above
  definite integral using the FTC and antiderivative otherF*)
```

Out[38]= $\frac{7}{3}$

```
(*Another example:*)
f[x_] := x^3 + 2 x^2 - 5 x + 6
Integrate[f[x], x](*Antiderivative / Indefinite integral*)
```

Out[51]= $6\,x - \frac{5\,x^2}{2} + \frac{2\,x^3}{3} + \frac{x^4}{4}$

In[52]:=
```
F[x_] := 6 x - 5 x^2/2 + 2 x^3/3 + x^4/4 + 17
```
$$F[x\_] := 6x - \frac{5x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4} + 17$$

In[54]:=
```
F'[x] == f[x]
```

Out[54]= True

```
(*Definite integral of f(x) from a=-2 to b=3 *)
```

In[55]:=
```
Integrate[f[x], {x, -2, 3}]
```

Out[55]= $\frac{685}{12}$

```
(*Difference of values of antiderivative F[b] - F[a] *)
F[3] - F[-2]
```

Out[56]= $\frac{685}{12}$

```
(*Define antiderivative / indefinite integral*)
```

$$F[x] = \int (x^3 + 2x^2 - 5x + 6)\, dx = 6x - \frac{5x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4} + 17$$

```
(*By the FTC, we know:*)
```

$$\int_{-2}^{3} (x^3 + 2x^2 - 5x + 6)\, dx = F[3] - F[-2]$$

```
(*Video 2: Another take on the Fundamental Theorem of Calculus*)
```

In[78]:=
```
HoldForm[g[x] = Integrate[t^2, {t, 0, x}]]
```

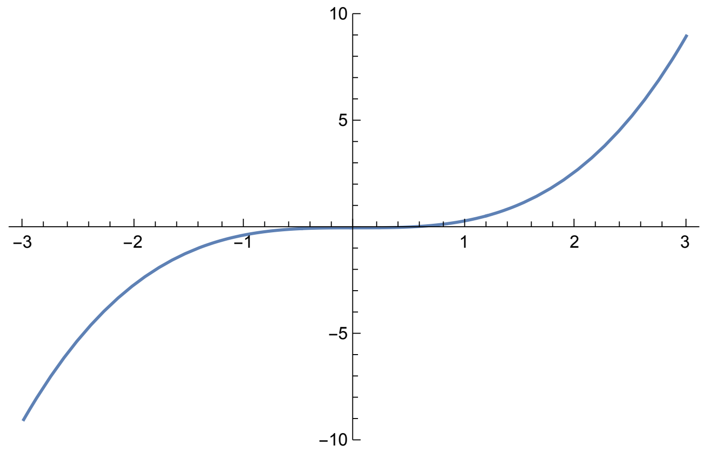
Out[78]= $g[x] = \int_0^x t^2\, dt$

In[79]:=
```
g[x_] := Integrate[t^2, {t, 0, x}]
```

In[87]:=
```
g[4]
```

Out[87]= $\frac{64}{3}$

In[86]:=
```
Plot[g[x], {x, -3, 3}]
```

Out[86]=


In[89]:=
```
g'[t]
```

Out[89]= t^2

```
(*The FTC can also be stated as:*)
```

$$D\left[\int_a^x f[t]\, dt, x\right] = f[x]$$

In[98]:=
```
Integrate[E^t + 5 t^2 - 6 t + 1, {t, 0, x}]
```

In[99]:= $g[x\_] := -1 + e^x + x - 3x^2 + \frac{5x^3}{3}$

In[105]:=
```
g'[x] == E^t + 5 t^2 - 6 t + 1 /. t → x
```

Out[105]= True

```
(*How about endpoints which are more complicated functions of x*)
```

$$g[x] = \int_{2x+1}^{7x} f[t]\, dt$$

$$f[t] = 5t + 1$$

In[108]:=
```
g[x_] := Integrate[5 t + 1, {t, 2 x + 1, 7 x}]
```

In[110]:=
```
g'[x]
```

Out[110]= 5 + 5 (49 x - 2 (1 + 2 x))

```
(*How do we compute that by hand?*)
```

$$\int_{2x+1}^{7x} f[t]\, dt = \int_a^{7x} f[t]\, dt - \int_a^{2x+1} f[t]\, dt$$

$$g'[x] = D\left[\int_{2x+1}^{7x} f[t]\, dt, x\right] = D\left[\int_a^{7x} f[t]\, dt, x\right] - D\left[\int_a^{2x+1} f[t]\, dt, x\right]$$

```
(*So compute each of the two derivatives
 on the right using the FTC and Chain rule*)

In[123]:= D[Integrate[5 t + 1, {t, a, 7 x}], x] - D[Integrate[5 t + 1, {t, a, 2 x + 1}], x]

Out[123]= 5 + 245 x - 10 (1 + 2 x)

In[125]:= Simplify[5 + 5 (49 x - 2 (1 + 2 x)) == 5 + 245 x - 10 (1 + 2 x)]

Out[125]= True

(*Recall how the Chain rule is applied: we are composing two functions,
the integral with endpoint y(x) and y(x)=7x*)

D[Integrate[5 t + 1, {t, a, y}], y](*Use FTC here!*)

1 + 5 y /. y → 7 x(*Replace y for what it is*)

Out[117]= 1 + 35 x

D[7 x, x](*Derivative of the inside function y(x)*)

Out[118]= 7

Expand[(1 + 35 x) * 7](*Chain rule: multiply the above to get answer*)

Out[120]= 7 + 245 x

In[126]:= D[Integrate[5 t + 1, {t, a, 7 x}], x]

Out[126]= 7 + 245 x

(*Similarly for the second one:*)

In[127]:= D[Integrate[5 t + 1, {t, a, 2 x + 1}], x]

Out[127]= 2 + 10 (1 + 2 x)

(*Then take the difference to find g' :*)

In[130]:= Simplify[
 g'[x] == D[Integrate[5 t + 1, {t, a, 7 x}], x] - D[Integrate[5 t + 1, {t, a, 2 x + 1}], x]]

Out[130]= True
```