



Laboratory Manual

Physics 167 and 169

Department of Physics and Astronomy
HERBERT LEHMAN COLLEGE

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Writing a laboratory report

OBJECTIVES

The main way to communicate scientific information today is through articles and reports in scientific journals. Traditionally these were distributed in print, but can now be read in digital format as well, as shown in table 1.

Table 1. Online resources for digitally distributed scientific publications

Resource	Topics	URL
arXiv	Physics, Mathematics, Computer Science	http://arxiv.org/
NASA Scientific and Technical Information (STI)	Astronomy, Aerospace Engineering	www.sti.nasa.gov/STI-public-homepage.html
SOA/NASA Astrophysics Data System	Astronomy, physics	adswww.harvard.edu/

In college physics, you write a laboratory report for each experiment that contains the essential information about the experiment. For scientific information, a consistent format is helpful to the reader (and your lab instructor). Each laboratory report you turn in contains a subset of the sections found in a professional scientific publication for experimental topics, as shown in table 2.

Table 2. A comparison of the sections of a laboratory report and a professional scientific publication

Laboratory Report	Professional Publication
1. Name, date and title of the experiment	1. Cover page: name, date, and title
2. Abstract	2. Abstract
	3. Introduction
	4. Methods and procedure
3. Data	5. Raw data and graphs
4. Calculations and analysis	6. Calculations and analysis
	7. Results
5. Conclusion	8. Discussion and conclusion

The content to include in each section is detailed below. Your lab instructor requires all five sections to evaluate your work, so be sure to include every section in every report.

A laboratory report must be typed. Photocopies of the manual are not accepted. Your laboratory instructor can tell you whether your laboratory report must be printed or can be delivered in a digital format such as email.

ABSTRACT

Describe in your own words what you did in the experiment and why. Your abstract should include one or two sentences each for Purpose, Methods and Conclusions.

- **Purpose:** What physical principle or law does this experiment test?
- **Methods:** What apparatus did you use? How did you analyze the data?
- **Conclusions:** Do your results support the physical law or principle? You should describe any significant experimental errors or uncertainties.

Note that the abstract should be no more than 5 or 6 sentences long, and should not include too much detail. The goal of the abstract is to sum up the experiment quickly and succinctly.

DATA

The data section includes all the raw data you collected in the laboratory without any calculation or interpretation. At a minimum, include the following information:

- A copy of the data table with all fields and rows filled with measurements.
- Any drawings or sketches you were required to make in the laboratory. You must deliver drawings with a printed lab report. You can take a digital photograph of your drawings and import it to a document as needed.

CALCULATIONS AND ANALYSIS

In the calculations and analysis section, you write out all of your calculations and results as explained in the instructions for the experiment. Be sure to answer all of the questions in the lab manual. Include the following information as instructed:

- The equations you used to make all calculations
- Tables of calculated values
- Graphs of the raw data or calculated values
- Average values, uncertainty, and % uncertainty calculations
- Error and % error calculations

CONCLUSION

In the conclusion section, interpret the results you obtained by analyzing the data. Include the following information:

- Do your data and calculations support the physical principle or law being tested?
- What are the important sources of experimental error and uncertainty?
- Are there ways you could have improved your experimental results?
- Also answer any specific questions posed by your lab instructor.

Introduction: Measurement and uncertainty

No physical measurement is ever completely precise. All measurements are subject to some uncertainty, and the determination of this uncertainty is an essential part of the analysis of the experiment.

Experimental data include three components: 1) the value measured, 2) the uncertainty, and 3) the units. For example a possible result for measuring a length is $3.6 \pm 0.2 \text{ m}$. Here 3.6 is the measured value, ± 0.2 specifies the uncertainty, and m gives the units (meters).

ERRORS AND UNCERTAINTIES

The accuracy of any measurement is limited. An *uncertainty* is our best estimate of how accurate a measurement is, while an *error* is the discrepancy between the measured value of some quantity and its true value. Errors in measurements arise from different sources:

- a) A common type of error is a blunder due to carelessness in making a measurement, for example an incorrect reading of an instrument. Of course these kinds of mistakes should be avoided.
- b) Errors also arise from defective or improperly calibrated instruments. These are known as systematic errors. For example, if a balance does not read zero when there is no mass on it, then all of its readings will be in error, and we must either recalibrate it, or be careful to subtract the empty reading from all subsequent measurements.
- c) Even after we have made every effort to eliminate these kinds of error, the accuracy of our measurements is still limited due to so-called statistical uncertainties. These uncertainties reflect unpredictable random variations in the measurement process: variations in the experimental system, in the measuring apparatus, and in our own perception! Since these variations are random, they will tend to cancel out if we average over a set of repeated measurements. To measure a quantity in the laboratory, one should repeat the measurement many times. The average of all the results is the best estimate of the value of the quantity.
- d) Besides the uncertainty introduced in a measurement due to random fluctuations, vibrations, etc., there are also so-called instrumental uncertainties which are due to the limited accuracy of the measuring instruments we use. For example, if we use a meter stick to measure a length, we can, at best, estimate the length to within about half of the smallest division on the stick or 0.5 millimeters. Beyond that we have no knowledge. It is important to realize that this kind of uncertainty persists, even if we obtain identical readings on repeated trials.

CALCULATING AVERAGES

There are several important steps we will follow to help us quantify and control the errors and uncertainties in our laboratory measurements.

Most importantly, in order to minimize the effect of random errors, one should always perform several independent measurements of the same quantity and take an average of all these readings. In taking the average the random fluctuations tend to cancel out. In fact, the larger the number of measurements taken, the more likely it is that random errors will cancel out.

When we have a set of n measurements x_1, x_2, \dots, x_n of a quantity x , our *best estimate* for the value of x is the average value \bar{x} , is defined as follows.

$$\text{Average value:} \quad \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (0.1)$$

The average value is also known as the mean value. Note that when making repeated measurements of a quantity, one should pay attention to the consistency of the results. If one of the numbers is substantially different from the others, it is likely that a blunder has been made, and this number should be excluded when analyzing the results.

REPORTING ERRORS

Quite often in these labs one has to compare a value obtained by measurement with a standard or generally accepted value. To quantify this one can compute the *percent error*, defined as follows.

$$\text{Percent error:} \quad \% \text{ Error} = \left| \frac{\text{measured value} - \text{accepted value}}{\text{accepted value}} \right| \times 100 \quad (0.2)$$

Sometimes one has to report an error when the accepted value is zero. You'll encounter this situation in experiment 3. The procedure to follow is described at the end of that experiment.

CALCULATING UNCERTAINTIES

To estimate the uncertainty associated with our best estimate of x , we begin by examining scatter of the measurements about the mean \bar{x} . Specifically, we start by determining the absolute value of the deviation of each measurement from the mean:

$$\text{Deviation:} \quad \Delta x_i = |x_i - \bar{x}| \quad (0.3)$$

Next we have to compare the deviation to the systematic or reading uncertainty due to limited accuracy of the instrument used. If this systematic uncertainty R is bigger than the deviation Δx_i , then the result of our measurements can be written as

$$\bar{x} \pm R \quad (0.4)$$

If, however, the deviation is larger than the systematic or reading error, then we must determine how big the random uncertainty in our measurements is. This is given by the standard deviation, defined as follows.

$$\text{Standard deviation: } \sigma = \sqrt{\frac{(\Delta x_1)^2 + (\Delta x_2)^2 + \dots + (\Delta x_n)^2}{n-1}} \quad (0.5)$$

The standard deviation has the following meaning: if we were to make one single additional measurement of the quantity x , there is 68% probability of obtaining a value which lies between $\bar{x} - \sigma$ and $\bar{x} + \sigma$. The uncertainty in the average value \bar{x} is smaller (that's the whole point of taking an average!). In fact the uncertainty in \bar{x} is the standard deviation divided by the square root of the number of measurements:

$$\text{Uncertainty in } \bar{x}: \quad U = \frac{\sigma}{\sqrt{n}} \quad (0.6)$$

Sometimes it is useful to express this as a percent uncertainty, defined as one hundred times the uncertainty divided by the average value.

$$\% \text{ uncertainty in } \bar{x}: \quad \% \text{ uncertainty} = \frac{U}{\bar{x}} \times 100 \quad (0.7)$$

EXAMPLE

To illustrate the calculation of \bar{x} and the associated uncertainty U , suppose we are measuring the length of a stick and have obtained, in five separate measurements, the results tabulated below.

Length x (cm)	Deviation Δx (cm)	$(\Delta x)^2$
54.84	0.43	0.1849
53.92	0.49	0.2401
54.46	0.05	0.0025
54.55	0.14	0.0196
54.30	0.11	0.0121
sum: 272.07		sum: 0.4592

From this information we can calculate

$$\text{Average: } \bar{x} = \frac{272.07 \text{ cm}}{5} = 54.41 \text{ cm}$$

$$\text{Standard deviation: } \sigma = \sqrt{\frac{0.4592}{4}} = 0.3388 \text{ cm}$$

$$\text{Uncertainty: } U = \frac{\sigma}{\sqrt{n}} = \frac{0.3388}{\sqrt{5}} = 0.1515 \text{ cm}$$

Thus our final result for the length is $54.41 \pm 0.15 \text{ cm}$.

SIGNIFICANT FIGURES

A number expressing the result of a measurement, or of computations based on measurements, should be written with the proper number of *significant figures*, which just means the number of reliably known digits in a number. The number of significant figures is independent of the position of the decimal point, for example 2.163 cm, 21.63 mm and 0.02163 m all have the same number of significant figures (four).

In doing calculations, all digits which are not significant can be dropped. (It is better to round off rather than truncate). A result obtained by multiplying or dividing two numbers has the same number of significant figures as the input number with the fewest significant figures.

EXAMPLE

Suppose that we want to calculate the area of a rectangular plate whose measured length is 11.3 cm and measured width is 6.8 cm. The area is found to be

$$\text{Area} = 11.3 \text{ cm} \times 6.8 \text{ cm} = 76.84 \text{ cm}^2$$

But since the width only has two significant figures we can round to two figures and report that the area is 77 cm^2 .

GRAPHING, SLOPE AND INTERCEPTS

In almost every laboratory exercise, you plot a graph based on the data measured or calculated. A graph lets you visualize the relation between two physical quantities. In plotting a graph, use the following steps:

1. Arrange the data into a table with two columns listing the values for the two measured or calculated quantities. For example, the first column could list the values for time and the second column could list the values for the average velocity.
2. Decide which of the two quantities to plot along each axis. Graphs have two perpendicular axes, the x-axis and the y-axis and by convention you plot the independent quantity along the x-axis and the dependent quantity along the y-axis.
3. Choose the scale for each axis to cover the range of variation of each quantity. You should choose the scale so that the final curve spans the largest area possible on the graph paper.
4. Label each axis with the quantity plotted on that axis and the units used.
5. Mark the main divisions along each axis.
6. Mark each data point on the graph using the values in each row of the data table. Data points must align with the value of each quantity on their respective axes. Make each data point clearly visible on the graph.
7. Fit and draw a smooth curve through the data points so that the curve comes as close as possible to most of data points. Do not force the curve to go exactly through all the points or through the origin of the coordinate system. The fact that not all points lie along the fitted curve just indicates that measurements are subject to some uncertainty.

In many cases the fitted curve is a straight line. The best straight line fit has nearly the same number of data points above and below the line. The equation for a straight line is given by

Straight Line $y = mx + b$ (0.8)

The quantity b is the intercept: it is the value of y when $x = 0$. The quantity m is the slope of the curve. Given two points on the straight line, $\{y_1 = mx_1 + b, y_2 = mx_2 + b\}$, called basis points, the slope is defined as the ratio of the change in y to the change in x between these points, as shown in equation 0.9.

Slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ (0.9)

Basis points are NOT experimental points. They should be chosen as far from each other as possible to increase the precision of m , as shown in figure 0.1.

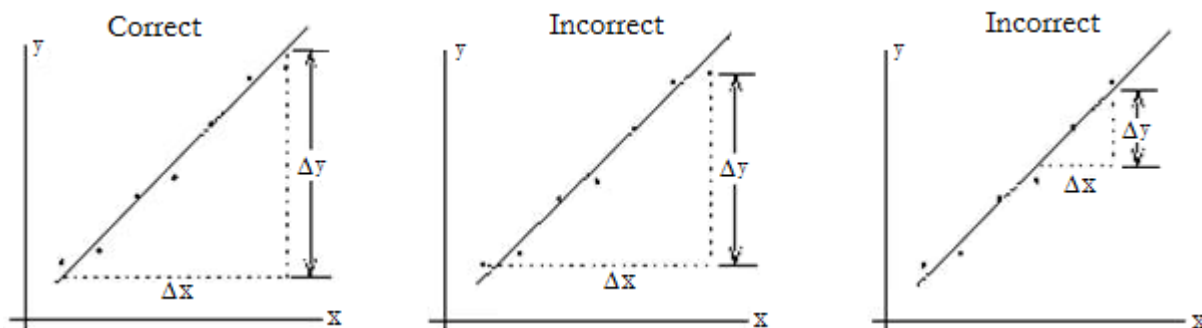


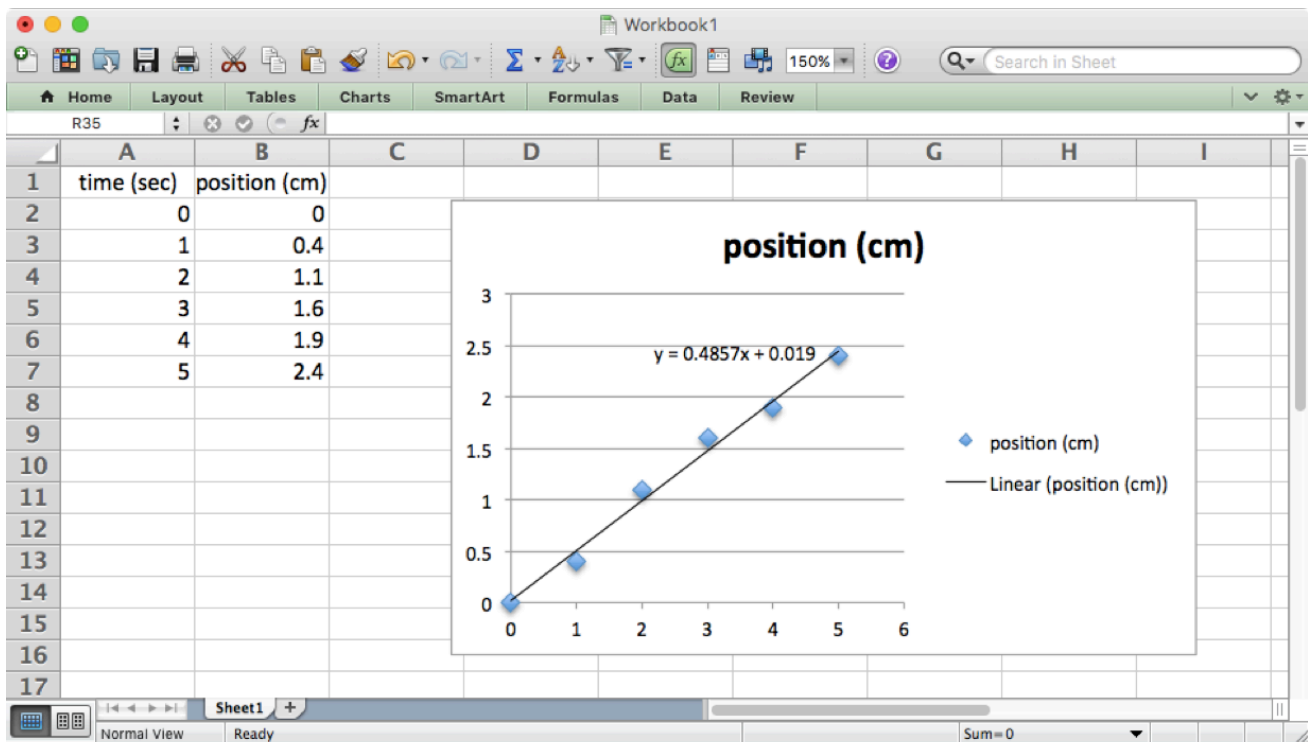
Figure 0.1 Choosing the correct basis points to calculate the slope

PLOTTING USING A COMPUTER

After you have some experience making plots by hand, it's easier and more accurate to let a computer do the work. Any standard spreadsheet or plotting software should work. The steps might depend on the type of software you're using. But in Excel, for example, you would start by entering your data in two columns. The first column gives the x values and the second column gives the corresponding y values. Select the data you want to plot, then

- Charts → Scatter → Marked Scatter will produce a plot of your data
- Chart Layout → Trendline → Linear Trendline will add a best-fit line to your plot
- To see the equation of the best-fit line go to Chart Layout → Trendline → Trendline Options... then in the dialog box that appears go to Options and check "Display equation on chart".

Here's a screenshot of a typical Excel plot.



PRACTICE CALCULATIONS

1. The accepted value of the acceleration due to gravity on Earth is $g = 980 \text{ cm/s}^2$. When trying to measure this quantity, we performed an experiment and got the following five values for g .

Trial	$g \text{ (cm/s}^2\text{)}$
1	1004
2	992
3	978
4	985
5	982

a) Find the average value and standard deviation of our measurements of g .

b) Find the uncertainty in our average value for g .

c) What is the percent error in our measurement of g ?

2. A box is moving along a frictionless inclined plane. Experimental measurements of velocity at various times are given below.

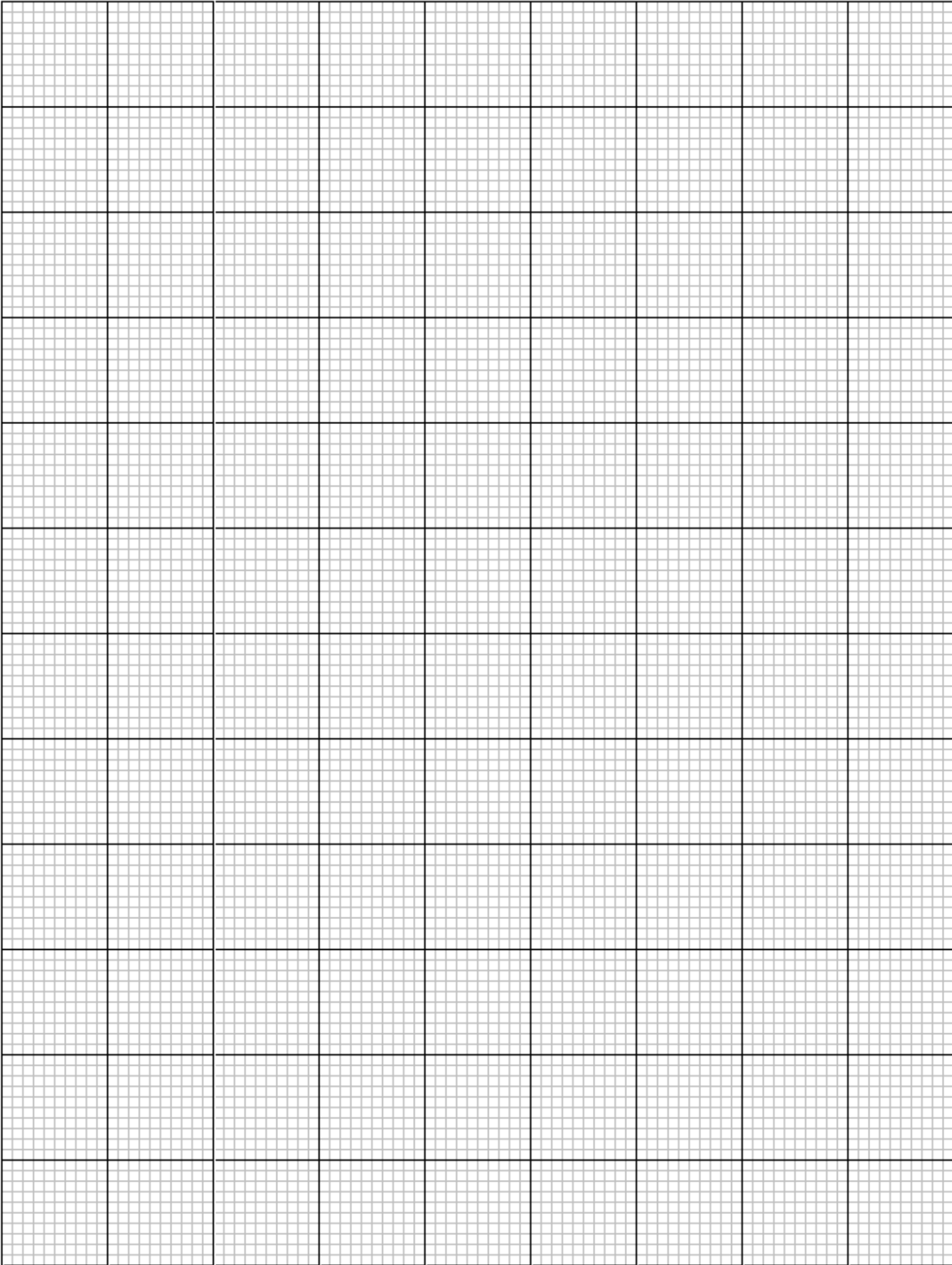
time $t \text{ (s)}$	velocity $v \text{ (m/s)}$
0	0
1	5.2
2	10.1
3	14.8
4	19.9
5	25

a) Plot a graph of v versus t . Can the data be represented by a straight line? (You can use the graph paper on the next page.)

b) Calculate the slope.

c) What physical quantity does this slope represent?

d) From your estimate of the slope, what would the velocity be at $t = 10 \text{ s}$?



Introduction: Units and conversions

Every measurement requires a choice of units. It's important to keep track of the units you're using, and to know how to convert between different common units. Here are a few examples.

The SI unit of length is the meter (m). But we sometimes measure lengths in centimeters (cm) or millimeters (mm). The conversions are

$$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$$

For example 70 cm is the same as 0.7 m , because

$$70 \text{ cm} = 70 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.7 \text{ m}$$

The SI unit of mass is the kilogram (kg), but we sometimes measure mass in grams (g). The conversion is

$$1 \text{ kg} = 1000 \text{ g}$$

For example 150 g is the same as 0.15 kg , because

$$150 \text{ g} = 150 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 0.15 \text{ kg}$$

It's also important to recognize the difference between mass (measured in kilograms) and weight (measured in Newtons). Weight is another name for the force of gravity. It's given by the formula $F = mg$ where m is the mass and $g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity. For example a mass of 150 g has a weight of 1.47 Newtons, because 150 g is the same as 0.15 kg and

$$F = 0.15 \text{ kg} \times 9.8 \text{ m/s}^2 = 1.47 \text{ kg m/s}^2 = 1.47 \text{ N}$$

Note that we had to convert the mass to kilograms in order to get the right answer in Newtons!

Experiment 11: Electrostatic Field

OBJECTIVES

An electrostatic field can be measured indirectly by measuring the electrostatic potential or voltage at different points in the field. In this experiment, you use a multimeter and probe to detect the voltage changes in an electrostatic field distributed across a 2-D surface. The objectives of this experiment are as follows:

1. To measure the voltage at different points in an electrostatic field
2. To find equipotential lines based on different voltage readings
3. To draw electrostatic field lines using equipotential lines

THEORY

The force between charged particles at rest is attributed to an electrostatic field E that surrounds static charges. A point with charge q in an electric field experiences an electrostatic force F_E that is proportional to q , as shown in equation 11.1.

$$\text{Electrostatic Force} \quad F_E = qE \quad (11.1)$$

Here, E is measured in V/m (volts per meter), q is measured in C (coulombs), and F_E is measured in N (newtons). The direction of the electrostatic force at any point in space coincides with the direction of the electrostatic field for a positive charge.

While it is possible to measure the force on a point charge, it is sometimes simpler to calculate an electrostatic field by measuring the work required to move charges through the field. The electrostatic potential or voltage difference V between two points is defined as the work W required to move a positive test charge q from one point to the other divided by the magnitude of q , as shown in equation 11.2.

$$\text{Voltage difference} \quad V_{BA} = V_B - V_A = \frac{W}{q} \quad (11.2)$$

Here, the work W is measured in Joules or $N \cdot m$, which is the force multiplied by distance. Because the force in this case is the electrostatic force F_E from equation 11.1, the voltage change depends on the electrostatic field E and the distance moved d , calculated by combining equations 11.1 and 11.2, as shown in equation 11.3.

$$\text{Voltage difference (E)} \quad V_{BA} = \frac{-F_E d}{q} = \frac{-qEd}{q} = -Ed \quad (11.3)$$

Here d is the component of the distance from point A to point B measured along the field lines. An equipotential line is a curve that has the same voltage everywhere. By definition if you move a charge along an equipotential line, so that the displacement is tangent to the curve, the voltage does not change. Then from equation 11.3 the electric field has no component along the displacement. This means the electric field is perpendicular to equipotential lines.

Because equipotential lines are perpendicular to electrostatic field lines, if you can find equipotential lines, you can also find electrostatic field lines. Figure 11.1 shows the equipotential lines and electrostatic field lines of an electric dipole.

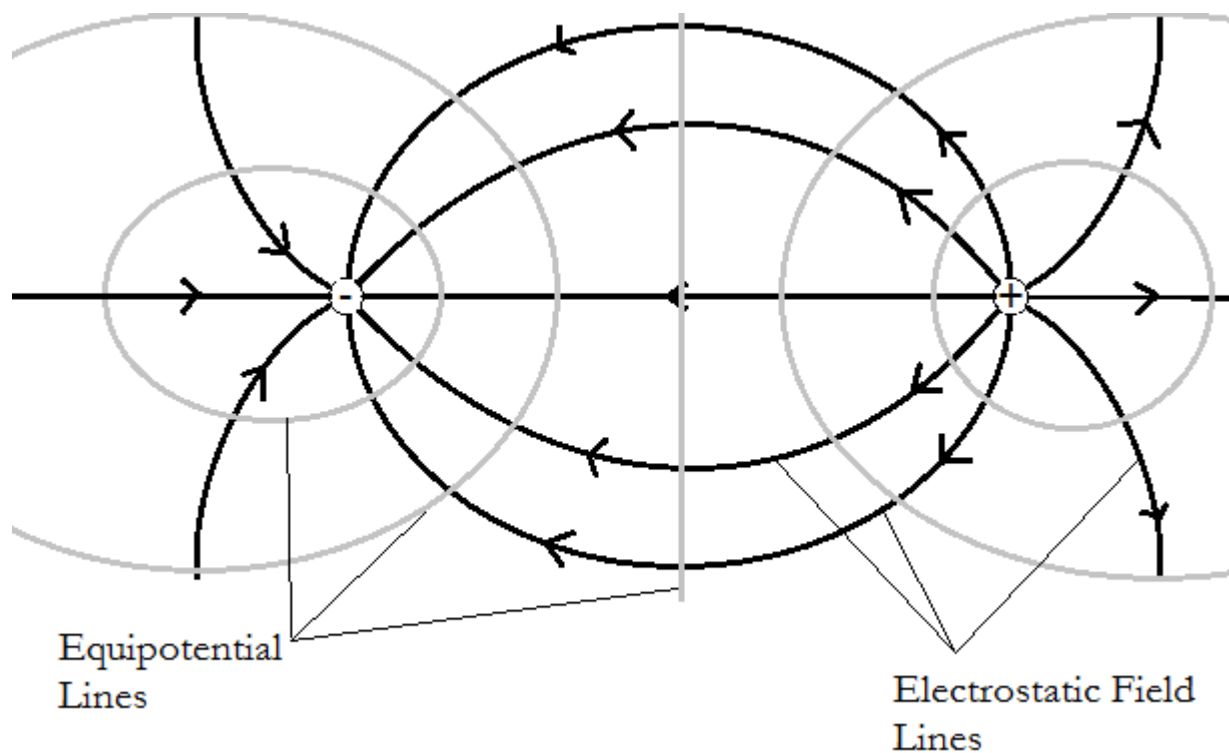


Figure 11.1 The electrostatic field lines and equipotential lines around an electric dipole

In this experiment, you measure the voltage at different points so that you can determine the equipotential lines. It is important to note that because one point cannot have two different values of V , equipotential lines never cross.

ACCEPTED VALUES

There are no accepted values for this lab except that the equipotential lines do not cross.

APPARATUS

- DC power supply
- field mapping board
- multimeter
- u-shaped probe
- wires

When the field mapping board is powered on, the board produces several static charges on a graphite plate that generate an electrostatic field on the board's surface. The terminals marked E1 through E7 across the top of the board provide a linear scale of voltages, ranging from lowest to highest. The multimeter is used to measure the voltage difference between one of the E# terminals and a point on the field mapping board. When the voltage reported by the multimeter is zero, it means the tip of the U-shaped probe is at the same electrostatic potential as the E# terminal the multimeter is connected to. So you can find one equipotential line for each E# terminal by marking points where the multimeter reads zero voltage and connecting those points with a curve.

SETTING UP THE FIELD MAPPING BOARD

The power supply provides Direct Current (DC) power to the field mapping board through two wires. The first step is setting up the board and measuring the voltage at the E terminals.

1. With the power supply OFF, plug the red wire into the positive out terminal (+) and plug a black wire into the ground terminal (\ominus), as shown in figure 11.2.

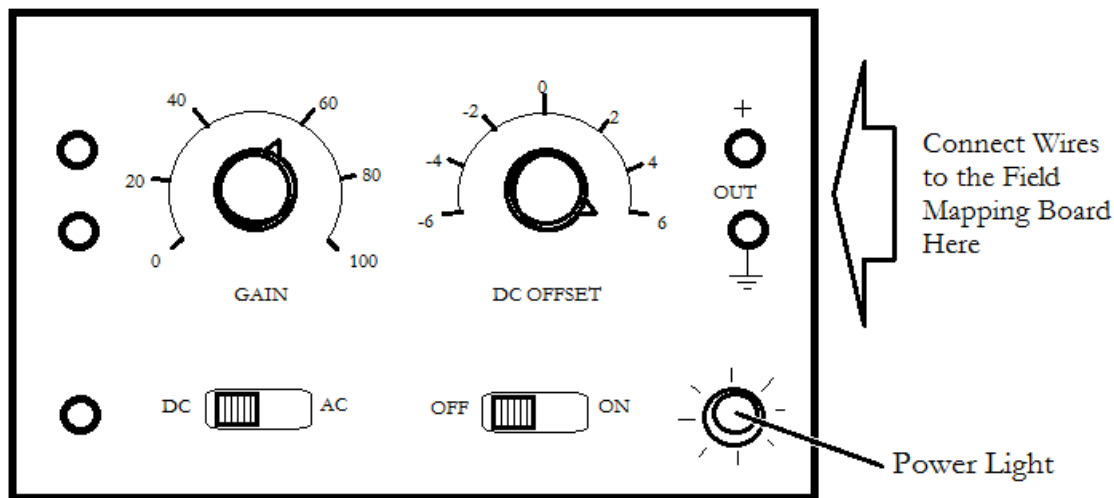


Figure 11.2 Setting up the DC power supply

2. On the power supply, set the AC/DC switch to DC. Turn the DC offset to the maximum positive value, which is past 6 in the clockwise direction. The GAIN knob does not significantly change the DC output, so leave it between 40 and 60.
3. Connect the black wire from the power supply to the left "Battery or Oscillator" terminal on the field mapping board and connect the red wire from the power supply to the right "Battery or Oscillator" terminal on the field mapping board, as shown in figure 11.3.

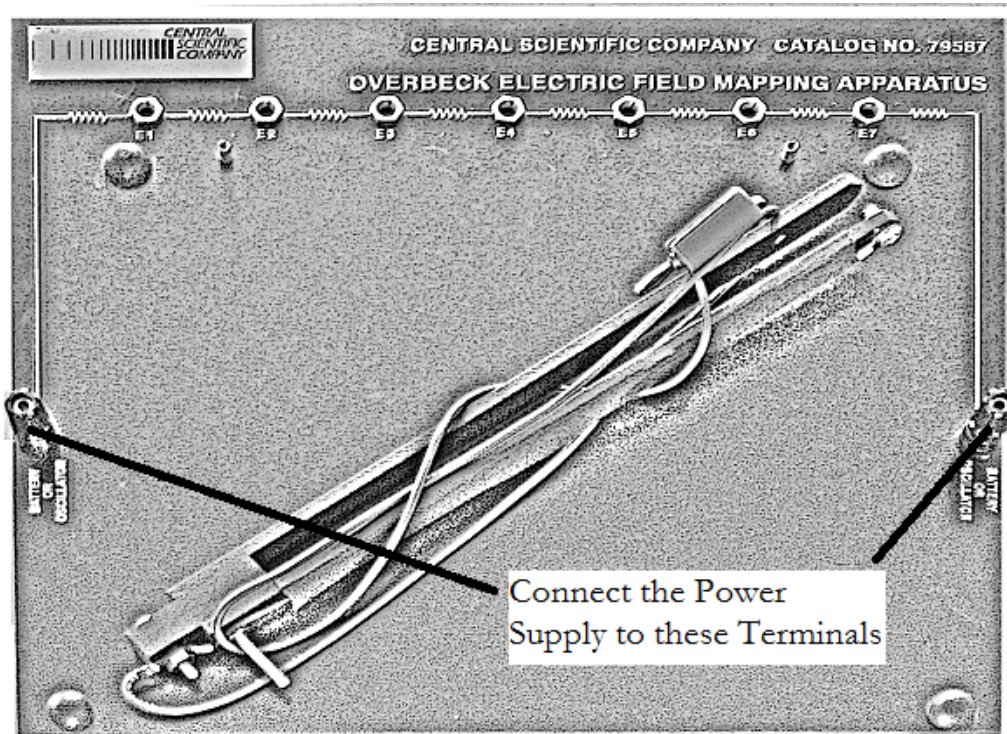


Figure 11.3 Connecting the power supply to the field mapping board

4. Connect a wire from the ground terminal \oplus of the power supply to the “COM” jack of the multimeter. Connect a second wire from the (+) terminal of the power supply to the “V” jack of the multimeter.
5. Switch the multimeter AC/DC switch to DC and set the multimeter dial to the 20 V DC scale in the “V” range of the dial. DO NOT turn the multimeter dial to any other settings.
6. Turn on the power supply. Adjust the DC Offset knob on the power supply until the multimeter reads 4.0 volts. This is the total voltage difference from one Battery or Oscillator jack on the field mapping board to the other.
7. Unplug the wire connecting the “V” jack of the multimeter and the (+) terminal of the power supply. Reconnect it so the “V” jack of the multimeter is connected to the “E1” plug on the field mapping board. The multimeter should read 0.5 volts. If you get a significantly different value, ask your lab instructor for help.
8. Repeat the process in step 7 for the E2 through E7 plugs. Each plug should be 0.5 volts above the preceding plug. This provides a linear scale of voltage differences from plug E1 through E7.
9. Unplug the wires from the multimeter. Plug the wire from the u-shaped probe into the “V” jack of the multimeter. Plug a wire into the “COM” jack of the multimeter and plug the other end of the wire into the terminal marked “E2” on the field mapping board, as shown in figure 11.4. Proceed to the procedure.

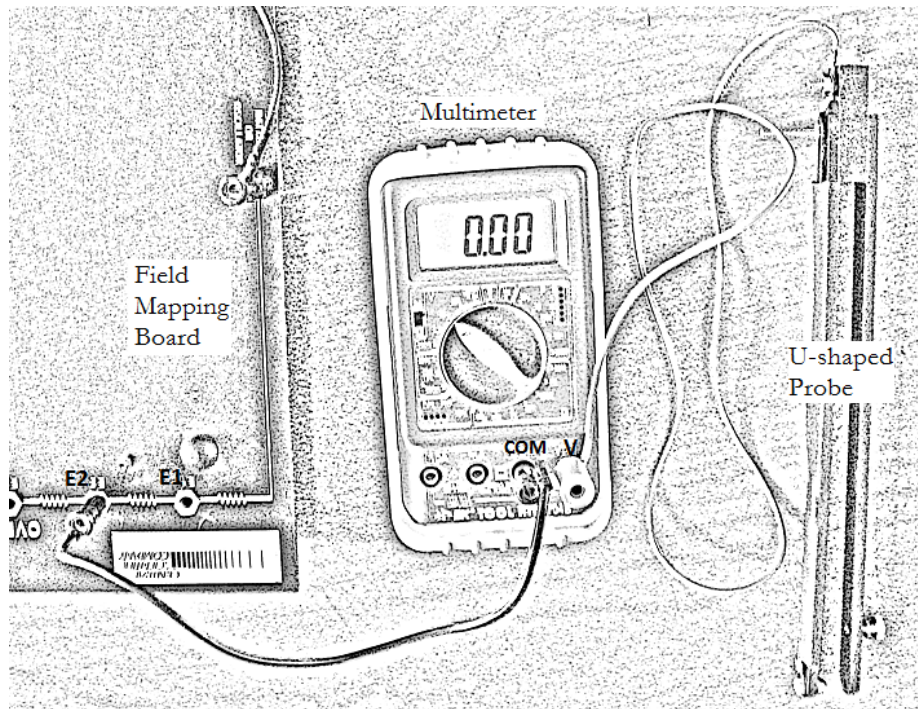


Figure 11.4 Connecting the multimeter and U-shaped probe to the field mapping board

PROCEDURE

1. Lay the field mapping board flat on the lab table with the terminals and rubber bumpers facing up.
2. Fasten a sheet of 8.5 x 11-inch paper to surface of the field mapping board. Secure the paper by slipping the corners of the paper under the four rubber bumpers.
3. Slide the U-shaped probe onto the field mapping board with the ball end facing the underside of the mapping board. Notice the screw below the probe acts as a support leg.
4. Switch the multimeter AC/DC switch to DC and set the multimeter dial to the 200mV DC scale in the “V” range of the dial. DO NOT turn the multimeter dial to any other settings. In particular, do not connect the multimeter to any power supply and switch the dial to a setting in the “ Ω ” scale; this damages the meter.
5. Turn on the power supply. The power light lights up when the power is on.
6. Gently glide the U-shaped probe over the paper until the reading of the multimeter is close to zero. Then, slide the U-shaped probe back and forth until you can clearly see a point where the voltage on reading of the multimeter switches from positive to negative. With a pencil, mark that point on the paper through the eye of the U-shaped probe and label the point E2 as you mark it. From the way we set up the field mapping board, we know this point is at a potential of 1.0 V.
7. Repeat step 5 at different places on the board until you have generated a series of zero-voltage points across the paper for E2, labeling each point E2.
8. Unplug the wire from terminal E2 and plug it into terminal E3. Repeat steps 5 and 6 for the new voltage, labeling each point with E3 as you mark it. Repeat this process with the wire plugged into terminals E4, E5, and E6. Be sure to label each point with the terminal code.
9. Turn the power supply off. The power light should turn off.

10. Create copies of the drawing and data points using a photocopier or digital camera. Distribute a copy of the drawing to each member of the lab group.

DATA

The data sheet in this experiment is the sheet of paper upon which you draw the equipotential lines. Experiments seldom produce perfect results, and the charges on the field mapping board typically do not produce a balanced dipole. The equipotential lines might align in unexpected ways but each line should form a closed loop or extend from one edge of the page to another, as shown in figure 11.5.

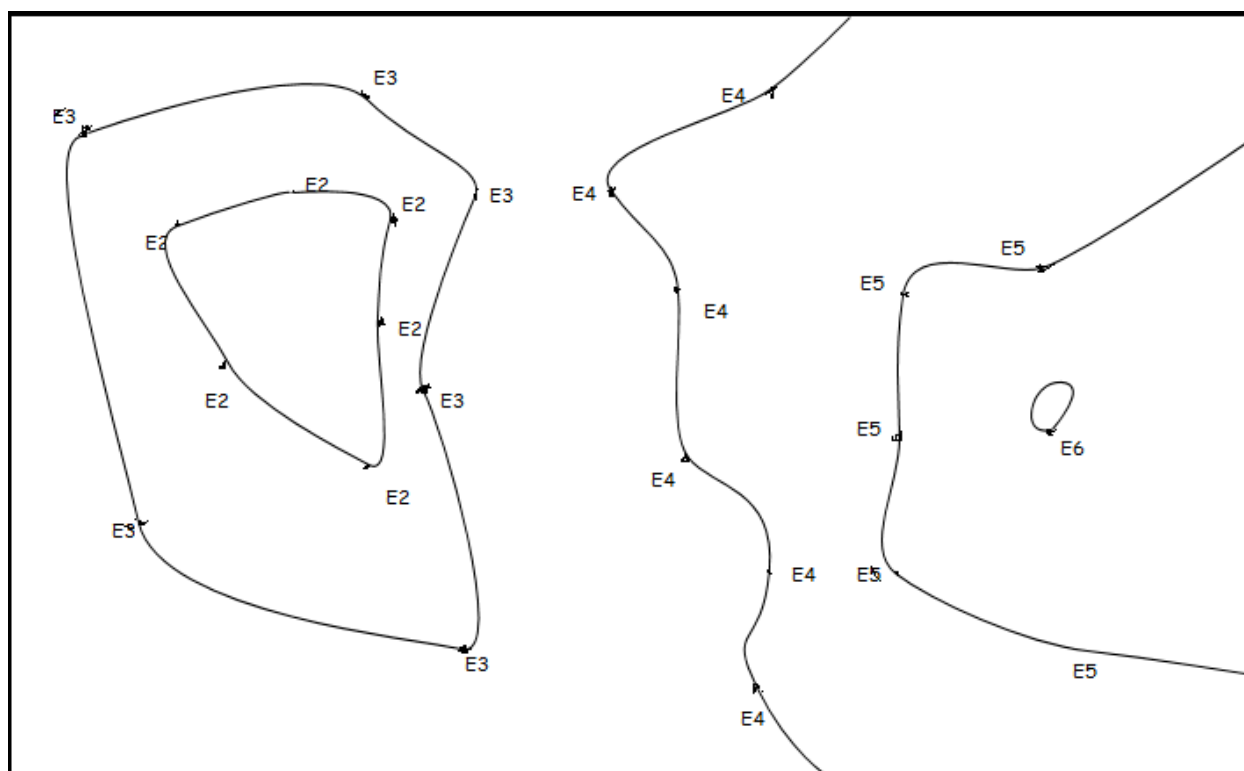


Figure 11.5 Equipotential lines form loops or proceed from one edge of the page to another

CALCULATION AND ANALYSIS

1. Draw a smooth curve that connects the dots labeled E2 on the data sheet using a pencil. This is the equipotential line for E2.
2. Repeat step 1 for each series of points E3 through E6.
3. Draw the electric field lines. Remember that the electric field is perpendicular to the equipotential lines.
4. Sketch your idea of the locations and signs of the charges on the electrostatic field board.
5. If a point charge is placed in an electric field and released from rest, will it travel along a field line?

Experiment 12: Ohm's Law

OBJECTIVES

The movement of charge in a conductor, referred to as current, is possible because electrons can move when a voltage is applied. A good conducting material transfers electrons easily and is said to have low resistance, while a poor conductor transfers electrons less readily and has high resistance. In many materials, it is found that current is directly proportional to voltage. Georg Ohm discovered this and formalized this relationship as Ohm's Law. In this experiment you measure the voltage, current, and resistance of circuits, and test Ohm's law by predicting one of these quantities based on the other two. The objectives of this experiment are as follows:

1. To measure current and voltage in a circuit
2. To calculate resistance using different methods given current and voltage measurements
3. To test Ohm's law

THEORY

When there is a potential difference V between two points in a circuit, a current flows. Experimentally it is found that for many materials if the temperature remains constant, the voltage V is proportional to the current I , as shown in equation 12.1.

$$\text{Ohm's Law} \quad V = IR \quad (12.1)$$

The proportionality constant R is the resistance between the two points considered.

ACCEPTED VALUES

The accepted value for the resistance R is measured directly from the resistor, which you do at the end of the procedure for this experiment.

APPARATUS

- resistor
- wires
- DC power supply
- multimeters
- fuse

Simple electric circuits consist of conducting wires, resistors, power supplies, and fuses. A circuit diagram is a model of a circuit that corresponds to the arrangement of the components, denoted by the following symbols:



DC power supplies provide a potential difference that causes current to flow through a closed circuit at a constant rate.



Conducting wires provide a medium through which charge can move.



Resistors provide resistance to charge moving.



Fuses are safety devices that break a circuit when a current threshold is exceeded

A simple example of a circuit diagram is shown in figure 12.1. A fuse is always included in the circuits to protect the equipment. When too high a current flows through a fuse it interrupts the current.

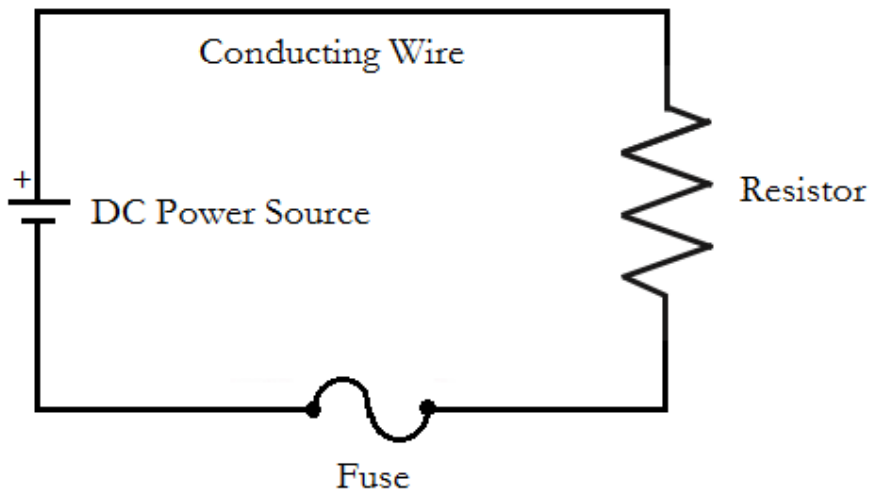


Figure 12.1 A simple circuit diagram displaying four essential components

In this experiment you measure the current and voltage at specific points on the circuit. To do this you'll use an instrument called a multimeter. You will connect two multimeters to the circuit at the same time, with one set up to measure current and the other set up to measure voltage.

Multimeters can be set for several functions.

- to measure direct (DC) or alternating (AC) current
- to measure voltage (V), current (A) or resistance (Ω)

They can also be set for the maximum value of the quantity to be measured. For an illustration see figure 12.2.

To use the multimeter to measure current (ammeter), set the meter to DC and 200 mA. Then connect the multimeter in series with the circuit, meaning that you plug the source of the current in in the mA socket and the destination of the current into the COM socket of the multimeter. For example, to measure the current before a resistor, plug the wire from the power supply into the mA socket and plug a wire connected to the resistor into the COM socket.

To use the multimeter to measure voltage (voltmeter), set the meter to DC and 20 V. Then connect the multimeter in parallel with the circuit, meaning that you plug the source of the current into the V socket and the destination of the current into the COM socket of the multimeter. For example, to measure the voltage across a resistor, plug a wire from the end of the resistor nearest the power supply into the V socket and plug a wire connected to the other end of the resistor into the COM socket.

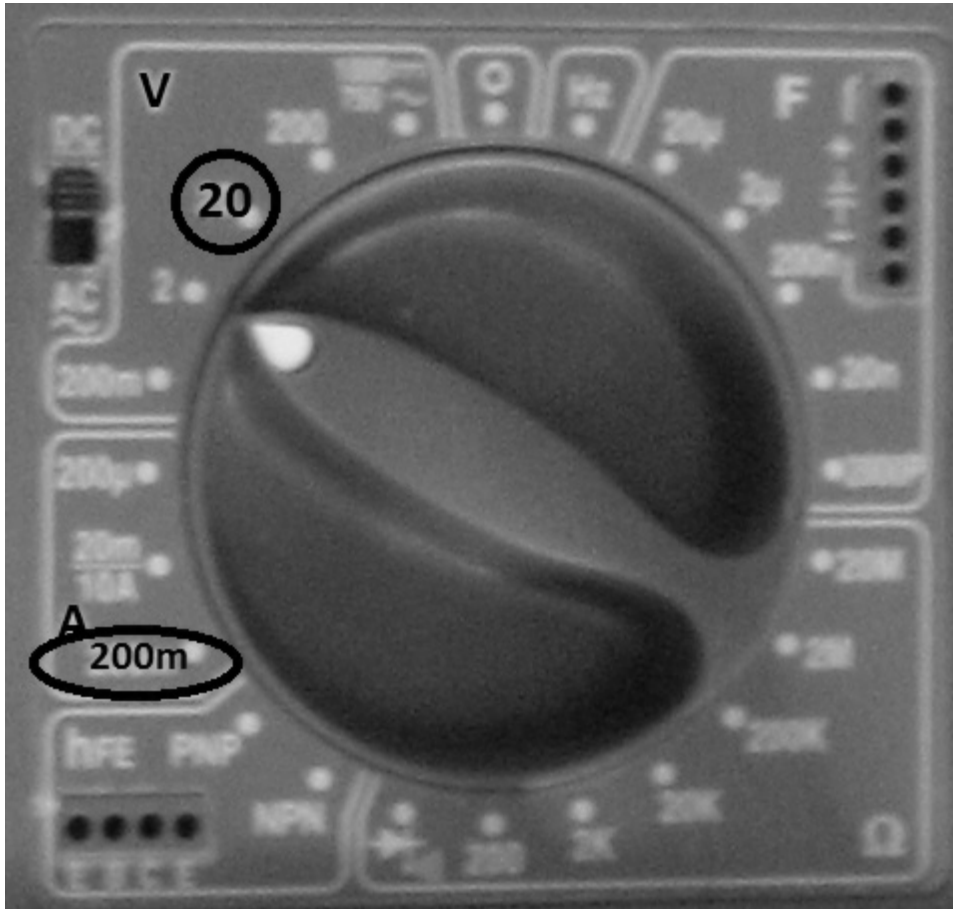




Figure 12.2 Locating the 20V and 200mA settings on the BK Toolkit 2704B multimeter.

In a circuit diagram, the voltmeter and ammeter are denoted by the following symbols:

- 
 Ammeters measure current and are setup in series
- 
 Voltmeters measure voltage and are setup in parallel

Ammeters are always connected in series so that the current to be measured flows through the instrument. Voltmeters are connected in parallel with the portion of the circuit across which the potential difference is measured. In figure 12.3, the circuit diagram for this experiment is shown with the ammeter in series with the resistor and the voltmeter in parallel with the resistor.

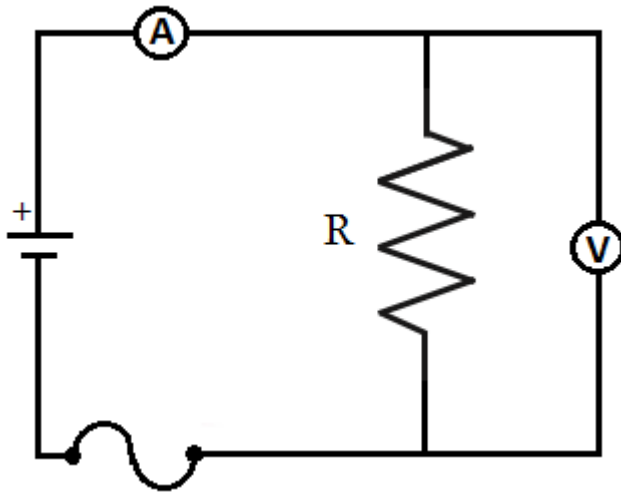


Figure 12.3 A circuit diagram with an ammeter setup in series and a voltmeter setup in parallel

Current in the circuit is supplied by the DC power supply, illustrated in figure 12.4. The current is varied by adjusting the DC OFFSET knob, which controls the voltage produced by the power supply. The zero position for this control is at twelve o'clock. Clockwise rotation increases the voltage (0 to + 6V), while counterclockwise rotation yields the opposite polarity (0 to - 6V). Use only positive voltages in this experiment.

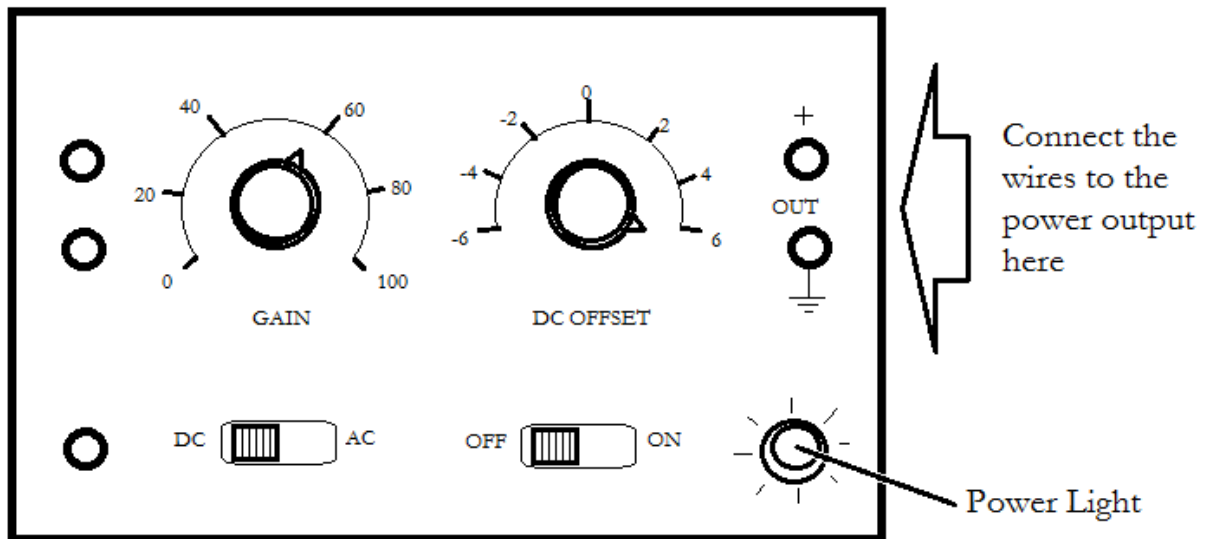


Figure 12.4 Vary current from the power supply by adjusting the DC offset knob

CALCULATION AND ANALYSIS

1. Calculate V/I for each pair of voltage current readings. Compute the average of your V/I values. This is one estimate for the resistance R .
2. Draw a graph showing the relation between voltage (y-axis) and current (x-axis). Draw the best fit straight line through the points on your graph. Calculate the slope of this line as another estimate for R .
3. Calculate the % error for the two resistance values you obtained, using the direct measurement you made in step 8 of the procedure as the accepted value. Explain whether your results support Ohm's law.

Experiment 13: Electric Circuits

OBJECTIVES

An electric circuit is a closed path of conducting material through which an electric current flows. In this experiment, you build circuits out of conductors and resistors and measure the currents through resistors in parallel and in series. The objectives of this experiment are as follows:

1. To build electric circuits with resistors in parallel and in series
2. To measure the current and voltage in a circuit
3. To calculate the current and voltage for resistors in parallel and in series

THEORY

Circuits may include combinations of several resistors. Two connected so that the same current flows through both are said to be in series, as shown in the circuit diagram in figure 13.1.

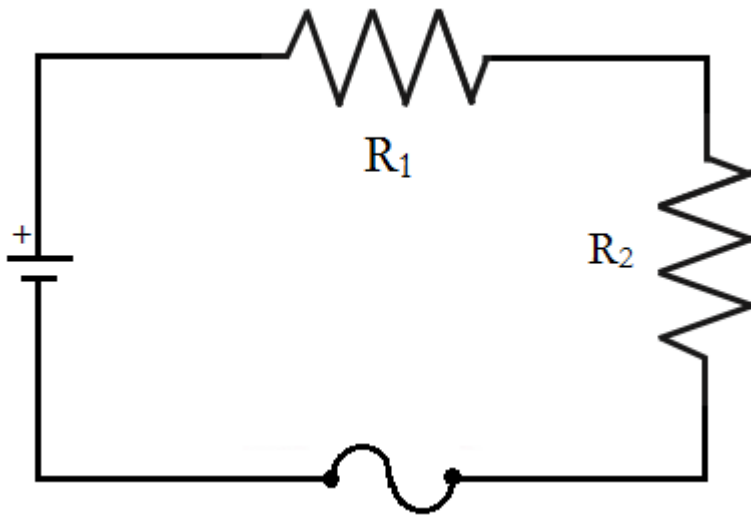


Figure 13.1 A circuit diagram showing resistors R_1 and R_2 in series

Resistors connected so that the sum of the currents through the two resistors is equal to the total current in the circuit are said to be in parallel, as shown in the circuit diagram in figure 13.2.

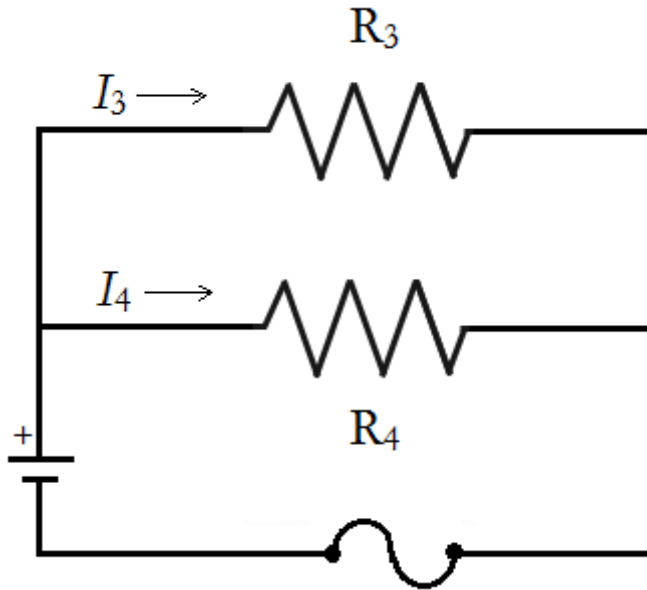


Figure 13.2 A circuit diagram showing resistors R_3 and R_4 in parallel

A series circuit showing an ammeter measuring the current before both resistors and a voltmeter measuring the voltage across resistor R_2 is shown in figure 13.3.

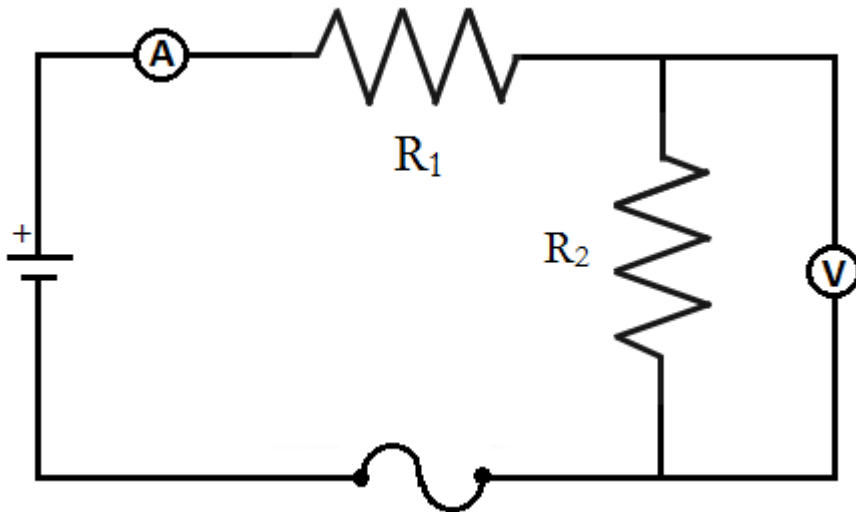


Figure 13.3 A circuit diagram with an ammeter set up in series and a voltmeter set up in parallel

The basic rules for analyzing circuits are known as Kirchhoff's laws. They give relations among currents and voltages as follows.

KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law states that the current at any junction in a circuit is the same going in to the junction as going out.

$$\text{Kirchhoff's Current Law} \quad \sum I_{in} = \sum I_{out} \quad (13.1)$$

This is equivalent to conservation of charge. It tells us that we neither gain nor lose electrical charge at any junction. Consequently, in a series circuit, the same current flows through each resistor, while in a parallel circuit the sum of the currents through the resistors equals the total current in the circuit.

KIRCHHOFF'S VOLTAGE LAW

Recall that electrical potential, or voltage, is the potential energy per unit charge. It does not change along any conductor that has zero resistance. If we go around any closed loop in a circuit, we must get back to the same potential we started from. This is Kirchhoff's voltage law, which states that the sum of the voltage changes ΔV around any closed loop in a circuit is zero.

$$\text{Kirchhoff's Voltage Law} \quad \sum \Delta V = 0 \quad (13.2)$$

Across a resistor the potential decreases as we follow the circuit in the direction of the current. Consequently the total decrease (drop) in voltage in a series circuit is the sum of the voltage drops across each resistor. For a parallel circuit, the input of each resistor is connected to a common conductor. Consequently in a parallel circuit the voltage drop is the same across each resistor.

ACCEPTED VALUES

The accepted values for the circuit with resistors in series (part A) is that the current at the end of the circuit is equal to the current at the beginning ($I_{initial} = I_1 = I_2$) and that the total voltage on the circuit is equal to the sum of the voltages across the resistors ($V_{total} = V_1 + V_2$).

The accepted values for the circuit with resistors in parallel (part B) is that the initial voltage equal to the voltages across either resistor ($V_{initial} = V_3 = V_4$) and that the total current on the circuit is equal to the sum of the currents through each resistor ($I_{total} = I_3 + I_4$).

APPARATUS

- power supply
- fuse
- multimeters
- resistors
- conducting wires

PROCEDURE

PART A. RESISTORS IN SERIES

1. Connect a simple series circuit as shown in figure 13.1. Have your lab instructor check the circuit before turning on the power supply. The voltmeter should be set to DC and 20 V, and the ammeter to DC and 200 mA.
2. Measure the voltage across resistor R_1 , across resistor R_2 , and across the series combination of R_1 and R_2 . Recall that the voltmeter is connected across (or in parallel with) resistors. Be sure the voltmeter is set for direct current voltage (DCV) on the 20 Volt scale. Record the voltages in the data table for Part A.
2. Measure the current entering resistor R_1 , between resistors R_1 and R_2 , and leaving resistor R_2 . Ammeters are connected in series with the resistor whose current is being measured. Be sure your ammeter is set for direct current amps (DCA) and the maximum current is set for 200 mA. Record the currents in the data table for Part A.

PART B. RESISTORS IN PARALLEL

1. Connect resistors R_3 and R_4 in parallel as shown in figure 13.2. Have your lab instructor check the circuit before turning on the power supply. Set the voltmeter and ammeter as before.
2. Measure the voltage across resistor R_3 , across resistor R_4 , and across the whole circuit. Record the voltages in the data table for Part B. (Note that these three measurements involve essentially the same connection to the circuit.)
3. Measure the current through the circuit from the power supply, through resistor R_3 , and through resistor R_4 . Again be sure your ammeter is in series with the circuit to be measured and that the meter is correctly set to DC. Record the currents in the data table for Part B.

DATA

Part A. Resistors in series

Position	Voltage (Volts)	Current (mA)
across R_1		
across R_2		
across R_1+R_2		
entering R_1		
between R_1 and R_2		
leaving R_2		

Part B. Resistors in parallel

Position	Voltage (Volts)	Current (mA)
across R_3		
across R_4		
across R_3+R_4		
entire circuit		
through R_3		
through R_4		

CALCULATION AND ANALYSIS**PART A. RESISTORS IN SERIES**

1. From your data for the series circuit, compare the three voltages you measured: across R_1 , R_2 , and the combination of R_1 and R_2 together. What simple relationship do you find?
2. Calculate the percent error for the sum of the voltages though R_1 and R_2 using the measured voltage across both resistors as the accepted value
3. Compare the three currents you measured through each resistor and through the entire circuit. What simple relation do you find? How would you interpret this result in terms of the Law of Conservation of Charge?
4. Calculate the percent error for the currents between R_1 and R_2 and leaving R_2 using the measured current entering R_1 as the accepted value.
5. The manufacturer claims that the multimeter has 0.5% accuracy. Compare the manufacturer's stated accuracy with the calculated % error and explain whether either of the results verify Kirchhoff's laws.

PART B. RESISTORS IN PARALLEL

6. Compare the voltage across the whole circuit with the voltage across each resistor individually. What relation do you find
7. What relationship do you find between the current through the whole circuit and the current through each resistor? How do you interpret this in terms of the law of conservation of charge?
8. Calculate the percent error for the sum of the currents though R_3 and R_4 using the measured current through the entire circuit as the accepted value
9. The manufacturer claims that the multimeter has 0.5% accuracy. Compare the manufacturer's stated accuracy with the calculated % error and explain whether either of the results verify Kirchhoff's laws.

Experiment 15: Force on a Current-Carrying Conductor in a Uniform Magnetic Field

OBJECTIVES

The force on a current carrying conductor in a uniform magnetic field depends on the length of the conductor and on the current flowing through it. To test this mathematical relationship, a good experiment must isolate each contributing component and vary it independently of the others. In this experiment, you measure the force on a conductor while varying only the length or only the current. You also measure the current required to produce a constant force on the conductor while varying its length. The objectives of this experiment are as follows:

1. To build a circuit in which you can vary the current and length of the wire
2. To measure the force on a current carrying wire in a magnetic field
3. To calculate the strength of the magnetic field for a permanent magnet

THEORY

If a current is sent through a conductor in a magnetic field there is a force on the conductor which is perpendicular to both the conductor and the direction of the field, shown in figure 15.1.

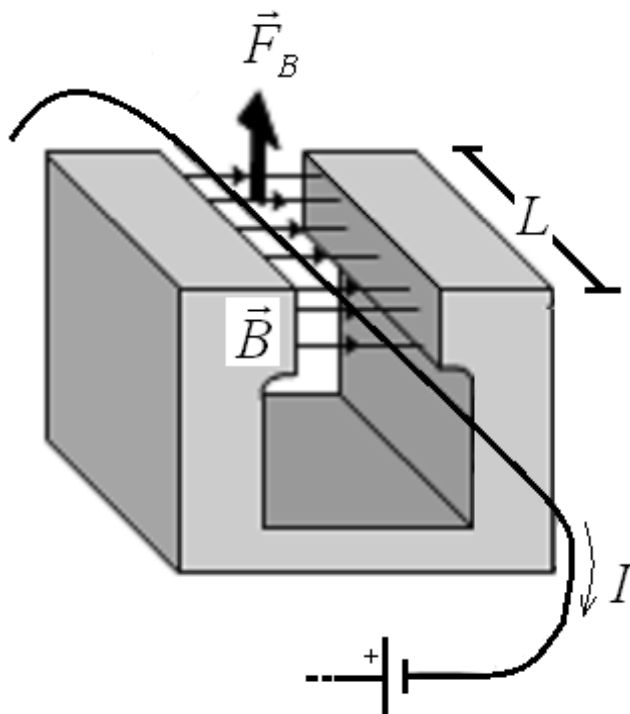


Figure 15.1 Magnetic force on a current-carrying wire

If the magnetic field is uniform and perpendicular to the conductor, the magnitude of this force is the product of the current on the wire, the length of the wire in the magnetic field, and the magnetic field, as shown in equation 15.1.

$$\text{Magnetic Force, Magnitude} \quad F_B = BIL \quad (15.1)$$

Here,

- B is the strength of the magnetic field in teslas. This is constant for a permanent magnet.
- L is the length of the conductor in meters
- I is the current moving through the conductor in amperes
- F_B is the magnitude of the magnetic force in newtons

In this experiment, the magnetic field is supplied by an Alnico permanent magnet. In permanent magnets, the magnetic field is constant and does not change throughout the experiment.

Opposing the magnetic force F_B on the conductor is the gravitational force F_G . By Newton's second law, the sum of these forces is zero when the conductor is stationary. By Newton's third law, there is a force on the magnet equal but opposite to the force on the conductor. The magnet is placed on the pan of a balance and you measure the force on the magnet rather than the equal and opposite force on the conductor.

You test the validity of equation 15.1 in three ways. The magnetic field of the permanent magnet B cannot be varied in this experiment.

- Part A: You determine the magnetic force F_B as a function of I when L and B are constant.
- Part B: You determine the magnetic force F_B as a function of L when I and B are constant.
- Part C: You determine the current I as a function of L when F_B and B are constant.

ACCEPTED VALUES

There is no accepted value for the strength of the magnetic field B generated by the magnet because all the magnets are different for each laboratory setup. You may compare the different values of B you calculate for each part and calculate variance and uncertainty, but you cannot calculate percent error.

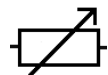
APPARATUS

- multimeter
- linear rheostats
- switch
- DC power supply
- magnetic force apparatus

In the diagram for the circuit you construct for this experiment, there are several specific components denoted by the following symbols:



Switches interrupt the current flowing in a conductor by physically interrupting the current.



Rheostats are resistors for which the resistance can be varied on a sliding scale.

\vec{B} Magnets through which a circuit passes have no standard symbol in circuit diagrams. Parallel lines above and below the conductor indicate where the wire passes through the magnetic field.

The circuit diagram for the magnetic force apparatus is shown in figure 15.2.

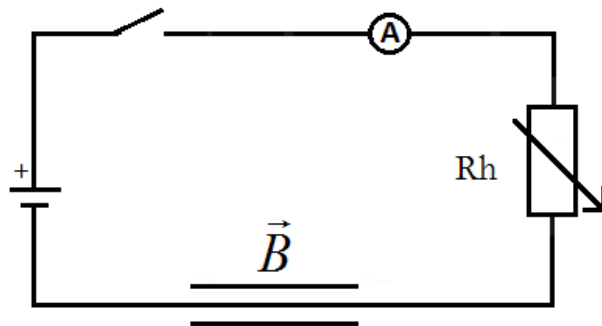


Figure 15.2 The circuit diagram for a circuit in which you can vary current through a magnetic field

The multimeter in this experiment is used as an ammeter to measure I . In part A, you vary I by changing the resistance on the circuit using two rheostats. Moving the sliders up and down the rheostats changes the resistance they apply to the circuit and by Ohm's law, this changes the current.

The magnetic force apparatus is equipped with a three-beam balance, which measures the gravitational force on the permanent magnet. You use the three-beam balance in a similar way to the platform balance by varying the position of the sliders on the beams until the pointer to the right of the beams aligns with fixed scale mark, as shown in figure 15.3.

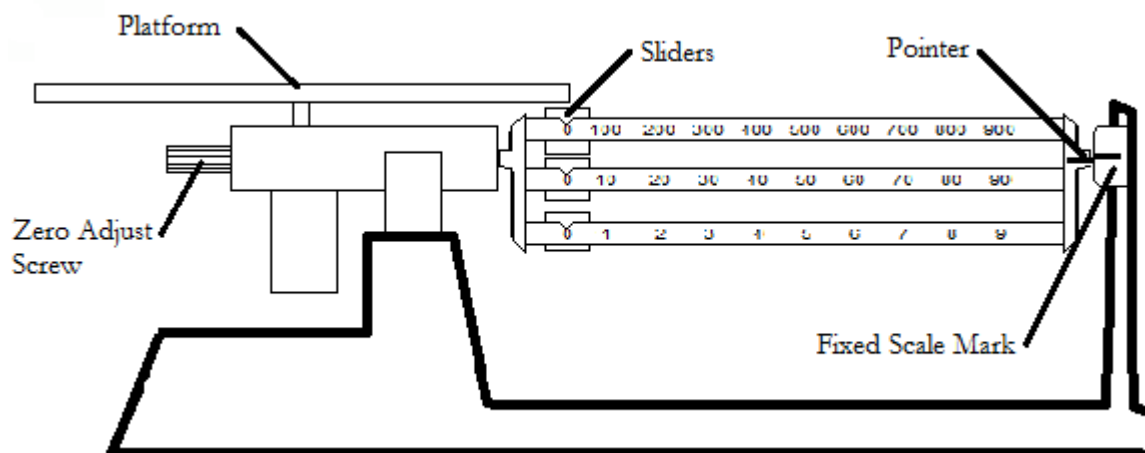


Figure 15.3 Measuring weight with the three-beam balance

PROCEDURE

1. Connect the circuit with the switch, multimeter, rheostats, and wire through the magnetic force apparatus in series. **DO NOT CONNECT TO THE POWER SUPPLY UNTIL THE CIRCUIT HAS BEEN CHECKED BY YOUR INSTRUCTOR.**
2. While the switch is open, adjust the balance for equilibrium using only the beam slider weights. When this balance is reached, be sure that the wire is parallel to the long side of the magnet and slightly below the top of the air gap. Record the balance measurement as the initial mass m_0 on the data sheet.

PART A: FORCE GENERATED BY VARYING CURRENT

3. Adjust the clips on the wire through the magnetic force apparatus so that the current is supplied to the entire length L of the wire. The entire length of the wire L is 0.152 meter. Record this length as the initial length L_0 on the data sheet for Part A.
4. Move the slider to the middle of each rheostat and close the switch. If this causes a repulsive force between the magnet and the wire, and the magnet moves downward, proceed to step 5. If the magnet and wire attract each other and the magnet lifts, open the switch, switch the clips connected the wire passing through the magnetic force apparatus, close the switch, and proceed to step 5.

CAUTION: To increase the current in the circuit, first decrease the resistance of the rheostat with the larger resistance. Do not reduce the resistance of the other rheostat until the first rheostat has been reduced to zero resistance.

5. Adjust the rheostats until the multimeter displays a current of 0.5 ampere and adjust the balance for equilibrium. Record the balance measurement in the mass (g) column in the 0.5 ampere row in the data table for Part A.
6. Repeat step 5 for currents of 1.00, 1.50, 2.00, 2.50 and 3.00 amperes.

PART B: FORCE GENERATED BY VARYING LENGTH

7. Change the clips so that the current is supplied to $5/6L$ and measure the balance equilibrium when the current is 2.00 amperes. Record the balance measurement in the mass (g) column in the 0.1267 meter row in the data table for Part B. Record 2.00 amperes as the value for the initial current I_0 .
8. Repeat step 7 for lengths $4/6L$, $3/6L$, $2/6L$, and $1/6L$ with a constant current of 2.00 amperes in each case. When changing the length, use the connections such that the length of wire through which the current is flowing is in the center of the magnetic field.

PART C: EQUILIBRIUM CURRENT VERSUS LENGTH

9. Set the balance reading at a value 2.00 grams greater than the equilibrium value you measured in step 2. The goal is to balance the resulting 2 grams worth of additional gravitational force against the magnetic force. Converting to kilograms, multiply 0.002 kg by $g=9.80 \text{ m/s}^2$ and record this as the constant gravitational force F_g on the data sheet for Part C.
10. Adjust the clips so that current is supplied to the entire length of the wire and adjust the current until the balance reaches equilibrium. Record the current measurement from the multimeter in the Current (A) column in the 0.1520 meter row in the data table for Part C.
11. Repeat step 10 for lengths $5/6L$, $4/6L$, $3/6L$, $2/6L$, and $1/6L$.

DATA

$$m_0 \text{ (g)} = \underline{\hspace{2cm}}$$

Part A: Force generated by varying current

$$L_0 \text{ (m)} = \underline{\hspace{2cm}}$$

Current (A)	Mass (g)
0.50	
1.00	
1.50	
2.00	
2.50	
3.00	

Part B: Force generated by varying length

$$I_0 \text{ (A)} = \underline{\hspace{2cm}}$$

length (m)	Mass (g)
0.1267	
0.1013	
0.0760	
0.0507	
0.0253	

Part C: Equilibrium current versus length

$$F_i \text{ (Newtons)} = \underline{\hspace{2cm}}$$

length (<i>m</i>)	Current(A)
0.1520	
0.1267	
0.1013	
0.0760	
0.0507	
0.0253	

CALCULATION AND ANALYSIS

1. Calculate the difference in force ΔF between the initial force and the measured force for each row in the data table for Part A. You should express ΔF in Newtons. It is given by the formula $\Delta F = (m - m_0)g$ where $g=9.80 \text{ m/s}^2$.
2. Plot a graph of the change in force ΔF (in Newtons) against current I (in Amperes) using your data from Part A. From the slope of the graph and the initial length L_0 , calculate the strength of the magnetic field using $B = (\text{slope}) / L_0$.
3. Calculate the difference in force ΔF in grams between the initial force and the measured force for each row in the data table for Part B. Again it is given by the formula $\Delta F = (m - m_0)g$ where $g=9.80 \text{ m/s}^2$.
4. Plot a graph of the change in force ΔF (in Newtons) against length L (in meters) using your data from Part B. From the slope of this graph and the initial current I_0 , calculate the strength of the magnetic field using $B = (\text{slope}) / I_0$.
5. Calculate the average value of $I \times L$ (ampere meters) using the data from the data table for Part C. From this average value and the constant gravitational force F_i , calculate the strength of the magnetic field using $B = F_i / (I \times L)$.
6. Calculate the average value (mean) of your three measurements of B .
7. Calculate the uncertainty in your average value for B using equation 0.6.

Experiment 16: The Specific Charge of the Electron

OBJECTIVES

The purpose of this experiment is to observe the trajectory of moving charges (electrons) in a uniform magnetic field and to calculate the charge-to-mass ratio of an electron. In this experiment a beam of electrons travels through a uniform magnetic field produced by a pair of Helmholtz coils. The magnetic field bends the beam into circular path, and by measuring the radius of the circle you can calculate the charge-to-mass ratio of a single electron. The objectives of this experiment are as follows:

1. To measure the radius of a circular electron beam in a magnetic field
2. To calculate the apparatus constant for a pair of Helmholtz coils
3. To calculate the ratio of charge to mass for an individual electron

THEORY

In this experiment two devices are used. The electrons are emitted by a heated cathode, also known as an electron gun, into a glass bulb filled with low pressure Hydrogen gas. Two identical coils of wire separated by a distance equal to their radius, called Helmholtz coils, produce a uniform magnetic field through the gas-filled bulb as shown in figure 16.1.

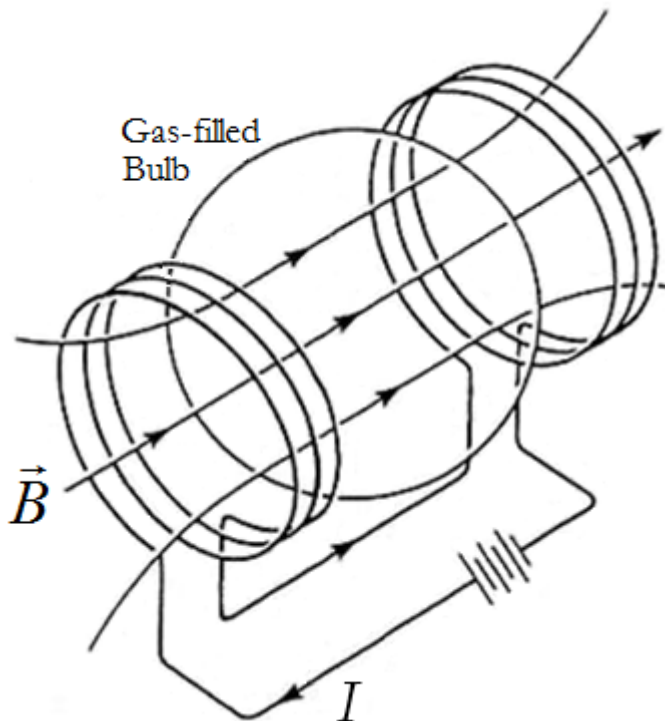


Figure 16.1 A magnetic field produced by Helmholtz coils through a gas-filled bulb

Electrons emitted by a heated cathode are accelerated in an electric field. They acquire kinetic energy in accord with equation 16.1.

$$\text{Kinetic energy (electrons)} \quad \frac{1}{2}mv^2 = eV \quad (16.1)$$

Here, m is the mass of the electron, v is magnitude of the velocity of the electron, e is the charge of the electron, and V is the anode potential/Voltage. The electrons are assumed to have an initial velocity of zero as they emerge through an opening in the anode. The tube can be turned so that the electron beam is directed at right angles to the magnetic field, which bends the beam into a circular path. The electrons gain no energy from the magnetic field because the magnetic force acts at right angles to the path of the electrons. Therefore, they move with constant speed v . The magnetic deflecting force is shown in equation 16.2.

$$\text{Magnetic Deflecting Force} \quad F_B = Bev \quad (16.2)$$

Here B is the strength of the magnetic field. Because this force acts radially, it is also a centripetal force, which is equivalent to the product of the mass and the velocity squared divided by the radius of the circular path, as shown in equation 16.3.

$$\text{Centripetal Force} \quad F_C = Bev = \frac{mv^2}{r} \quad (16.3)$$

Here r is the radius of the circular path the electrons follow. Solving equation 16.3 for r yields equation 16.4.

$$\text{radius} \quad r = \frac{mv}{Be} \quad (16.4)$$

To eliminate v from equation 16.4, square equation 16.4 and substitute the quantity of v^2 from equation 16.1. After some algebra, the final value for the ratio of charge to mass of the electron e/m is shown in equation 16.5.

$$\text{Ratio of charge to mass} \quad \frac{e}{m} = \frac{2V}{r^2 B^2} \quad (16.5)$$

The magnetic field produced by a pair of Helmholtz coils is a function of the current I passing through them. A single Helmholtz coil of radius R produces the magnetic field along the axis of symmetry at distance x from the center, as shown in equation 16.6.

$$\text{Magnetic field (one coil)} \quad B = \frac{\mu_0}{2} \frac{NR^2}{(R^2 + x^2)^{3/2}} I \quad (16.6)$$

Here, N is the number of turns in the coil and μ_0 is the magnetic permeability of a vacuum ($4\pi \times 10^{-7}$ Tm/A). The pair of Helmholtz coils in the apparatus are separated by a distance R from each other, so halfway between them $x=R/2$. At this distance, the magnetic field in the region between the coils is nearly uniform. Substituting $x=R/2$ into equation 16.6 and doubling the result for the pair of coils, the apparatus constant is calculated using equation 16.7.

$$\text{Apparatus Constant} \quad B = CI \rightarrow C = \frac{\mu_0}{(5/4)^{3/2}} \frac{N}{R} \quad (16.7)$$

From equations 16.5 and 16.7, e/m is the calculated quantity in equation 16.8.

$$\text{Ratio of charge to mass} \quad \frac{e}{m} = \frac{2V}{r^2 I^2 C^2} \quad (16.8)$$

ACCEPTED VALUES

The accepted value for the ratio of charge to mass of an electron to three significant figures is 1.756×10^{11} Coulombs/kilogram. The manufacturer claims the voltmeter and ammeter are accurate to within $\pm 2.5\%$.

APPARATUS

- specific charge apparatus
- transparent half-meter stick

The bulb contains hydrogen gas at a pressure of about 10 mm Hg. As the electron beam passes through the gas it leaves a glowing track. The bulb is placed between two coils of wire which are separated by a distance equal to their radius. The apparatus is self-contained, as shown in figure 16.2.

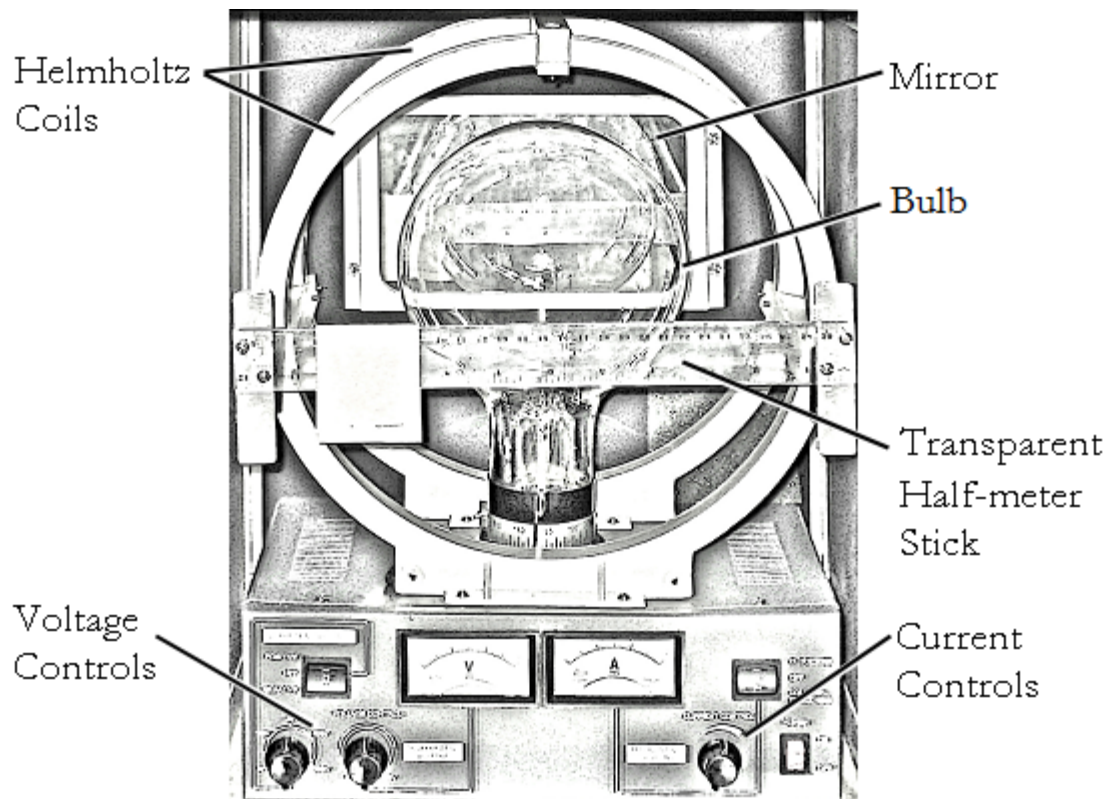


Figure 16.2 The specific charge apparatus

This particular arrangement of coils, known as Helmholtz coils, produces a very uniform magnetic field in the space between the coils. To measure the diameter of the electron beam, measure each edge of the beam individually using the transparent half-meter stick on the front of the apparatus. By positioning your eyes level with the half-meter stick, align the edge of the electron beam with the edge of the reflection of the electron beam in the mirror behind the CRT, as shown in figure 16.3.

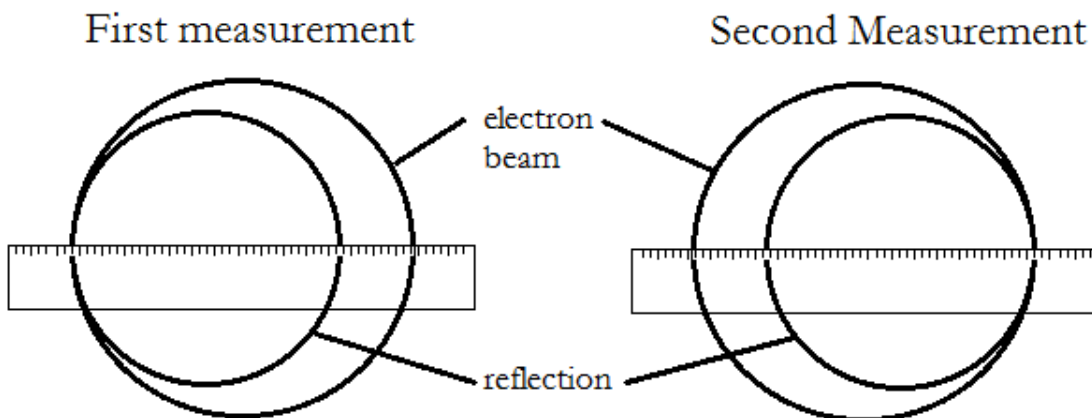


Figure 16.3 Aligning the electron beam and reflection to measure the beam diameter.

The difference between the readings on the left and right is the diameter of the circular path of the electron beam $2r$.

PROCEDURE

1. Set the anode voltage to 150 V and the Helmholtz coil current to 1 A.
2. Increase the filament current slowly until the electron beam appears. If the beam is not clearly visible, increase the filament current slightly. If the electron beam is helical, rotate the tube until it is circular. If the beam does not appear, reverse the connections to the Helmholtz coils.
3. Measure the diameter of the path of the electrons using the procedure described in Figure 16.3. Record your result in the data table.
4. Repeat step 3 with:
 - Helmholtz coil current = 1.5 amperes, anode voltage $V = 150$ volts
 - Helmholtz coil current = 1.0 amperes, anode voltage $V = 200$ volts
 - Helmholtz coil current = 1.5 amperes, anode voltage $V = 200$ volts

DATA

anode voltage, V (V)	Helmholtz coil current, I (A)	beam diameter, $2r$ (m)
150	1.0	
150	1.5	
200	1.0	
200	1.5	

CALCULATION AND ANALYSIS

1. Calculate the value of C in equation 16.7 for the apparatus in this experiment. For this apparatus $R=0.14$ m and $N = 140$.
2. Calculate e/m from each row in the data table using equation 16.8.
3. Calculate the average value of e/m , the uncertainty in the average value, and the % uncertainty in the average value using equations 0.1, 0.6, 0.7.
4. Calculate the % error in your average value of e/m . Do your measurements support the accepted value to within the uncertainty?
5. What effect would you expect Coulomb electrical forces between electrons to have on the beam? Explain.

Experiment 17: Refraction

OBJECTIVES

The transmission of light across a boundary between two media is accompanied by a change in both the speed and wavelength of the wave. This can result in a change of direction at the boundary, a phenomenon known as refraction. In this experiment you measure the change in direction of light beams as they refract or reflect at a boundary to determine the index of refraction of a transparent object. The objectives of this experiment are as follows:

1. To measure the angles of incidence and refraction at a boundary between media
2. To observe total internal reflection at a boundary between media
3. To calculate the critical angle of a boundary between media

THEORY

Snell's law describes the relationship between the angle of incidence of a beam of light as it intersects a new transparent medium and the angle of refraction as enters that transparent medium.

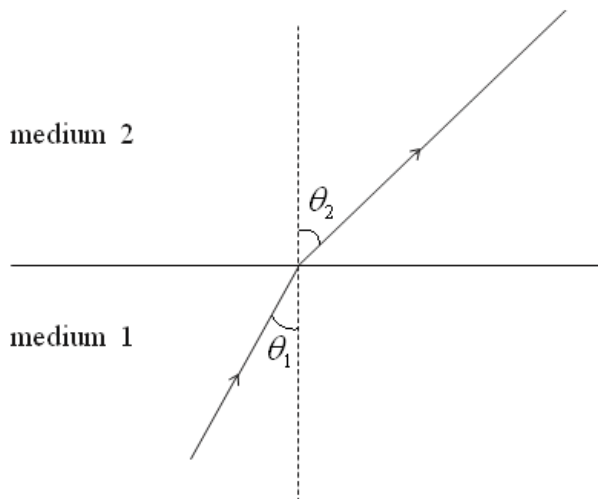


Figure 17.1 angles of incidence for a light as it intersects a new medium

Snell's law quantifies the relationship as shown in equation 17.1.

$$\text{Snell's Law} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (17.1)$$

Here,

- n_1 is the index of refraction of the initial medium
- n_2 is the index of refraction of the new medium
- θ_1 is the angle of incidence
- θ_2 is the angle of refraction

The index of refraction of a medium is the ratio of the speed of light in a vacuum to the speed of light in the medium, as shown in equation 17.2.

$$\text{Index of Refraction} \quad n = \frac{c}{v} \quad (17.2)$$

Here $c = 3 \times 10^8$ m/s, the speed of light in a vacuum, and v is the speed of light in the medium. For air to a very good approximation $n = 1$.

The critical angle is the angle of incidence at which light begins to reflect, rather than refract, when it enters a new medium that has a lower index of refraction. This happens when the angle of refraction reaches 90° . Substituting $\theta_2 = 90^\circ$ in equation 17.1 and solving for θ_1 calculates the critical angle as shown in equation 17.3.

$$\text{Critical Angle} \quad \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad (17.3)$$

A prism is a transparent medium composed of two non-parallel polished surfaces. When light travels through a prism such that it enters one side and exits another, the total deviation δ , in the ray leaving the second side from the original angle of incidence is dependent on the index of refraction, n , and the angle between the two sides of the prism α , as shown in figure 17.2.

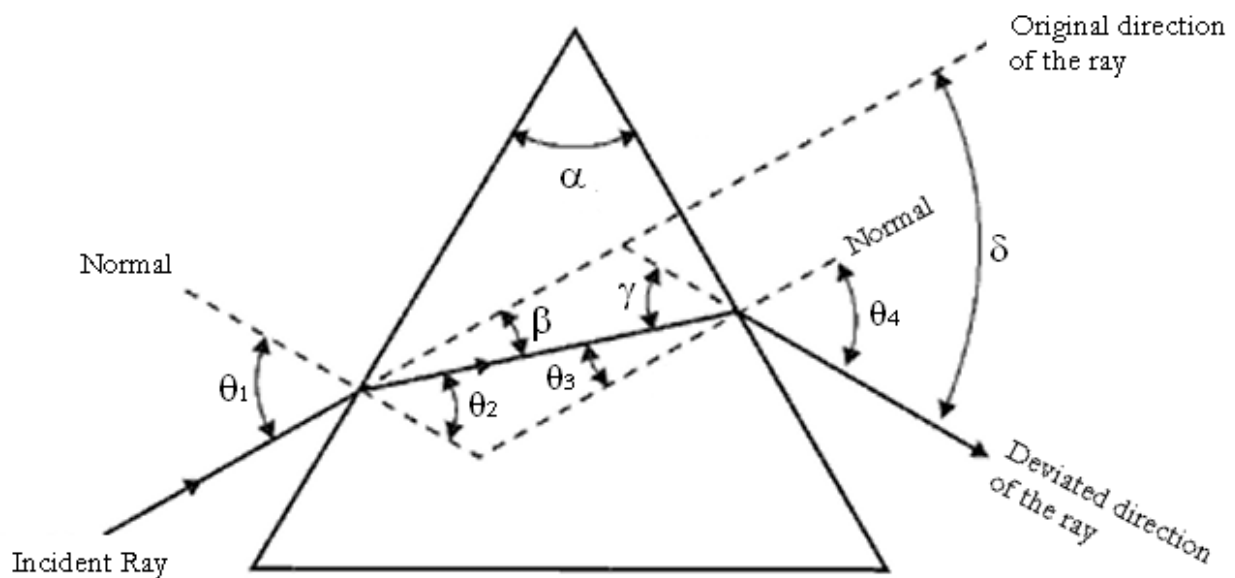


Figure 17.2 Total deviation of a light ray through a prism

The value of δ depends on the angle of incidence of the light, θ_1 , at the first face of the prism. When the light passes symmetrically through the prism so that the angle of refraction at the second face is equal to angle of incidence at the first face, $\theta_3 = \theta_1$, then δ has a minimum value δ_m . If the light enters the prism from a near vacuum medium, for example air with an index of refraction of 1.00, then the index of refraction is calculable in terms of the angle between the two sides of the prism α and the total deviation δ , as shown in equation 17.4.

$$\text{Index of Refraction} \quad n = \frac{\sin \frac{\delta_m + \alpha}{2}}{\sin \frac{\alpha}{2}} \quad (17.4)$$

ACCEPTED VALUES

The accepted value for the index of refraction for Lucite is 1.50.

APPARATUS

- Straight filament lamp
- protractor
- Lucite prism
- ruler

PROCEDURE

1. Place a sheet of blank paper on your desk. Arrange the light source and prism so that a narrow beam of light is incident on the prism as shown in figure 17.3.

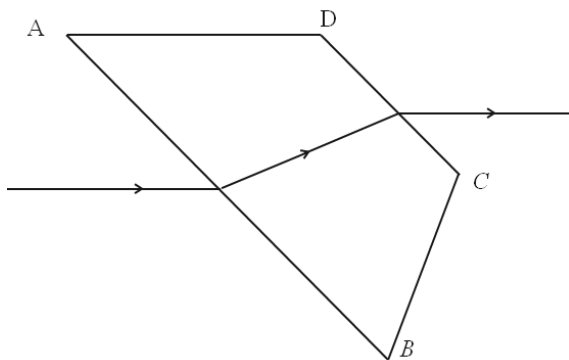


Figure 17.3 Trace of a light ray refracting at two surfaces.

2. Adjust the prism so that the angle of incidence on the first surface is between 45° and 60° . Using two dots made with a sharp pencil to determine the direction of a straight line, trace the outline of the prism and the ray of light onto the paper.
3. Repeat steps 1 and 2 with a second surface with an angle of incidence greater than 60° .
4. Arrange the prism and the beam of light as in figure 17.4 so that the angle of refraction at the second surface is 90° . Trace, as in step 2, the outline of the prism and the ray of light.

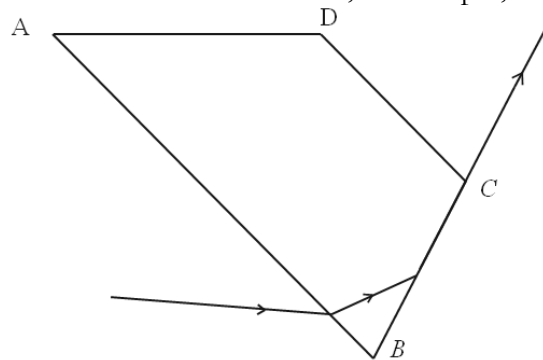


Figure 17.4 Trace of a light ray refracting and then reflecting internally

5. Arrange the prism and the beam of light as in figure 17.5. Rotate the prism until the angle of deviation δ is a minimum and then trace the outline of the prism and the beam of light. Repeat, using the other angle of the prism which is less than 90° as the refracting angle.

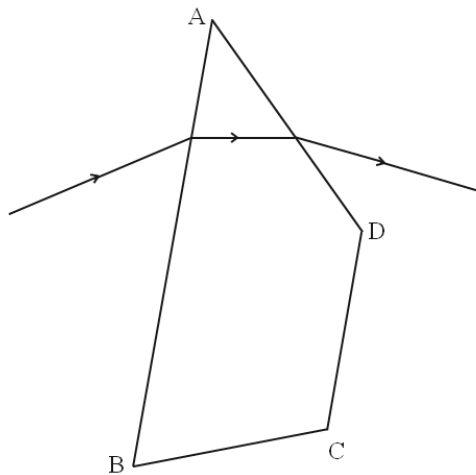


Figure 17.5 Trace of a light ray through the angle of minimum deviation

DATA

The data sheet for this lab is the blank sheet of paper with tracings of the light rays and prism. From this sheet, you measure angles using a protractor, arranged in a data table like the following.

Measurement	Value (in degrees)
Angle of incidence (first surface)	
Angle of refraction (first surface)	
Angle of incidence (second surface)	
Angle of refraction (second surface)	
Critical Angle	
Angle of minimum deviation (narrow end)	
Angle of prism (narrow end)	
Angle of minimum deviation (wide end)	
Angle of prism (wide end)	

CALCULATION AND ANALYSIS

1. Measure the angles of incidence and refraction at both surfaces of the prism in the tracings of procedures step 2 and 3. Calculate the index of refraction for the Lucite prism from these measurements.
2. Measure the critical angle from the tracing of procedure step 4. Calculate the index of refraction for the Lucite prism from the critical angle.
3. Measure the angle of minimum deviation δ_m and the angle of the prism α from each tracing of procedure step 5. Calculate the index of refraction for the Lucite prism from these angles.
4. Find the average (mean) value for the index of refraction of the prism.
5. Calculate the velocity of light in the prism.

Experiment 18: Mirrors and Lenses

OBJECTIVES

Lenses and mirrors bend light in a predictable fashion. In this experiment you'll study concave mirrors and converging and diverging lenses and see what types of images they form. The objectives of this experiment are as follows:

1. To study the imaging properties of mirrors and lenses.
2. To measure the image distances and heights produced by various arrangements of mirrors and lenses.
3. To calculate focal lengths and magnifications.

THEORY

When you measure all distances from the mirror or lens, the object (source) distance is d_o , the image distance is d_i , and the focal length is f . The focal point is the position of the image for incident rays parallel to the axis. For mirrors and lenses the equation relating these distances is given by equation 18.1.

$$\text{Mirror and Lens Equation} \quad \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad (18.1)$$

The sign of f is specific to each type of mirror or lens. For concave mirrors and converging lenses, f is positive. For convex mirrors and diverging lenses, f is negative.

An image is said to be inverted if its orientation is opposite to that of the object. Otherwise it is said to be upright. The magnification of the image is given by equation 18.2.

$$\text{Magnification} \quad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad (18.2)$$

Real images actually have light passing through them focused by the mirror or lens from the source; otherwise the image is virtual. Review these topics from your textbook.

ACCEPTED VALUES

The accepted value for the focal length of a converging lens is the image distance when the object is very far away. This is the only accepted value needed in this lab.

APPARATUS

- optical bench
- illuminated object
- screens
- converging and diverging lenses
- concave mirror
- plane mirror
- half meter stick

The optical bench aligns mirrors, lenses, and screens along a fixed one-dimensional axis. The illuminated object must be aligned along this axis to produce measurable images.

CONCAVE MIRROR

For a concave spherical mirror the focal distance f , measured from the mirror, is half the radius of curvature of the sphere from which the mirror is made. A real inverted image forms at a distance d_i from the mirror when the object is at a distance d_o . Light rays reflected by a concave mirror are shown in figure 18.1, first for the case $d_o = d_i = 2f$, then for the case $d_o > 2f$.

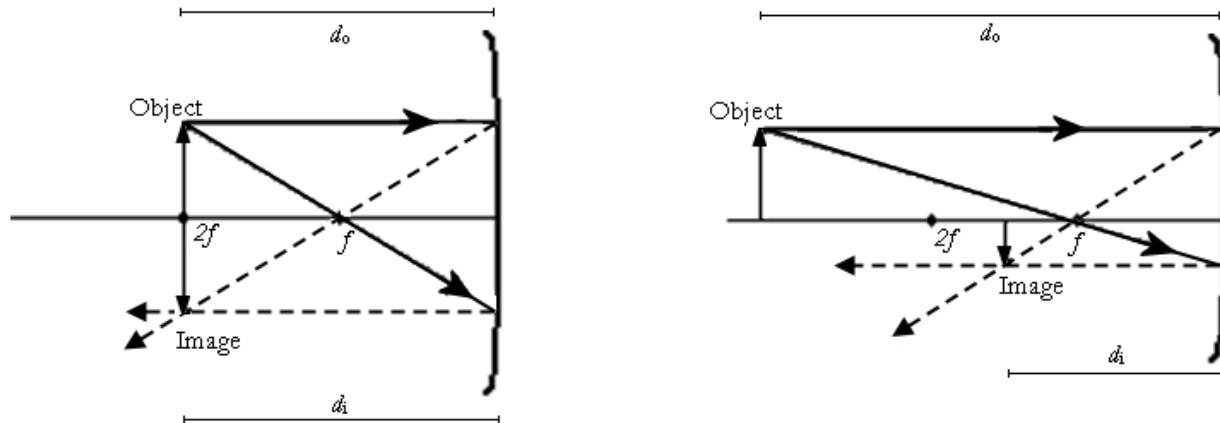


Figure 18.1 light rays reflected by concave mirrors

CONVERGING LENS

Converging lenses, which are thicker at the center than at the edges, focus incident light rays to form a real inverted image on the opposite side of the lens from the object when the object distance d_o is greater than f . When d_o is less than f , a virtual upright image forms on the same side of the lens as the object, as shown in figure 18.2, and the image distance d_i is negative

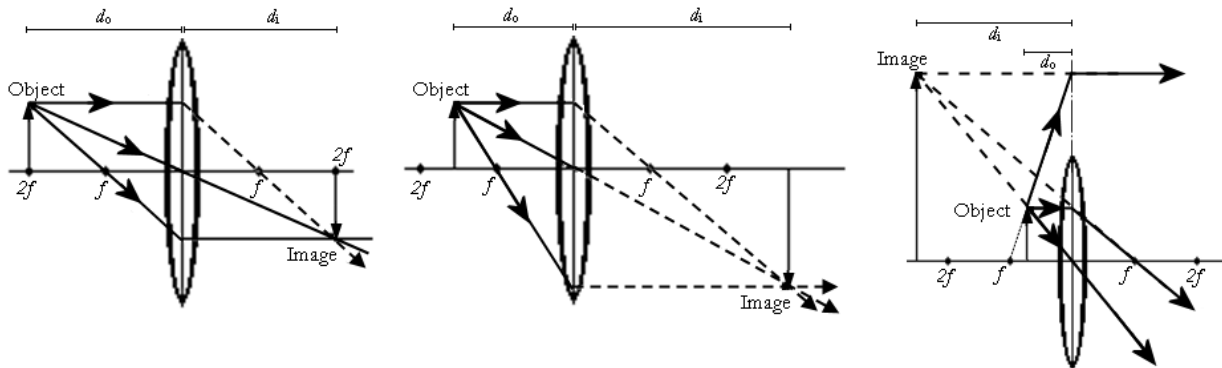


Figure 18.2 light rays refracted through converging lenses

DIVERGING LENS

Diverging lenses, which are thinner in the center than at the edges, spread incident light rays as they pass through the lens. These rays appear to diverge from a point on the same side of the lens as the object. For these lenses, the image is virtual and the focal length and the image distance are negative. These virtual images are upright and on the same side of the lens as the object.

Because diverging lenses make virtual images it's not so easy to measure their focal length. One way to do it is with a converging lens. Suppose a converging lens is used to make an image of an object

on a screen. Now suppose a diverging lens is placed between the converging lens and the screen. If the distance from the diverging lens to the screen is exactly the focal length of the diverging lens, then after going through the diverging lens the light rays are parallel to the axis. If a mirror is placed in front of the screen it will reflect these parallel rays back and form an image at the location of the original object. This is shown in figure 18.3, where we've moved the mirror away from the focal point for clarity.

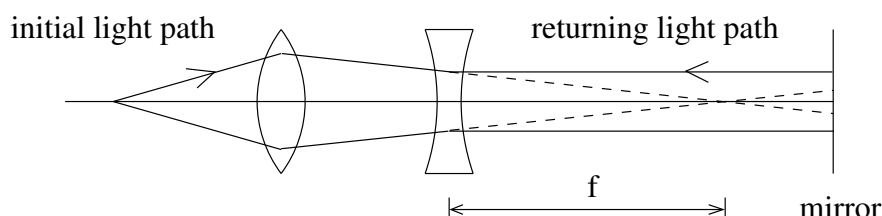


Figure 18.3 light rays refracted through diverging and converging lenses

With such a system we can determine the focal length of a diverging lens: it is the distance from the diverging lens to the screen, when everything is arranged to form an image at the position of the original object.

PROCEDURE

For each part of the experiment, draw a graph with a horizontal line representing the optical axis perpendicular to the lens or mirror. Record the position of each component of the system in the data tables. The distance for any object, image, or focal point is always measured from the lens or mirror.

PART A: CONCAVE MIRROR

1. Arrange the illuminated object and the concave mirror so that a clear image forms at the same position as the object. For this arrangement the image and object distances are the same.
2. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part A.
3. Arrange the illuminated object, half-screen and concave mirror so that a real image is formed with the object distance approximately twice the image distance.
4. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part A.

PART B: CONVERGING LENS

5. Arrange the illuminated object, converging lens and screen so that a real image is formed with the object distance approximately equal to the image distance.
6. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part B.
7. Arrange the object, lens, and screens so that the real image is formed with the object distance approximately twice the image distance ($d_o \approx 2d_i$).
8. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part B.
9. Keeping the object and screen positions the same as in part 7, move the lens toward the object until an image is again formed on the screen.

10. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part B.
11. Choose a distant object, such as a building far from the laboratory or a window in a separate wing of Gillet Hall, and form an image of a distant object with the converging lens and the screen. The image distance is equal to the focal length of the lens, because a very distant object is produced by rays that are essentially parallel.
12. Measure the image distance and image orientation, and record them in the data table for Part B.

PART C: DIVERGING LENS

13. Adjust the illuminated object, screen, and converging lens so that an image is formed on the screen approximately 30 cm from the lens. Place the diverging lens between the converging lens and the screen, and place a mirror in front of the screen. Move the diverging lens until an image forms at the position of the object.
14. Measure the distance from the diverging lens to the screen as the focal length in the data table for Part C.

DATA

Part A: Concave Mirror

Arrangement	Object Distance (cm)	Image Distance (cm)	Object Height (cm)	Image Height (cm)	Image Orientation
Projected back to object distance					
Projected back to 2x object distance					

Part B: Converging Lens

Arrangement	Object Distance (cm)	Image Distance (cm)	Object Height (cm)	Image Height (cm)	Image Orientation
image and object distance ~ the same					
object at ~ 2x image distance					
Reformed image after moving lens toward the object					
Distant object	∞				

Pat C: Diverging Lens

Arrangement	Focal Length
diverging lens between converging lens and screen, image reflected off a mirror onto the plane of the object	

CALCULATION AND ANALYSIS

1. Calculate the focal length of the concave mirror twice, using the data from the two rows of the data table for part A.
2. Calculate the magnification of the concave mirror for each object distance analyzed.
3. Calculate the focal length of the converging lens three times, using the data from the first three rows of the data table for part B.
4. Use the image distance for a distant object as the accepted value for the focal length of a converging lens. Calculate the % error for the focal lengths you obtained in step 3.
5. Report the focal length of the diverging lens.

Experiment 19: The Grating Spectrometer

OBJECTIVES

A diffraction grating is a way to measure the wave properties of light. The grating produces a diffraction pattern that depends on the wavelength of the light that passes through it. In this experiment you observe the diffraction pattern produced by light from excited Mercury atoms. This allows you to determine the emission spectrum of Mercury. The objectives of this experiment are as follows:

1. To observe the emission spectrum of Mercury using a diffraction grating
2. To measure the angles to various lines in the diffraction pattern
3. To calculate some of the wavelengths of light emitted by Mercury

THEORY

A diffraction grating consists of a surface with regularly-spaced parallel slits through which light can pass. Light passing through different slits travels different distances from the grating to the screen (or spectrometer telescope), as shown in figure 19.1.

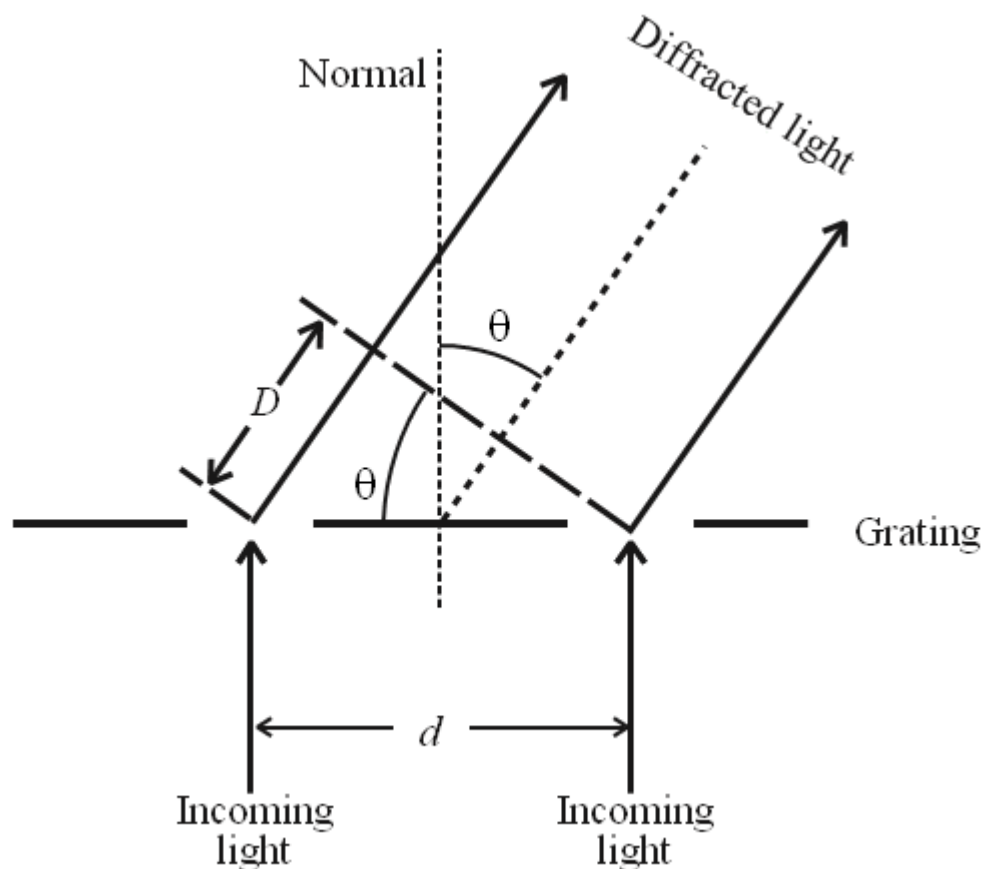


Figure 19.1 Diffraction angles for light passing through a diffraction grating

When the light reaches the screen there can be constructive interference, which produces a bright line, or destructive interference, in which case no light is observed.

Consider two slits with a separation distance d . The light passing through the slits is observed at an angle θ with respect to the normal to the diffraction grating. As shown in figure 19.1, the ray to the left travels an extra distance D . D is calculable using trigonometry, as shown in equation 19.1.

$$\text{Extra distance traveled (light)} \quad D = d \sin \theta \quad (19.1)$$

Constructive interference occurs when the path difference for the two rays is exactly an integer number of wavelengths, as shown in equation 19.2.

$$\text{Constructive Interference} \quad D = n\lambda = d \sin \theta_n \quad (19.2)$$

Here

- n is an integer (0, 1, 2, ...)
- θ_n is the corresponding angle at which constructive interference occurs
- $n\lambda$ is the path difference
- The gratings have 500 lines per mm so $d = \frac{1 \text{ mm}}{500} = 2 \times 10^{-6} \text{ m}$

The angle for constructive interference depends on the wavelength λ , so by measuring the angle you can determine the wavelength of the light. You observe light from Mercury atoms, which is a mixture of different wavelengths (colors). You measure the angle for constructive interference from a grating for which we know the line spacing d and use equation 19.2 to calculate the wavelength, λ , of the light. The telescope is initially set perpendicular to the grating ($\theta = 0$) where all wavelengths (colors) of the beam constructively interfere, which appears as a bright white line.

To observe the separated colors the spectrum, swing the telescope away from $\theta = 0$ until you observe separate color lines from violet to red, which comprise the $n = 1$ angular position for the lines of the spectrum. The angular position of the telescope for the first appearance of a given color line to the left of the initial position is a poor estimate of θ_n for that spectrum line. To get a more accurate estimate, also measure the angular position of the telescope for the first appearance of same color line to the right of the initial position. Half of the difference between the angular positions of these two observations is a good estimate of θ_n for that color. Measuring the angular position to the left and then to the right of the perpendicular direction corrects for any error in positioning the grating perpendicular to the collimator beam. This process must be repeated for each color line in each spectrum, $n=1$ and $n=2$, which correspond to the first order and second order constructive interference patterns of the light. The $n=2$ spectrum is more spaced out than the $n=1$ spectrum.

ACCEPTED VALUES

In one fringe, you can typically make out four distinct color bands, similar to the spectrum shown in figure 19.2.



Figure 19.2 A typical diffraction pattern for one fringe from the light emitted by Mercury

A Mercury lamp emits light at specific wavelengths. The colors and wavelengths of the strongest visible lines in the Mercury spectrum are:

Color	Wavelength (nm)	Intensity
Violet	404.6	moderate
	407.8	weak
blue-violet	435.8	strong
green	546.1	strong
yellow	577.0	strong
	579.0	strong

APPARATUS

- Spectrometer
- Diffraction grating
- mercury arc lamp

The grating is clamped in a rotating table so that it is perpendicular to the collimator which focuses the light from the lamps. The telescope is initially set perpendicular to the grating ($\theta = 0$) where all wavelengths (colors) of the beam constructively interfere. The goniometer angular scales display different initial angles through the magnifying glasses, neither of which may be equal to zero, as shown in figure 19.3. Make sure you choose one of the goniometer angular scales to use for the entire experiment. We will call the angle measured on the goniometer scale φ . For each line you will measure the angle twice, once to the right (φ) and once to the left (φ'). The deflection angle θ is half the absolute value of the difference,

$$\text{Deflection angle} \quad \theta = \left| \frac{\varphi - \varphi'}{2} \right| \quad (19.3)$$

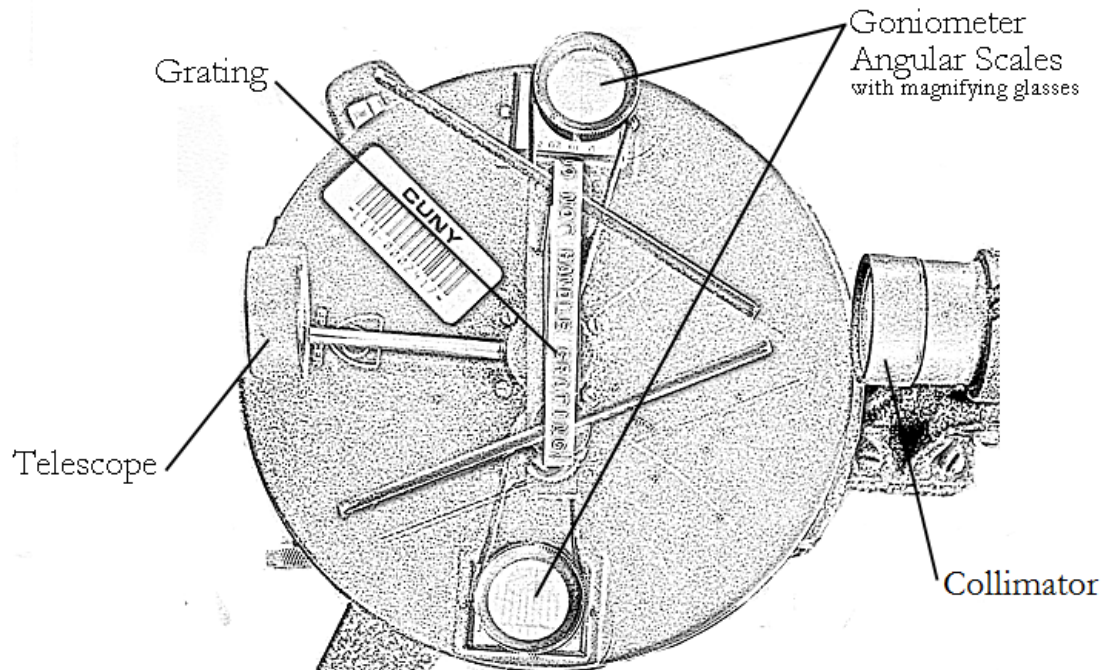


Figure 19.3 An overhead view of the parts of the diffraction grating spectrometer

PROCEDURE

1. Position the telescope so that it is perpendicular to the grating. Move it until you see a bright white band in the cross hairs. Important: choose one of the windows and use it to measure all of your angles. That is, make sure you always read the angle from the same goniometer angular scale of the apparatus!
2. Swing the telescope to the right until you observe the separated colors of the spectrum.
3. Set the telescope cross hairs to the first mercury line to the right. Record its color. Then read the angle from your chosen goniometer angular scale and record the angle in your data table as φ_1 .
4. Swing the telescope to find the same color line to the left. Read the angle from your chosen goniometer angular scale and record the angle in your data table as φ'_1 .
5. Repeat procedures 3 through 5 for the other colors. Take care that you are observing the first line of each color on either side.
6. Now measure the angles (left and right) for the second appearance of just the green and yellow lines. Record the angles as φ_2 and φ'_2 .

DATA

Spectrum Measurements

color	φ_1	φ'_1	φ_2	φ'_2

CALCULATION AND ANALYSIS

1. Calculate the wavelength for each of the lines you measured. Recall that for each line you measured the angle twice. To get the deflection angle θ use equation 19.3.
2. Try to match your calculated wavelength values with the values in the table of accepted values. Calculate the percentage error for each wavelength calculation.

Appendix: Algebra and Trigonometry Review Topics

REFRESHING

High-School Mathematics

and beyond..

Numbers and symbols

Symbolic and numeric calculations

Physics students have to be able to operate with symbols (such as a , b , x , y , Q , etc.) that stand for numbers. Calculations should be done, as a rule, in a symbolic form, and the analytical (that is, symbolic) result should be obtained. Only after that concrete numbers should be plugged into the analytical result, to obtain the numerical result. Doing this is not a big problem since all operations on numbers can be done on symbols as well. Symbolic operations have important advantages, however, such as better overview of the manipulations, possibility of backtracking and checking, possibility of using multiple sets of numerical values without a necessity of doing similar calculations many times, possibility of operating with quantities, numerical values of which are unknown but irrelevant.

Basic identities

$$a + b = b + a$$

$$a + (b + c) = (a + b) + c = (a + c) + b = a + b + c$$

$$ab = ba$$

$$a(bc) = (ab)c = (ac)b = abc = acb = bca = \dots$$

$$a(b + c) = ab + bc$$

$$(a + b)^2 = a^2 + 2ab + b^2 \text{ (binomial formula)}$$

(operations in brackets are performed first)

Fractions

$$a/b = \frac{a}{b} = a \times \frac{1}{b} = a \times b^{-1}$$

$$\frac{b}{a} = \frac{b}{c} \times \frac{c}{a} = ab \frac{1}{c} = ab/c$$

$$\frac{ac}{bd} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

Inserted for convenience

Compound Fractions

Fractions containing fractions are sometimes confusing, such as

$$\frac{a}{\frac{b}{\frac{c}{d}}} \text{ or } \frac{\frac{a}{b}}{\frac{c}{d}}$$

To avoid confusion, we can distinguish between external and internal fractions and make external fractions longer and/or bolder. If we divide

$$a \text{ or } \frac{a}{b} \text{ over } \frac{c}{d}$$

we write

$$\frac{a}{\frac{b}{\frac{c}{d}}}$$

internal fractions
external fraction

and simplify the fractions as follows

$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{ac}{b}$$

$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{a}{\frac{b}{c} \cdot \frac{1}{d}} = \frac{a}{\frac{bd}{c}} = \frac{ac}{bd}$$

Manipulations above can be justified using powers instead of reciprocals:

$$\frac{a}{\frac{b}{\frac{c}{d}}} = a \left(\frac{b}{\frac{c}{d}} \right)^{-1} = a \frac{c}{b} = \frac{ac}{b}$$

$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{a}{b} \left(\frac{c}{d} \right)^{-1} = \frac{a}{b} \frac{d}{c} = \frac{ad}{bc}$$

Fractions can also be written as

$$\frac{a}{\frac{b}{c}} = a/b$$

Expression a/bc can be confusing. If the writer means $a/(bc)$, it should be written explicitly so. Otherwise it means

$$a/bc = (a/b)c = \frac{a}{b}c = \frac{ac}{b}$$

according to all programming languages.

Exponents

Products of several equal numbers can be represented by powers of these numbers, such as

$$\underbrace{a \times a \times \dots \times a \times a}_{b \text{ times}} = a^b$$

Here b is the exponent and a is the base. Bases and exponents can be, in fact, any numbers, not necessary natural. In particular, negative exponents are used for reciprocals, such as

$$a^{-b} = \frac{1}{a^b}$$

and fractional exponents represent roots

$$a^{1/2} = \sqrt{a}$$

Properties of powers:

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^1 = a, \quad a^0 = 1$$

Examples:

$$\frac{3 \times 5 \times 3 \times 5 \times 3}{5 \times 3 \times 5 \times 5} = 3^{3-1} 5^{2-3} = 3^2 5^{-1}$$

$$(\sqrt{2})^2 = (2^{1/2})^2 = 2^{1/2 \times 2} = 2^1 = 2$$

$$\sqrt{2^2} = (2^2)^{1/2} = 2^{2 \times 1/2} = 2^1 = 2$$

Scientific notation for numbers

In physics one has frequently to deal with very large and very small numbers. The best way to write these numbers is using the scientific notation. For instance, the scientific notation for 12345.678 is 1.2345678×10^4 . The exponent 4 shows that we have moved the decimal point by 4 positions to the left. Here one sees more clearly how large the number is, its order of magnitude is 10^4 . For this reason, the factor in front (the so-called simple part) should be kept maximally close to 1. The number 9123.456 is better to write as 0.9123456×10^4 than as 9.123456×10^3 because the order of magnitude of this number is 4 and not 3. Similarly the number 0.000001234 is written in the scientific notation as 1.234×10^{-6} , where the exponent -6 shows that we have moved the decimal point by six positions to the right. Remember that negative powers describe reciprocals, so that in this case we divide 1.234 by 10 six times.

Example: The mass of electron m_e is approximately 0.911×10^{-30} kg, difficult to write in the usual notation!

When several numbers are multiplied or divided, one can operate the simple parts and powers of 10 independently:

$$\frac{1.2 \times 10^5 \times 3.4 \times 10^7 \times 0.68 \times 10^{-21}}{0.56 \times 10^{12} \times 4.4 \times 10^{-30}} = \frac{1.2 \times 3.4 \times 0.68}{0.56 \times 4.4} 10^{5+7-21-12+30} = 1.13 \times 10^9$$

For an order-of-magnitude estimation, you can simplify the task and drop all simple parts, that yields 10^9 in the example above.

Algebraic equations

Solving physical problems usually involves solving algebraic equations and systems of equations. In most cases these equations are linear.

An example of a linear equation (usually we use a, b, c, \dots for knowns and x, y, z, \dots for unknowns):

$$ax + b = c$$

Equations remain valid if the same quantity is added or subtracted to their right-hand side (rhs) and left-hand side (lhs) and if both rhs and lhs are multiplied or divided by the same quantity. This can be used to isolate unknowns in one of the sides of the equation, that is to solve the equation. For the equation above it is done in the following way:

$$ax + b = c \quad \Rightarrow \quad ax + b - b = c - b \quad \Rightarrow \quad ax = c - b \quad \Rightarrow \quad x = \frac{c - b}{a}$$

(Frame your final result!)

Solving physical problems, we use standard notations adopted in physics rather than just a, b, c and x, y, z . One should understand which quantities are known and which are unknown. If, for instance, acceleration a has to be found from Newton's second law $F = ma$, then we consider a as the unknown and solve the algebraic equation as follows:

$$F = ma \quad \Rightarrow \quad a = \frac{F}{m}$$

The framed expression is our final symbolic, or so-called analytical, result. We now plug numerical values for F and m into it and obtain our final numerical result for a .

Systems of algebraic equations

In many cases one has to solve systems of linear algebraic equations such as

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

where x and y are unknowns. One of different ways to do it is, say, to

- (i) find y from the first equation;
- (ii) plug the result into the second equation;
- (iii) solve the second equation for x ;
- (iv) plug the result for x into the expression for y obtained in (i)

that is

$$a_1x + b_1y = c_1 \Rightarrow y = \frac{c_1 - a_1x}{b_1}$$

$$a_2x + b_2y = c_2 \Rightarrow a_2x + b_2 \frac{c_1 - a_1x}{b_1} = c_2 \Rightarrow \left(a_2 - a_1 \frac{b_2}{b_1} \right) x + c_1 \frac{b_2}{b_1} = c_2$$

$$\Rightarrow x = \frac{\frac{b_2}{b_1} c_2 - c_1 \frac{b_2}{b_1}}{a_2 - a_1 \frac{b_2}{b_1}} = \frac{c_2 b_1 - c_1 b_2}{a_2 b_1 - a_1 b_2} = x$$

(do not forget to simplify your results!)

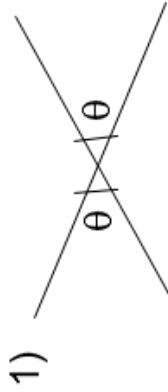
$$y = \frac{c_2 a_1 - c_1 a_2}{b_2 a_1 - b_1 a_2}$$

and then we perform (iv) that after simplification yields

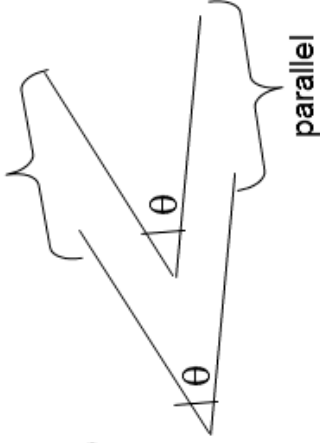
Remember that in a well-behaved system of equations the number of unknowns is equal to the number of equations. You should always perform the count of unknowns and equations before you start with solving a system of equations. 8

Lines, Angles, and Triangles

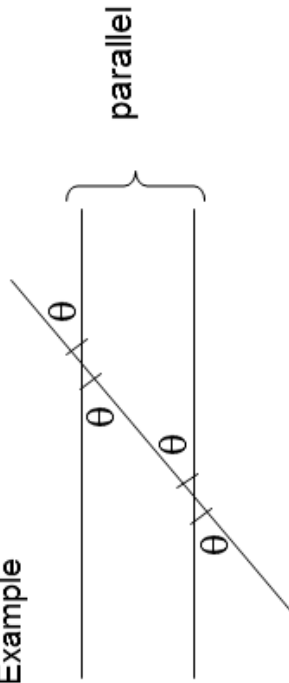
Angles are the same if



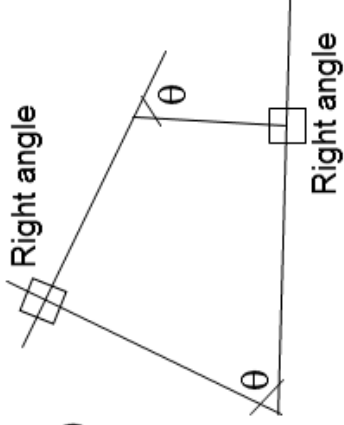
2)



Example

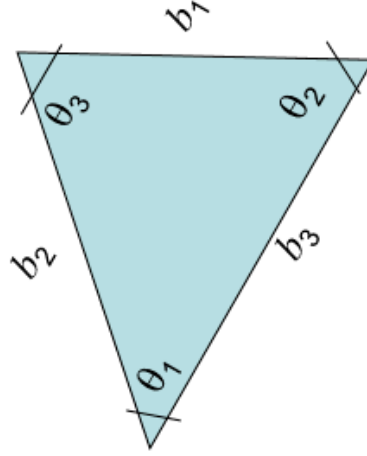
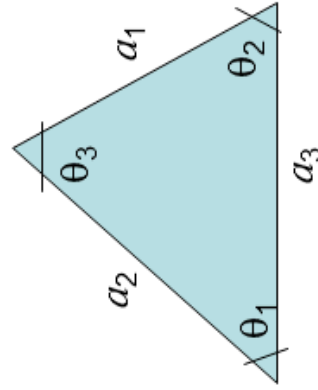


3)



Similar triangles

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$



Angles

Angles are usually denoted by Greek letters such as $\alpha, \beta, \theta, \varphi$, etc.

Angles can be measured in

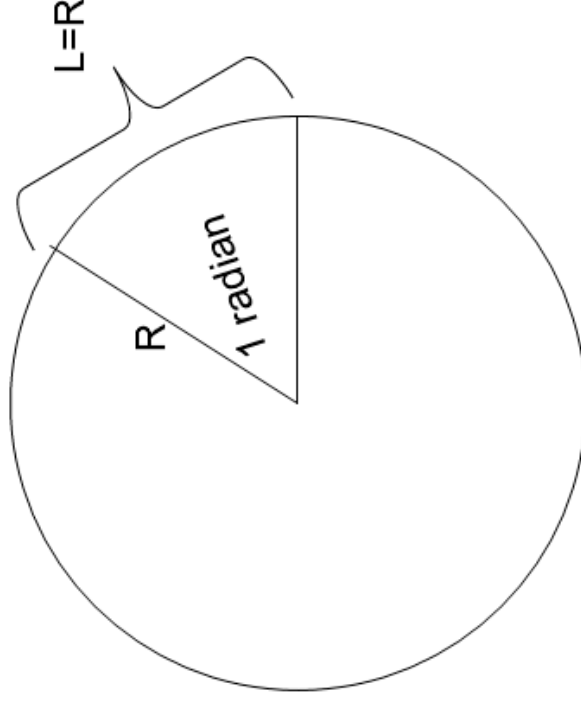
- degrees (seldom used in physics)
- revolutions (360°) (used in engineering)
- radians (mostly used in physics)

Radian is such an angle, for which the length of the arc is equal to the radius. In other words, the angle in radians is given by L/R and it is dimensionless.

Revolution corresponds to $L = 2\pi R$, thus $360^\circ = 2\pi$ radians and

$$1 \text{ radian} = 360^\circ / (2\pi) = 57.3^\circ$$

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ radian}$$

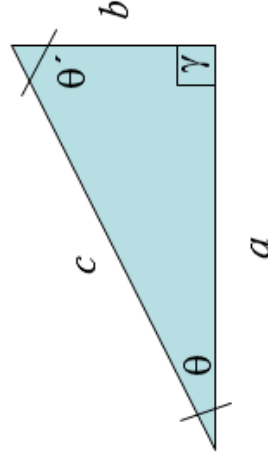


Trigonometric functions

Properties of triangles with right angle ($\gamma = \pi/2$):

$$\theta + \theta' = \pi/2 = 90^\circ$$

$$c^2 = a^2 + b^2 \quad (\text{Pythagoras theorem})$$



For these triangles one defines trigonometric functions as follows:

$$\sin \theta = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}, \quad \cos \theta = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}, \quad \tan \theta = \frac{b}{a} = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{a}{b} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$

sin and cos are called, as a group, sinusoidal functions. They satisfy

$$\sin^2 \theta + \cos^2 \theta = 1$$

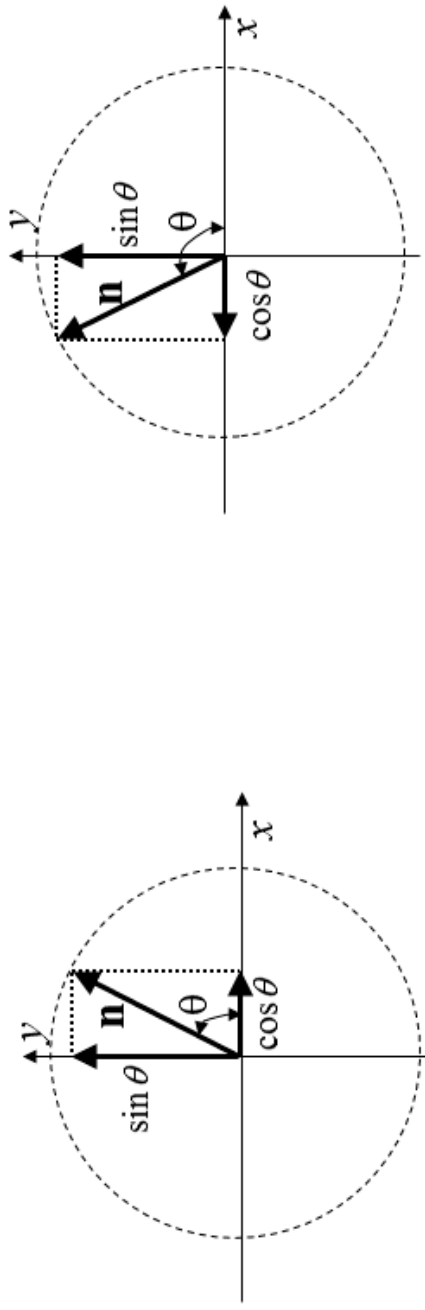
On the other hand, one can define trigonometric functions for the angle θ' of the triangle in the same way:

$$\sin \theta' = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}, \quad \cos \theta' = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}, \quad \tan \theta' = \frac{a}{b}, \quad \cot \theta' = \frac{b}{a}$$

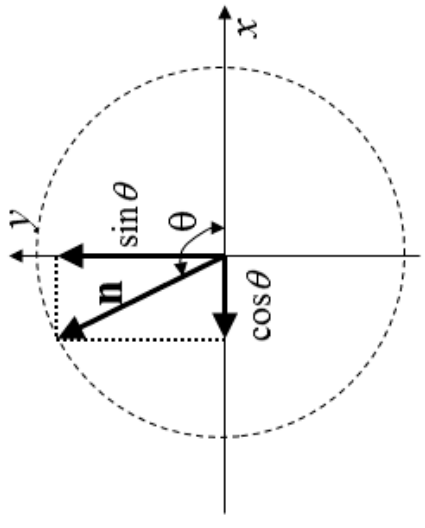
Comparison with the above yields

$$\sin \theta' = \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta, \quad \cos \theta' = \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta, \quad \tan \theta' = \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta, \quad \dots \quad 11$$

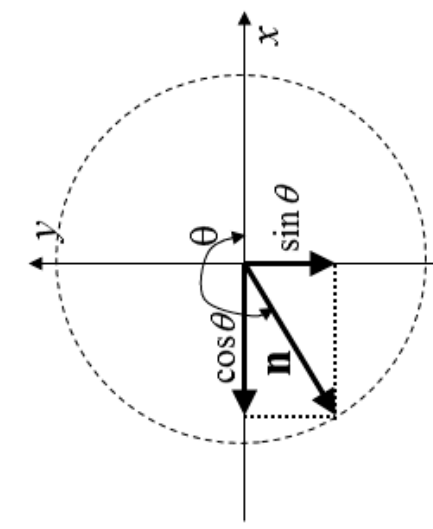
Above we have defined trigonometric functions with the help of triangles, so that θ is limited to the interval $0 \leq \theta \leq \pi/2$. One can, however, define trigonometric functions for arbitrary arguments with the help of components of a two-dimensional unit vector \mathbf{n} ($|\mathbf{n}| = 1$) as follows



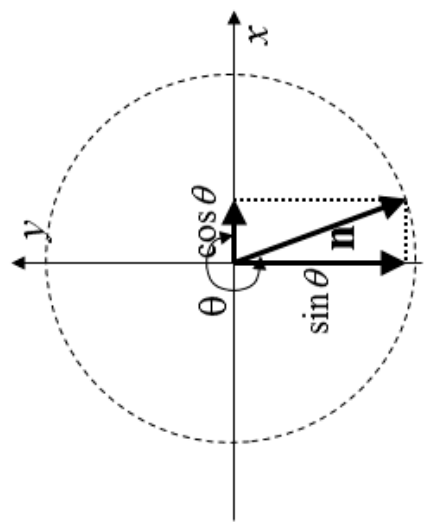
$$\cos \theta > 0, \sin \theta > 0$$



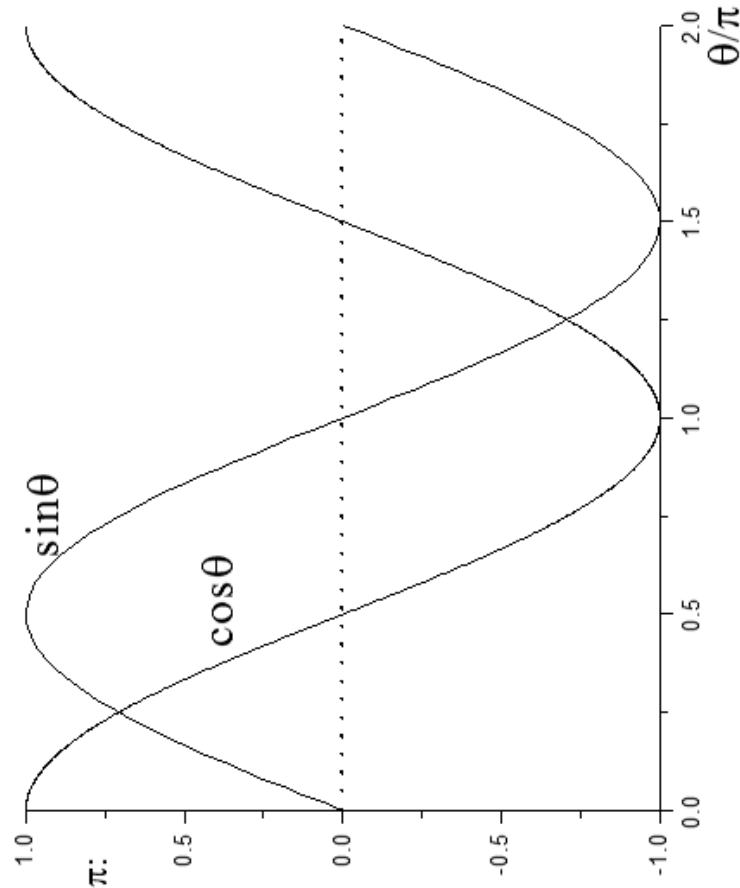
$$\cos \theta < 0, \sin \theta > 0$$



$$\cos \theta < 0, \sin \theta < 0$$



$$\cos \theta > 0, \sin \theta < 0$$



Trigonometric functions are periodic with period 2π :

\sin and \cos differ by an argument shift and there are a lot of corresponding relations

Symmetry:

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

Inverse trigonometric functions

$$\sin \theta = x \Rightarrow \theta = \arcsin x$$

$$\cos \theta = x \Rightarrow \theta = \arccos x$$

$$\tan \theta = x \Rightarrow \theta = \arctan x$$

$$\cot \theta = x \Rightarrow \theta = \text{arccot } x$$

Scalars and Vectors

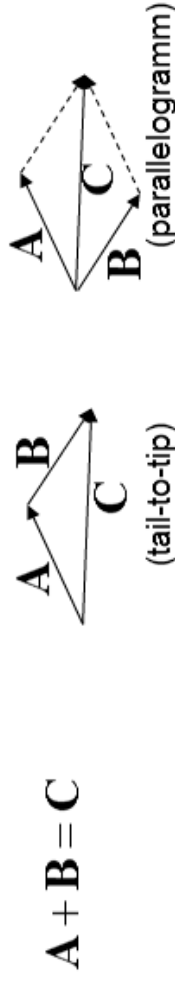
Most of the physical quantities are scalars or vectors.

Scalars are objects that are represented by numbers, such as time t , mass m , electric charge q or Q , temperature T or T . Some scalar quantities are always positive or nonnegative, such as mass m , volume V , kinetic energy E_k , absolute temperature T , etc. Most of scalar quantities can be either positive or negative, such as electric charge q or Q , time t , etc.

Vectors are mathematical and/or physical objects that are characterized by (i) their magnitude or absolute value or length and (ii) their direction in space. Many physical quantities are vectors, such as position, velocity, force, electric and magnetic fields, etc. Vectors can be added, subtracted, and multiplied. Vectors can be divided by a scalar but one cannot divide by vector. Vectors are denoted by symbols with overhead arrows (\vec{A}) in handwritten texts and by boldface symbols (\mathbf{A}) in printed texts.

Magnitude (length) of a vector A is denoted as $|A|$ or simply as A . Vectors of unit length, $|A| = 1$, describe directions. Each vector can be represented in the form $A = A\mathbf{n}$, where $A > 0$ and \mathbf{n} is a unit vector directed along A .

Addition of vectors can be done graphically with the help of either the tail-to-tip rule or the parallelogram rule



Subtraction of vectors:

$$A - B = C$$



because $A = B + C$

Vectors and Coordinate Systems

To perform operations on vectors numerically, it is convenient to introduce a coordinate system. The latter is defined by the origin O and three mutually perpendicular axes x , y , and z . The tail of the vector \mathbf{A} is in the origin of the coordinate system. We project the vector \mathbf{A} onto the axes of the coordinate system by drawing the three lines from its tip towards all three axes perpendicularly to the latter. As the result, \mathbf{A} is represented as the sum of three vectors:

$$\mathbf{A} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z$$

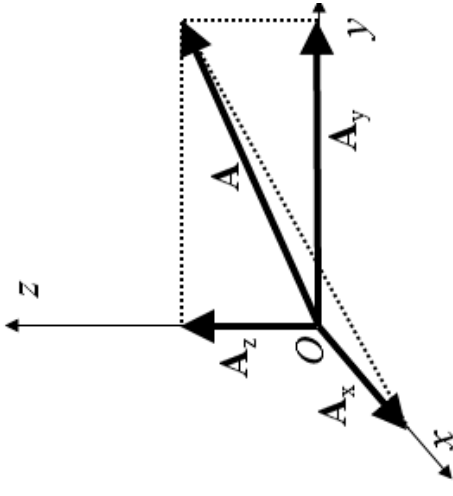
Here we have introduced the unit vectors \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z , ($|\mathbf{e}_x| = 1$ etc.) that are directed along different axes. The scalar quantities A_x , A_y , and A_z are components of the vector \mathbf{A} in this coordinate system or its projections on the axes of this coordinate system. Note that components of a vector can be both positive and negative.

With the above definitions, one obtains many useful formulas. Addition of vectors can be done as

$$\mathbf{A} + \mathbf{B} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z + B_x \mathbf{e}_x + B_y \mathbf{e}_y + B_z \mathbf{e}_z = (A_x + B_x) \mathbf{e}_x + (A_y + B_y) \mathbf{e}_y + (A_z + B_z) \mathbf{e}_z$$

$$\mathbf{A} + \mathbf{B} = \mathbf{C} = C_x \mathbf{e}_x + C_y \mathbf{e}_y + C_z \mathbf{e}_z \Rightarrow C_x = A_x + B_x, \quad C_y = A_y + B_y, \quad C_z = A_z + B_z$$

That is, to add vectors, one has just to add their components, and similar for subtraction.



Multiplication or division of a vector by a positive scalar changes its length but does not change its direction. If \mathbf{A} is a vector and $\varphi > 0$ is a scalar, then $\mathbf{B} = \varphi \mathbf{A} = \varphi A \mathbf{n} = B \mathbf{n}$, that is, $B = \varphi A$. Multiplication of a vector by a negative scalar additionally inverts its direction. In components one obtains

$$\mathbf{B} = \varphi \mathbf{A} = \varphi A_x \mathbf{e}_x + \varphi A_y \mathbf{e}_y + \varphi A_z \mathbf{e}_z = B_x \mathbf{e}_x + B_y \mathbf{e}_y + B_z \mathbf{e}_z$$

thus one has simply to multiply components of the vector by the scalar: $B_x = \varphi A_x$, etc., for both signs of φ .

The length of a vector can be obtained in components from the Pythagoras theorem:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Using trigonometric functions, one can express components (projections) of a vector as

$$A_x = A \cos \alpha, \quad A_y = A \cos \beta, \quad A_z = A \cos \gamma$$

where α , β , and γ are the angles between vector \mathbf{A} and the axes x , y , and z , respectively. In particular, for a vector that is confined to a plane (that is, has only two components) one has

$$A_x = A \cos \theta, \quad A_y = A \sin \theta$$

