Prof. Anchordoqui

Problems set # 8 **Physics 411** October 25, 2007

1. A particle moves in an elliptical orbit in an inverse-square-law central-force field. If the ratio of the maximum angular velocity to the minimum angular velocity of the particle in its orbit is n, then show that the eccentricity of the orbit is

$$e = \frac{\sqrt{n}-1}{\sqrt{n}+1}.$$

2. Use Kepler's results to show that the gravitational force must be central and that the radial dependence must be $1/r^2$. Thus, perform an inductive derivation of the gravitational force law.

3. For a particle moving in an eliptical orbit with semimajor axis a and eccentricity e show that

$$\langle (a/r)^4 \,|\cos\theta|\rangle = e/(1-e^2)^{5/2},$$

where the slanted brackets denote a time average over one complete period.

4. An Earth satellite moves in an elliptical orbit with period τ, eccentricity e, and semi-major axis a. Show that the maximum radial velocity of the satellite is $2\pi ae/(\tau\sqrt{1-e^2})$.

5. An Earth satellite has a perigee of 300 km and an apogee of 3,500 km above the Earth surface. How far is the satellite above the Earth when (a) it has rotated 90° around the Earth from perigee and (b) it has moved halfway from perigee to apogee?

6. An Earth satellite has speed of 28,070 km/h when it is at its perigee of 220 km above the Earth's surface. Find the apogee distance, its speed at apogee, and its period of revolution.

7. Show that the most efficient way to change the energy of an eliptical orbit for a single short engine thrust is by firing the rocket along the direction of travel at perigee.

8. A spacecraft in an orbit about Earth has the speed of 10,160 m/s at perigee of 6,680 km above the Earth surface. What speed does the spacecraft has at apogee of 42,200 km?

9. What is the minimum scape velocity of a spacecraft from the moon?

10. The minimum and maximum velocities of a moon rotating around Uranus are $v_{\rm min} = v - v_0$ and $v_{\rm max} = v + v_0$. Find the eccentricity in terms of v and v_0.