PROBLEMS # 7

(1) Using Hohmann transfers equations

$$\Delta v = \Delta v_1 + \Delta v_2$$

$$\Delta v = v_{t_1} - v_1 + v_2 - v_{t_2}$$

$$\Delta v_1 = \sqrt{\frac{2GM_\oplus}{r_1}\left(\frac{r_2}{r_1+r_2}\right)} - \sqrt{\frac{GM_\oplus}{r_1}}$$

$$\Delta v_2 = \sqrt{\frac{GM_\oplus}{r_2}} - \sqrt{\frac{2GM_\oplus}{r_2}\left(\frac{r_1}{r_1+r_2}\right)}$$

Substituting $GM_\oplus = 6.67\times10^{-11}\,Nm^2/kg^2\ 5.98\times10^{24}\,kg$

r_1 = initial height above the center of the earth $\Rightarrow$ $r_1 = 2R_\oplus$

r_2 = final height above the center of the Earth $\Rightarrow$ $r_2 = 3R_\oplus$

$R_\oplus = 6.37\times10^6\,m \Rightarrow \Delta v = 1020\,m/s$

(2)

$GM_{\odot} = 6.67 \times 10^{-11}\ Nm^2/kg^2\ \ 1.99 \times 10^{30}\ kg$

r_1 = mean Earth sun distance $\sim 1.5 \times 10^{11}$ m

r_2 = mean Venus sun distance $\sim 1.08 \times 10^{11}$ m

$\Delta v = \Delta v_1 + \Delta v_2$

where Δv_1 and Δv_2 are as in previous problem

$\Delta v = -5275$ m/s. The answer is negative because $r_2 < r_1$, so the rocket must be fired in the direction opposite to the motion (the satellite must be slowed down).

$\Delta v = 5275$ m/s, opposite to direction of motion

The time is

$$T = \pi \sqrt{\frac{1}{GM_\odot}}\, a_t^{3/2} = \pi \sqrt{\frac{1}{GM_\odot}} \left(\frac{r_1 + r_2}{2}\right)^{3/2}$$

$T = 147$ days

(3) For a flyby we must calculate the quantity Δv_1 for transfers to Venus and Mars.

$$\Delta v_1 = v_{t_1} - v_1$$

$$= \sqrt{\frac{2GM_\odot}{r_1}\left(\frac{r_2}{r_1 + r_2}\right)} - \sqrt{\frac{GM_\odot}{r_1}}$$

r_1 = mean Earth-Sun distance $= 150 \times 10^9$ m

r_2 = mean $\begin{pmatrix} \text{Venus} \\ \text{Mars} \end{pmatrix}$ -Sun distance $= \begin{pmatrix} 108 \\ 228 \end{pmatrix} \times 10^9$ m

Substituting gives

$\Delta v_{Venus} = -2.53$ km/s

$\Delta v_{Mars} = 2.92$ km/s

where the negative sign for Venus means the velocity kick is opposite to the earth's orbital motion

Thus, a Mars flyby requires a larger Δv than a Venus flyby.

(4) We use Hohmann transfers

$$\Delta v = \Delta v_1 + \Delta v_2$$

$$= v_{t_1} - v_1 + v_2 - v_{t_2}$$

where

$$v_{t_1} = \sqrt{2GM_\oplus \left(\frac{r_2}{r_1 + r_2}\right)^{1}}$$

$$v_1 = \sqrt{\frac{GM_\oplus}{r_2}}$$

Substituting

$GM_\oplus = 6.67 \times 10^{-11}\ Nm^2/kg^2\ 5.98 \times 10^{24}\ kg$

$r_1 = 200\,km + r_\oplus = 6.37 \times 10^6\ m + 2 \times 10^5\ m$

$r_2 =$ mean Earth-Moon distance $= 3.84 \times 10^8\ m$

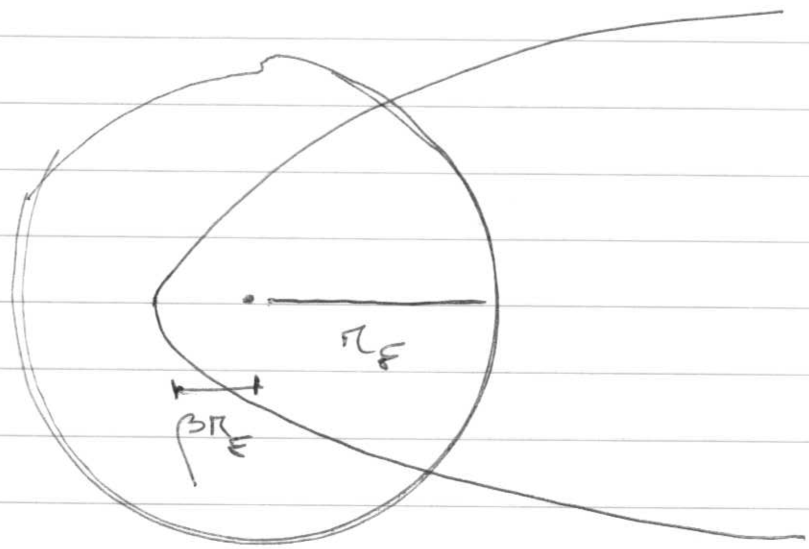
gives

$$\Delta v = 3966\ m/s$$

The time of transfer is

$$T = 429{,}000\ s \simeq 5\ \text{days}$$

(5)



The total energy per unit mass of an object around the sun is

$$\mathcal{E} = \frac{\dot{r}^2 + r^2\dot{\theta}^2}{2} - \frac{GM}{r} \qquad (1)$$

The orbit of the comet is parabolic $e=1$, so the equation of the orbit is

$$\frac{r_c}{r} = 1 - \cos\theta \qquad (2)$$

$$r(\theta = \pi) = \beta r_E \qquad (3)$$

$$r_c = \frac{h^2}{GM} = 2\beta r_E \qquad (4)$$

Since the orbit is parabolic, the total energy is zero,

$$\dot{r}^2 = \frac{2GM}{r} - r^2\dot{\theta}^2 \quad (5)$$

$h = r^2\dot{\theta} \Rightarrow$

$$\dot{r}^2 = \frac{2GM}{r} - \frac{h^2}{r^2} \quad (6)$$

From (4)

$h^2 = 2\beta r_E GM$

Replacing into (6)

$$\dot{r}^2 = 2GM\left(\frac{1}{r} - \frac{\beta r_E}{r^2}\right) \quad (7)$$

$$\dot{r}^2 = 2GM\left(\frac{r - \beta r_E}{r^2}\right)$$

$$dt = \frac{1}{\sqrt{2GM}} \frac{r}{\sqrt{r-\beta r_E}} dr \qquad (8)$$

$$T = 2 \int_{\beta r_E}^{r_E} \frac{1}{\sqrt{2GM}} \frac{r}{\sqrt{r-\beta r_E}} dr$$

$$= \sqrt{\frac{2}{GM}} \int_{\beta r_E}^{r_E} \frac{r}{\sqrt{r-\beta r_E}}$$

$$= \sqrt{\frac{2}{GM}} \left[-\frac{2\left(-2\beta r_E - r\right)}{3} \sqrt{r-\beta r_E} \right]_{\beta r_E}^{r_E} \qquad (9)$$

from which

$$T = \sqrt{\frac{2}{GM}} \left[\frac{2}{3} r_E^{3/2} \left(2\beta+1\right) \sqrt{1-\beta} \right] \qquad (10)$$

Now the period and the radius of the Earth are related by

$$T_E^2 = \frac{4\pi^2}{GM} r_E^3$$

$$\Rightarrow \quad r_E^{3/2} = \frac{\sqrt{GM}}{2\pi} T_E \qquad (11)$$

Substituting (11) into (10)

$$T = \sqrt{\frac{2}{GM}} \left[\frac{2}{3} \frac{\sqrt{GM}}{2\pi} T_E (2\beta+1) \sqrt{1-\beta} \right]$$

$$= \frac{1}{3\pi} \sqrt{2(1-\beta)} \, (1+2\beta) \, T_E$$

$T_E = 1\,yr$. Now, $\beta = r_{MERCURY}/r_E = 0.387$

$$\Rightarrow \quad T = \frac{1}{3\pi} \sqrt{2(1-0.387)} \, (1+2\times 0.387) \times 365\,\text{days}$$

$$\Rightarrow \quad T = 76 \text{ days.}$$