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Problems set # 6 **Physics 411** October 18, 2007

1. If the field vector is independent of the radial distance within a sphere, find the function describing the density $\rho = \rho(r)$ of the sphere.

2. Assuming that air resistance is unimportant, calculate the minimum velocity a particle must have at the surface of the Earth to escape from the Earth gravitational field. Obtain a numerical value for the result. (This velocity is called the scape velocity).

3. A particle is attracted toward a center of force according to the relation $F = -mk^2/x^3$. Show that the time required for the particle to reach the force center from a distance d is d^2/k.

4. A particle falls to the Earth starting from rest at a great height. Neglect air resistance and show that the particle requires approximately 9/11 of the total time of fall to traverse the first half of the distance.

5. Compute directly the gravitational force on a unit mass at a point exterior to a homogeneous sphere of matter.

6. Calcualte the gravitational potential due to a thin rod of length l and mass M at a distance R from the center of the rod and in the direction perpendicular to the rod.

7. Calculate the gravitational field vector due to a homogeneous cylinder at exterior points on the axis of the cylinder.

8. Calculate the potential due to a thin circular ring of radius a and mass M for points lying in the plane of the ring and exterior to it. The result can be expressed as an eliptical integral. Assume that the distance from the center of the ring to the field point is large compared with the radius of the ring. Expand the expression for the potential and find the first correction term.

9. A planet of density ρ_1 (spherical core radius R_1) with a thick spherical cloud of dust (density ρ_2 and radius R_2) is discovered. What is the force on a particle of mass m placed within the dust cloud?

10. A particle is dropped into a hole drilled straight through the center of the Earth. Neglecting rotational effects, show that the particle's motion is simple harmonic if you assume the Earth has uniform density. Show that the period of oscillation is about 84 min.

11. Show that the ratio of maximum tidal heights due to Moon and Sun is given by

$$\frac{M_{\rm Moon}}{M_\odot} \left(\frac{R}{D}\right)^3$$

and that this value is 2.2. Here R is the distance between the Sun and the Earth, D is the distance to the Moon, $M_\odot$ is the Sun mass and $M_{\rm Moon}$ is the mass of the Moon.

12. The orbital revolution of the Moon about the Earth takes about 27.3 days and is in the same direction that the Earth rotation (24 h). Use that information to show that high tides occur everywhere on Earth every 12 h and 26 min.

13. Determine how much greater the gravitational field strength g is at the pole than at the equator. Assume a spherical Earth. If actual difference is $\Delta g = 52$ mm/s^2, explain the difference. How might you calculate this difference between the measured result and your calculation.