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Problems set # 4 **Physics 411** October 4, 2007

1. Consider a simple harmonic oscillator. Calculate the time averages of the kinetic and potential energies over one cycle and show that these quantities are equal.

2. A body of uniform cross sectional area $A = 1$ cm^2 and of mass density $\rho = 0.8$ g/cm^3 floates in a liquid of density $\rho = 1$ g/cm^3 and at equilibrium displaces a volume $V = 0.8$ cm^3. Show that the period of small oscillations about the equilibrium point is given by $\tau = 2\pi\sqrt{V/gA}$, where g is the gravitational field strength. Determine the value of τ.

3. A simple pendulum consists of a mass m suspended from a fixed point by a weightless, extensionless rod of length l. Obtain the equation of motion and, in the approximation that $\sin\theta = \theta$, show that the natural frequency is $\omega_0 = \sqrt{g/l}$, where g is the gravitational field strength. Discuss the motion in the event that the motion takes place in a viscous medium with retarding force $2m\sqrt{gl}\dot{\theta}$.

4. A particle of mass m is at rest at the end of a spring (force constant $= k$). At $t = 0$ a constant force is applied to the mass and acts for a time t_0. Show that after the force is removed, the displacement of the mass from its equilibrium position $x = x_0$, is: $x - x_0 = F/k\,[\cos\omega_0(t - t_0) - \cos\omega_0 t]$, where $\omega_0^2 = k/m$. Neglect friction effects!

5. If the amplitude of a damped oscillator decreases to $1/e$ of its initial value after n periods, show that the frequency of the oscillator must be approximately $[1-(8\pi^2n^2)^{-1}]$ times the frequency of the corresponding undamped oscillator.

6. For a lightly damped oscillator, show that $Q \approx \omega_0/\Delta\omega$, where $\Delta\omega$ represents the frequency interval between the points on the amplitude resonance curve that are $1/\sqrt{2}$ of the maximum amplitude.

7. An electrical circuit consists of a resitor R a capacitor C conected in series to a source of alternating emf. Find the expression for the current as a function of time and show that it decreases to zero as the frequency of the alternating emf approaches zero.