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Problems set # 4 **Physics 303** September 30, 2014

1. N identical, ideal Carnot refrigerators are in series. In each cycle, the n-th refrigerator absorbs heat Q_n at temperature T_n, and emits heat Q_{n+1} at temperature T_{n+1}. The emitted heat Q_{n+1} is then absorbed by the $(n+1)$-th refrigerator, and so on. Each refrigerator requires 3 J of work to run, per cycle. You are given that $Q_1 = 1_{\rm J}$, and $T_1 = 1$ K. Express the answers below in the appropriate units (J or K). *(i)* What is Q_n, for all $n \geq 1$? *(ii)* What is T_n, for all $n \geq 1$?

2. A Carnot engine operates on 1 kg of methane (CH_4), which we will consider to be an ideal gas. Take $\gamma = 1.35$. The ratio of the maximum volume to the minimum volume is 4 and the cycle efficiency is 25%. Find the entropy increase of the methane during the isothermal expansion.

3. The low temperature specific heat of a diamond varies with temperature according to

$$c_v = 1.88 \times 10^6 \left(\frac{T}{\theta}\right)^3 \ \mathrm{J\,kilomole^{-1}\,K^{-1}},$$

where $\theta = 2230$ K. What is the entropy change of 1 kg of diamond when it is heated at constant volume from 4 K to 300 K.

4. *(a)* Derive an expression for the entropy of an ideal gas: *(i)* as a function of T and V; *(ii)* as a function of T and P. Assume that the specific heats of the gas are constants. *(b)* An ideal monatomic gas undergoes a reversible expansion from specific volume v_1 to specific volume v_2. *(i)* Calculate the change in specific entropy Δs if the expansion is isobaric. *(ii)* Calculate Δs if the process is isothermal. *(iii)* Which is larger? By how much?

5. A kilomole of an ideal gas undergoes a revrsible isothermal expansion from a volume of 5 liters to a volume of 10 liters at a temperature of 20° C. *(i)* What is the change in the entropy of the gas? Of the universe? *(ii)* What are the corresponding changes of energy if the process is a free expansion?