

1. Derive the density of states  $\rho(\varepsilon)$  as a function of  $\varepsilon$  for a free electron gas in one-dimension. (Assume periodic boundary conditions or confine the linear chain to some length  $L$ ). Then calculate the Fermi energy  $\varepsilon_F$  at zero temperature for an  $N$  electron system.
2. Calculate the average energy per particle,  $\varepsilon$ , for a Fermi gas at  $T = 0$ , given that  $\varepsilon_F$  is the Fermi energy. Consider two cases separately, non-relativistic and relativistic.
3. For a system of electrons, assumed non-interacting, show that the probability of finding an electron in a state with energy  $\Delta$  above the chemical potential  $\mu$  is the same as the probability of finding an electron absent from a state with energy  $\Delta$  below  $\mu$  at any given temperature  $T$ .