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Problems set # 12 **Physics 303** November 25 and December 2, 2014

1. A spherical black body of radius r at absolute temperature T is surrounded by a thin spherical and concentric shell of radius R, black on both sides. Show that the factor by which this radiation shield reduces the rate of cooling of the body (consider space between spheres evacuated, with no thermal conduction losses) is given by the following expression $aR^2/(R^2 + br^2)$, and find the numerical coefficients a and b.

2. The solar constant (radiant flux at the surface of the earth) is about 0.1 W/cm^2. Find the temperature of the sun assuming that it is a black body.

3. Estimate the temperature of the sun's surface given that the sun subtends an angle θ as seen from the earth's and the earth's surface temperature is T_0. (Assume the earth's surface temperature is uniform, and that the earth reflects a fraction, ε of the solar radiation incident upon it.) Use your result to obtain a rough estimate of the sun's surface temperature by putting in "reasonable" values for all parameters.

4. Consider an idealized sun and earth, both black bodies, in otherwise empty space. The sun is at a temperature of $T_\oplus = 6000$ K and heat transfer by oceans and atmosphere on the earth is so effective as to keep the earth surface uniform. The radius of the earth is $R_\oplus = 6 \times 10^8$ cm, the radius of the sun is $R_\odot = 7 \times 10^{10}$ cm, and the earth radius distance is $d_{\oplus-\odot} = 1.5 \times 10^{13}$ cm. *(i)* Find the temperature of the earth. *(ii)* Find the radiation force on the earth. *(iii)* Compare these results with those for an interplanetary "chondrule" in the form of a spherical, perfectly conducting black-body with a radius of $R = 0.1$ cm, moving in a circular orbit around the sun with a radius equal to the earth-sun distance $d_{\oplus-\odot}$.

5. Making reasonable assumptions, estimate the surface temperature of Neptune. Neglect any possible internal source of heat. What assumptions have you made about the planet's surface and/or atmosphere? [*Hint:* Astronomical data which may be helpful: radius of sun = 7×10^5 km; radius of Neptune = 2.2×10^4 km; mean sun-earth distance = 1.5×10^8 km; mean sun-Neptune distance = 4.5×10^9 km; temperature of the sun = 6000 K; rate at which sun's radiation reaches earth = 1.4 kW/m^2; Stefan-Boltzmann constant = 5.7×10^{-8} W/m^{2}K^4.]

6. Consider a photon gas enclosed in a volume V and in equilibrium at temperature T. The photon is a massless particle, so that $\varepsilon = pc$. *(i)* What is the chemical potential of the gas? *(ii)* Determine how the number of photons in the volume depends upon the temperature. *(iii)* One may write the energy density in the form

$$\frac{\overline{U}}{V} = \int_0^\infty \rho(\omega) d\omega.$$

Determine the form of $\rho(\omega)$, the spectral density of the energy. *(iv)* What is the temperature dependence of the energy $\overline{U}$?

7. Consider a gas of non-interacting, non-relativistic, identical bosons. Explain whether and why the Bose-Einstein condensation effect that applies to a three-dimensional gas applies also to a two -dimensional gas and to a one-dimensional gas.

8. The universe is pervaded by 3K black body radiation. In a simple view, this radiation arose from the adiabatic expansion of a much hotter photon cloud which was produced during the big bang. *(i)* Why is the recent expansion adiabatic rather than, for example, isothermal? *(ii)* Write down an integral which determines how many photons per cubic centimeter are contained in this cloud of radiation. Estimate the result within an order of magnitude. *(iii)* Show that a freely expanding blackbody radiation remains described by the Planck formula, but with a temperature that drops in proportion to the scale expansion. *(iv)* If in the next 10^{10} yr the volume of the universe increases by a factor of 2, what then will be the temperature of the blackbody radiation.