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Problems set # 11 **Physics 303** November 18, 2014

1. The partition function of classical particles in 3 dimensions is dened as

$$Z_{\text{class}} = \int d^3p \int d^3r \ \exp[-\beta E(\boldsymbol{p}, \boldsymbol{r})],$$

where $E(\boldsymbol{p}, \boldsymbol{r})$ is the particles energy. Note that this expression has the unit of (momentum $\times$ distance)^3, unlike the quantum partition function that is dimensionless. Define the density of states of a free classical particle in a box of volume V. By comparing it with the density of states for a quantum particle in a rigid box, find the missing factor in Z_{class} that would make the classical partition function match the quantum one. This will define a quantum-mechanical "cell" in the phase space of a classical particle. Show that this quantum-mechanical aspect does not contribute into the internal energy and heat capacity of the classical particles.

2. Using the distribution function

$$f(\boldsymbol{p}, \boldsymbol{r}) = \frac{1}{Z_{\text{class}}} \ \exp[-\beta E(\boldsymbol{p}, \boldsymbol{r})]$$

for classical particles with gravity, find the dependence of particle's concentration n and pressure P as the function of the height. Set the minimal height (the earth level) to zero. Calculate the heat capacity of this system and compare it with the one for free particles.

3. Consider two interacting Ising spins, *i.e.* a model of two coupled spins with the Hamiltonian

$$\hat{H} = -g\mu_B B(S_{1,z} + S_{2,z}) - J S_{1,z} S_{2,z},$$

where B is the external magnetic field and J is the so-called exchange interaction, ferromagnetic for $J > 0$ and antiferromagnetic for $J < 0$. The energy levels of this system are given by

$$\varepsilon_{m_1 m_2} = -g\mu_B B(m_1 + m_2) - J m_1 m_2,$$

where the quantum numbers take the values $-S \leq m_1, m_2 \leq S$. Write down the expression for the partition function of the system. Can it be calculated analytically for a general S? If not, perform the calculation for $S = 1/2$ only. Calculate the internal energy, heat capacity, magnetization induced by the magnetic field, and the magnetic susceptibility (in the zero field limit). Analyze ferro- and antiferromagnetic cases.

4. Consider classical particles with the potential energy $V(\boldsymbol{r}) = kr^2/2$ in 3 dimensions. Calculate the partition function, internal energy and heat capacity.