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Problems set # 10 **Physics 303** November 11, 2014

1. A model thermodynamic system in which the allowed non-degenerate states have energies $0, \varepsilon, 2\varepsilon, 3\varepsilon, \cdots$ consists of four particles with total energy $U = 6\varepsilon$. Identify the possible distribution of particles, evaluate $\Omega = \sum_k \omega_k$ and work out the average occupation numbers for the various energy levels. *(i)* When the particles are gaseous bosons; *(ii)* when the particles are gaseous fermions.

2. Consider a system of N distinguishable particles, at temperature T, with available energy levels ε_1 and $\varepsilon_2 > \varepsilon_1$ (only two available non-degenerate energy levels). *(i)* Determine the equilibrium values of the occupation numbers N_1 and N_2, and the energy U of the system, as a function of temperature. *(ii)* Show that the specific heat is given by

$$C_V = Nk \left(\frac{\Delta}{kT}\right)^2 \frac{e^{-\Delta/kT}}{(1 + e^{-\Delta/kT})^2},$$

where $\Delta = \varepsilon_2 - \varepsilon_1$. Examine the low temperature and high temperature behavior of C_V/Nk, and sketch it as a function of kT/Δ.

3. Show that for a system of a large number N of bosons at very low temperatures (such that they are all in the non-degenerate lowest energy state $\varepsilon = 0$) the chemical potential varies with temperature according to

$$\lim_{T \to 0} \mu = -\frac{kT}{N}.$$

4. *(i)* For a system of localized distingushable oscillators, Boltzmann statistics applies. Show that the entropy S is given by

$$S = -k \sum_J N_j \ln \left(\frac{N_j}{N}\right).$$

(ii) Substitute the Boltmann distribution in the previous result to show that

$$S = \frac{U}{T} + Nk \ln Z.$$

(iii) Show that

$$S = Nk \left[\frac{\theta/T}{e^{\theta/T} - 1} - \ln(1 - e^{-\theta/T})\right],$$

where $\theta = \hbar\omega/k$. Examine the behavior of S as $T \to 0$ and $T \to \infty$. [*Hint:* For part *(i)*, use $\omega = \omega_{\rm MB}$ (keeping the $N!$ in the numerator) and use Stirlings approximation. Note that the result of part *(i)* can be written as $S = -Nk \sum_j f_j \ln f_j$, where $f_j = Nj/N$ is the probability of occupying level j.]

5. The partition function of an Einstein solid is

$$Z = \frac{e^{-\theta_{\rm E}/2T}}{1 - e^{\theta_{\rm E}/T}},$$

where $\theta_{\rm E}$ is the Einstein temperature. Treat the crystaline lattice as an assembly of 3N distinguishable oscillators. *(i)* Calculate the Helmholtz function F. *(ii)* Calculate the entropy S. Show that the entropy approaches zero as the temperature goes to absolute zero. Show that at high temperatures, $S \approx 3Nk[1 + \ln(T/\theta_{\rm E})]$. Sketch $S/3Nk$ as a function of $T/\theta_{\rm E}$.