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Problems set # 1 **Physics 303** September 2, 2014

1. A process on an ideal gas is defined by $P = AT^b$. *(i)* Express this process in terms of (P, V) and (V, T). *(ii)* Calculate compressibility and thermal expansivity in this process. *(iii)* What is the limitation on b? *(iv)* For which values of b this process becomes a known process? *(v)* Find adiabatic values of the two thermodynamic coeficients above.

Solution: *(i)* Use the equation of state of the ideal gas, $PV = nRT$, to obtain: $P = A(\frac{PV}{nR})^b$. This can be represented in the simplified form $P^{1-1/b}V = nR/A^{1/b}$. Alternatively, write $nRT/V = AT^b$, which can be simplified to $T^{b-1}V = nR/A$. *(ii)* The compressibility coeficient is given by $\kappa = -\frac{1}{V}\frac{dV}{dP}$. Use $V \propto P^{-(1-1/b)}$, to find $\kappa = (1-\frac{1}{b})\frac{P^{-(1-1/b)-1}}{V} = (1-\frac{1}{b})\frac{1}{P}$. The thermal expansion coeficient is defined by $\beta = \frac{1}{V}\frac{dV}{dT}$. Use $V \propto T^{-(b-1)}$, to find $\beta = \frac{1-b}{T}$. *(iii)* Since mechanical stability requires $\kappa_T > 0$, the condition on b is $b > 1$ or $b < 0$. *(iv)* For $b \to 0^-$, the process is isobaric; for $b \to 1$, isochoric; for $b \to \pm\infty$ isothermic, and for $b = \gamma/(\gamma-1) > 1$ adiabatic. *(v)* From the latter you can obtain the adiabatic thermodynamic coeficients $\kappa_S = \frac{1}{\gamma P}$ and $\beta_S = -\frac{1}{(\gamma-1)T}$, which can be compared with $\kappa_T = \frac{1}{P}$ and $\beta_P = \frac{1}{T}$. Note that $\beta_S < 0$ because in the adiabatic process the volume decreases and the temperature increases.

2. A process on an ideal gas is defined by $P = AT^2$, with $A =$ constant. Calculate the received work and heat upon changing the temperature from T_1 to T_2. Assume $C_V =$ constant.

Solution: Use the equation of state of the ideal gas to express P in terms of V as $P = \frac{(nR)^2}{AV^2}$ and integrate $W_{12} = \int_1^2 P\,dV = \int_1^2 \frac{(nR)^2}{AV^2}dV = \frac{(nR)^2}{A}(\frac{1}{V_1} - \frac{1}{V_2})$. Then express V via T: $V = \frac{nRT}{P} = \frac{nR}{AT}$, and substitute it into the work, $W_{12} = nR(T_1 - T_2)$. To calculate the heat, use the first law of thermodynamics in the form $U_2 - U_1 = Q_{12} - W_{12}$. Using the internal energy for a perfect gas, $U = C_V T + \text{constant}$, and the result for the work obtained above, you straightforwardly find $Q_{12} = C_V(T_2 - T_1) + W_{12} = (C_V - nR)(T_2 - T_1)$.

3. System A is in equilibrium and has $V_A = 2$ m^3 and $P_A = 0.01$ bar. System B is in equilibrium, and has $V_B = 3$ m^3 and $P_B = 0.02$ bar. Systems A and B are put in thermal contact with each other, and it is found that they are also in thermal equilibrium with each other. Suppose that the densities of each system are very dilute. Moreover, suppose that the gas in each system happens to be $N_2 0$ (nitrous oxide). Throughout in what follows, suppose that the systems remain closed, i.e. they do not leak or exchange gasses. *(i)* Compute the ratio M_A/M_B of the masses of the gas in each container. *(ii)* System A is kept in contact with system B. The volume V_A is slowly changed to $V'_A = 4$ m^3, and the pressure P_A is changed to $P'_A = 0.03$ bar. The volume V_B is unchanged, $V'_B = 3$ m^3. What should the new pressure P'_B be, in order for the systems A and B to remain in thermal equilibrium? *(iii)* Is the temperature of the systems, in their final state of part *(ii)*, hotter or colder than they were in their initial state? Compute the ratio $T_{\text{final}}/T_{\text{initial}}$, where the temperatures are measured in Kelvin.

Solution: *(i)* Use the equation of state of the ideal gas to write $n_i = \frac{P_i V_i}{R T_i}$, which implies $\frac{m_A}{m_B} = \frac{n_A M}{n_B M} = \frac{n_A}{n_B} = \frac{P_A V_A / T_A}{P_B V_B / T_B} = \frac{1}{3}$. *(ii)* Use the equation of state of the ideal gas to write $T_i = \frac{P_i V_i}{n_i R}$. To be in thermal equilibrium, system A and system B must be at the same temperaure: $T'_A = T'_B$. Hence, $\frac{P'_A V'_B}{n_a} = \frac{P'_B V'_B}{n_B}$, yielding $P'_B = \frac{P'_A V'_A}{n_A} \frac{n_B}{V'_B} = 0.12$ bar. *(iii)* $\frac{T_{\text{final}}}{T_{\text{initial}}} = \frac{P'_A V'_A}{P_A V_A} = \frac{P'_B V'_B}{P_B V_B} = 6$.

4. A tank of volume 10 m^3 contains nitrous oxide at a pressure 1000 Pa and temperature of 20° C. Assume that it behaves like an ideal gas. *(i)* How many kilomoles of N_2O are in the tank? *(ii)* How many kilograms? *(iii)* Find the pressure if the temperature is increased to 50° C. *(iv)* At a temperature of 20°C, how many kilomoles should be withdrawn from the tank for the pressure to become 100 Pa?

Solution: *(i)* Use the equation of state of the ideal gas to obtain $n = \frac{PV}{RT} = 0.0041$ kilomole; *(ii)* $m = nM = 0.0041(14 \times 2 + 16) = 0.81$ kg; *(iii)* $P = \frac{nRT}{V} = 1102$ Pa; *(iv)* $n'/n = P'/P$, which implies $n' = 0.1\ n$, yielding $\Delta n = 0.9n = 0.0037$ kiomole.

5. A glass bottle of nominal capacity 250 cm^3 is filled brim full of water at 20° C. If the bottle and contents are heated to 50° C, how much water spills over? (For water, $\beta = 0.21 \times 10^{-3}\ K^{-1}$. Assume that the expansion of the glass is negligible.)

Solution: $\Delta V = \beta V_0 \Delta T = 1.6\ cm^3$.

6. *(i)* Calculate the heat capacity in the process $P = AT^b$ of an ideal gas, expressing it as a function of T. *(ii)* Analyze different cases of b.

Solution: *(i)* Use the first law of thermodynamics, $dU = \delta Q - PdV$, to obtain the infinitesimal heat received: $\delta Q = dU + PdV = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$. Next, use the isochoric heat capacity $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ and the fact that for the ideal gas $\left(\frac{\partial U}{\partial V}\right)_T = 0$ to simplify the previous expression to $\delta Q = C_V dT + PdV$. After that, expess dV through dT. To this end, use $P = AT^b$ and the equation of state of the ideal gas to write $V = \frac{nR}{A} T^{1-b}$, which leads to $dV = \frac{nR}{A}(1-b)T^{-b}dT$. Since $P = AT^b$, it follows that $\delta Q = C_V dT + nR(1-b)dT$ and hence $C = \frac{\delta Q}{dT} = C_V + nR(1-b)$. *(ii)* For the isobaric process, $b = 0$, it follows that $C = C_P = C_V + nR$ (Mayer's relation). For the isochoric process, $b = 1$, it follows that $C = C_V$. For the isothermic process, $b \to \pm\infty$, it follows that $C \to \mp\infty$. For the adiabatic process, $b = \gamma/(\gamma - 1)$, where $\gamma = C_P/C_V$, it follows that $C = C_V - \frac{nR}{\gamma - 1} = \frac{C_V\,(C_P/C_V - 1) - nR}{(\gamma - 1)} = \frac{C_P - C_V - nR}{\gamma - 1}$, and taking into account Mayer's relation $C = 0$.