Prof. Anchordoqui

Problems set # 3 **Physics 411** September 27, 2007

1. *(i)* Describe how to determine whether an equilibrium is stable or unstable when

$$(d^2U/dx^2)_0 = 0.$$

(ii) Write the criteria for determining whether an equilibrium point is stable or unstable when all derivatives up through order n, $(d^nU/dx^n)_0 = 0$.

2. A particle is under the influence of a force $F = -kx + kx^3/\alpha^2$, where k and α are constants and k is positive. Determine $U(x)$ and discuss the motion. What happens when $E = (1/4)k\alpha^2$?

3. Consider a particle moving in the region $x > 0$ under the influence of the potential

$$U(x) = U_0 \left(\frac{a}{x} + \frac{x}{a}\right) ,$$

where $U_0 = 1$ J and $a = 2$ m. Plot the potential, find the equilibrium points, and determine whether they are maxima or minima.

4. A water droplet falling in the atmosphere is spherical. Assume that as the droplet passes through a cloud, it acquires mass at a rate proportional to kA where $k > 0$ is a constant and A its cross sectional area. Consider a droplet of initial radius r_0 that enters a cloud with velocity v_0. Assume no resistance force and show (a) that the radius increases linearly with time, and (b) that if r_0 is negligible small then the speed increases lineraly with the time within the cloud.

5. A rocket in outer space in a negligible gravitational field starts from rest and accelerates uniformly at a until its final speed is v. The initial mass of the rocket is m_0. How much work does the rocket's engine do?

6. To perform a rescue, a lunar landing craft needs to hover just above the surface of the moon, which has a gravitational acceleration $g/6$. The exhaust velocity is 2×10^3 m/s, but fuel amounting to only 20% of the total mass may be used. How long can the landing craft hover?

7. A rocket has an initial mass of 7×10^4 kg and on firing burns its fuel at a rate of 250 kg/s. The exhaust velocity is 2.5×10^3 m/s. If the rocket has a vertical ascent from resting on the Earth, how long after the rockets engines fire will the rocket lift off? What is wrong with the design of this rocket?

8. A new single stage rocket is developed in the year 2020, having a gas exhaust velocity of 4×10^3 m/s. The total mass of the rocket is 10^5 kg, with 90% of its mass being fuel. The fuel burns quickly in 100 s at a constant rate. For testing purposes, the rocket is launched vertically at rest from the Earth surface. (a) Neglect air resistance and assume the acceleration of gravity is constant. Determine how high the launched object can reach above the surface of the Earth. (b) If the object has a radius of 20 cm and the air resistance is proportional to the square of the object's speed with $c_W = 0.2$, determine the maximum height reached. Assume the density of air is constant. (c) Now, also include the fact that the acceleration of gravity decreases as the object soars above the Earth. Find the height reached. (d) Now add the effects of the decrease of air density with altitude to the calculation. We can very roughly represent the air density by $\log_{10}(\rho) = -0.05\, h + 0.11$ where ρ is the air density in kg/m^3 and h is the altitude above the Earth in km. Determine how high the object now goes.