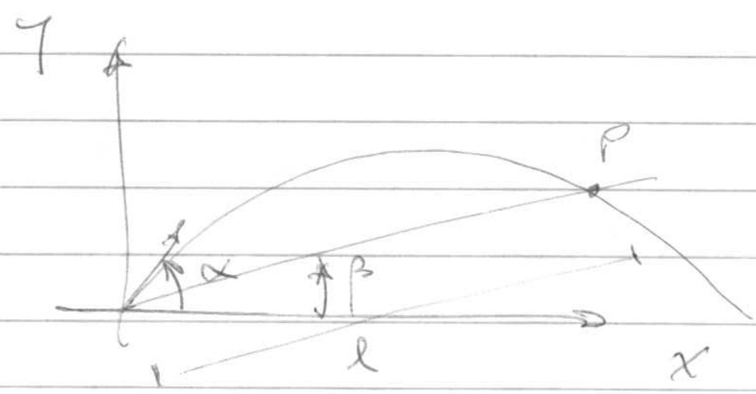
PROBLEMS # 2

(1)



The equation of motion is

$$\vec{F} = m\vec{a}$$

The gravitational force is the only applied force; therefore

$$F_x = m\ddot{x} = 0$$

$$F_y = m\ddot{y} = -mg$$

Integrating these equations and using the initial conditions

$$\dot{x}(t=0) = v_0 \cos\alpha$$

$$\dot{y}(t=0) = v_0 \sin\alpha$$

we find

$$\dot{x}(t) = v_0 \cos\alpha$$

$$\dot{y}(t) = v_0 \sin\alpha - g t$$

So the equations for x and y are

$$x(t) = v_0 t \cos\alpha$$

$$y(t) = v_0 t \sin\alpha - \frac{1}{2} g t^2$$

Suppose it takes a time t_0 to reach the point P. Then

$$l \cos\beta = v_0 t_0 \cos\alpha$$

$$l \sin\beta = v_0 t_0 \sin\alpha - \frac{1}{2} g t_0^2$$

Eliminating l between these equations

$$\frac{1}{2} g t_0 \left[t_0 - \frac{2 v_0 \sin\alpha}{g} + \frac{2 v_0 \cos\alpha \tan\beta}{g} \right] = 0$$

from which

$$t_0 = \frac{2 v_0}{g} (\sin\alpha - \cos\alpha \tan\beta)$$

(E) The static frictional force has the approximate value $f_{max} = \mu_s N$

$$\vec{F} = \vec{F_g} + \vec{N}$$

y-direction

$$-F_g \cos\theta + N = 0$$

x-Direction

$$-f_s + F_g \sin\theta = m\ddot{x}$$

The static frictional force f_s will be some value $f_s \le f_{max}$ required to keep $\ddot{x} = 0$. However, as the angle θ of the plane increases, eventually the static frictional force will be unable to keep the block at rest.

At that angle θ', f becomes

$$f_s(\theta = \theta') = f_{max} = \mu_s N = \mu_s F_g \cos\theta$$

and

$$m\ddot{x} = F_g \sin\theta - f_{max}$$

$$m\ddot{x} = F_g \sin\theta - \mu_s F_g \cos\theta$$

$$\ddot{x} = g(\sin\theta - \mu_s \cos\theta)$$

Just before the block starts to slide the acceleration $\ddot{x} = 0 \Rightarrow$

$$\sin\theta - \mu_s \cos\theta = 0$$

$$\tan\theta = \mu_s = 0.4$$

$$\theta = 22^\circ$$

(ii) The kinetic friction becomes approximately

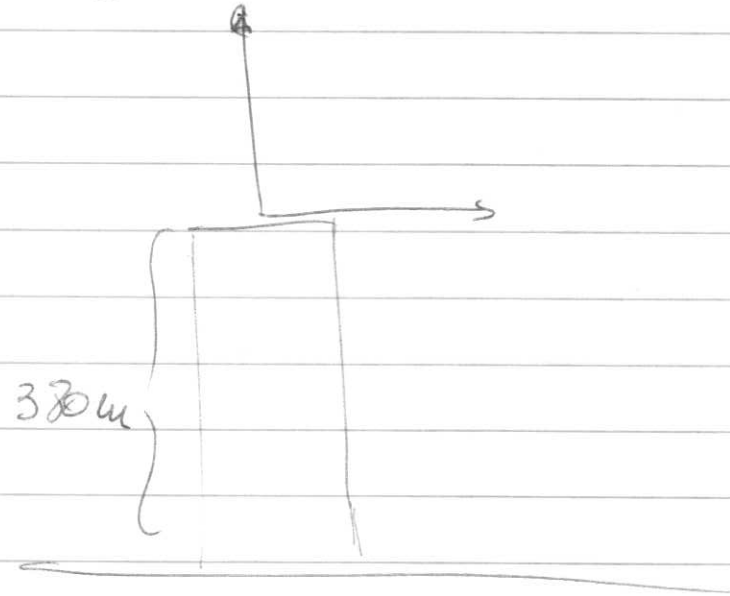
$$f_k = \mu_k N = \mu_k F_g \cos\theta$$

and

$$m\ddot{x} = F_g \sin\theta - f_k = mg(\sin\theta - \mu_k \cos\theta)$$

$$\ddot{x} = g(\sin\theta - \mu_k \cos\theta) = 0.24g$$

(3) Take origin at the top of Empire State building



$$m\ddot{y} = -mg$$

$$\frac{dv}{dt} = -g \Rightarrow v = v_0 - gt$$

$v_0 = 0$ from initial condition

$$\frac{dy}{dt} = -gt \Rightarrow y = y_0 - \frac{1}{2}gt^2$$

with $y_0 = 0 \Rightarrow$ from initial condition

$y_f = 380m$ $\quad t = \sqrt{\frac{2y_f}{g}} = 8.8s$; (ii) $v = 86 m/s$

(iii) $\frac{dv}{dt} = -g - kv$

$$\int_0^t dt = \int_0^v \frac{-1}{g+kv}\, dv$$

$u = g + kv \quad du = k\,dv$

$$\Rightarrow t = \left(-\frac{1}{k}\right)\int_{u_1}^{u_2} du$$

$-kt = \ln(g + kv) - \ln g$

$-kt = \ln\left(\frac{g+kv}{g}\right)$

$$e^{-kt} = \frac{g+kv}{g}$$

$\Rightarrow$ for $t = 0 \Rightarrow v = 0 \Rightarrow 1 = 1.$

$$v = \frac{g}{k} e^{-kt} - \frac{g}{k}$$

$$\frac{dy}{dt} = v = \frac{g e^{-kT}}{k} - \frac{g}{k}$$

$$\Rightarrow \int_0^y dy = \int_0^t \left[\frac{g e^{-kt}}{k} - \frac{g}{k} \right] dt$$

$$\Rightarrow y = -\frac{g e^{-kt}}{k^2} \Bigg|_0^t - \frac{gt}{k} \Bigg|_0^t$$

$$y = -\frac{g e^{-kt}}{k^2} + \frac{g}{k^2} - \frac{gt}{k}$$

$y = 0$ at $t = 0$

(iv)

If we neglect air resistance

$v = 19.6$ m/s

with the correction of retarding force

$v = 18.94$ m/s

$$\frac{19.6 - 18.94}{19.60} \times 100\% \simeq 3\%$$

(4)

Angular momentum must be conserved during the process. We are using the concept of moment of inertia learn in elementary physics to relate angular momentum L to angular velocity: $L = I\omega$

The initial angular momentum $L_0 = I\omega_0$ must be equal to the angular momentum L (fan + mouse) after the mouse jumps on. The velocity of the outside edge is $v = \omega R$

$$L = I\omega + mvR = \frac{v}{R}\left(I + mR^2\right)$$

$$L = L_0 = I\omega_0$$

$$\frac{v}{R}\left(I + mR^2\right) = I\frac{v_0}{R}$$

$$\frac{v}{v_0} = \frac{I}{I + mR^2}$$

and

$$\frac{\omega}{\omega_0} = \frac{I}{I + mR^2}$$

(5) (a) If there is no air resistance, then conservation of mechanical energy can be used. Subscript 1 represent the glider when at launched, and subscript 2 represent the glider at landing. The landing location is the zero for the potential energy PE (x=0). We have $y_1 = 500$ m and $y_2 = 0$.

$$v_1 = 500 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 138.9 \text{ m/s}$$

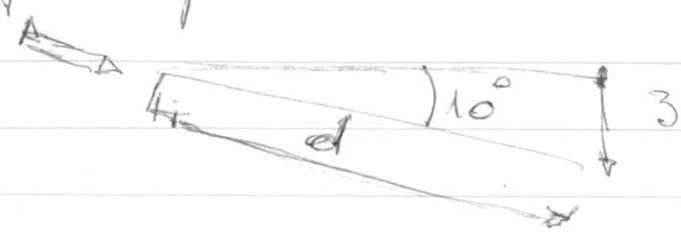
Solve for v_2

$$E_1 = E_2 \Rightarrow \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2$$

$$v_2 = \sqrt{v_1^2 + 2 g y_1} = 296 \text{ m/s} \left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}} \right)$$

$$= 1067 \text{ km/h} \approx 1.1 \; 10^3 \text{ km/h}$$

(b) Now include the work done by the non-conservative force. Consider the diagram of the glider



Calculate the work done by friction

$$W_{NC} = F_{fr}\, d \cos 180^\circ = -F_{fr}\, d = -F_{fr} \frac{3500\ m}{\sin 10^\circ}$$

Use the same subscript representations as above with y_1, v_1, y_2 as before and

$$v_2 = 200\ km/h \left(\frac{1\ m/s}{3.6\ km/h}\right) = 55.56\ m/s$$

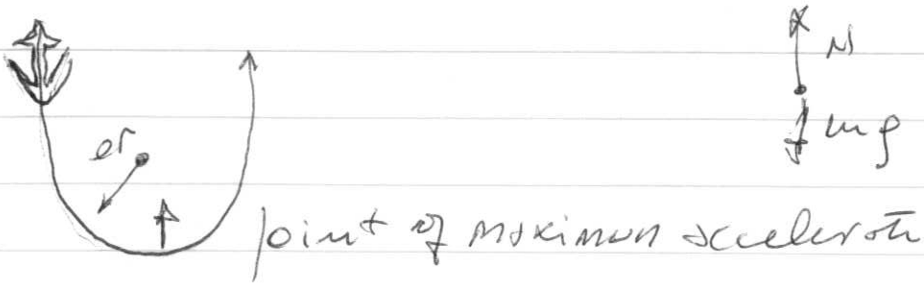
Write the energy conservation equation and solve for the frictional force

$$W_{NC} + E_1 = E_2 \implies$$

$$f_{fr}\, d + \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2 + m g y_2$$

$$\implies f_{fr} = m \frac{(v_1^2 - v_2^2 + 2 g y_1)}{2d} \approx 2 \times 10^3\ N$$

(e)



point of maximum acceleration

(a) From the force diagram we have $(N - mg)\,\hat{e}_r = \left(\frac{mv^2}{R}\right)\hat{e}_r$

The acceleration that the pilot feels is

$\bar{a} = \left(\frac{N}{m}\right)\hat{e}_r = g + (v^2/R)\;\hat{e}_r$, which has a maximum magnitude at the bottom of the manoeuver

(b) If the acceleration felt by the pilot must be less than 9g, then we have

$$R \geq \frac{v^2}{8g} = \frac{(3 \times 340\ m\,s^{-1})^2}{8\ \ 9.8\ m/s} = 13\ km$$

A circle smaller than that will result in pilot blackout.

(7) The equation of motion of the particle is

$$m\frac{dv}{dt} = -mk\left(v^3 + a^2 v\right) \quad (1)$$

Integrating

$$\int \frac{dv}{v(v^2+a^2)} = -k \int dt \qquad (2)$$

$$\frac{1}{2a^2} \ln\left[\frac{v^2}{a^2+v^2}\right] = -kt + c \qquad (3)$$

Therefore, we have

$$\frac{v^2}{a^2+v^2} = c' e^{-At} \qquad (4)$$

where $A = 2a^2 k$ and c' is a new constant. We can evaluate c' by using the initial condition, $v = v_0$ at $t = 0$.

$$c' = \frac{v_0^2}{a^2+v_0^2} \qquad (5)$$

Substituting (5) into (4) and rearranging, we have

$$v = \left[\frac{a^2 c' e^{-At}}{1 - c' e^{-At}}\right]^{1/2} = \frac{dx}{dt} \qquad (6)$$

Now, in order to integrate (6), we introduce $u \equiv e^{-At}$ so that $du = -Au\,dt$

Then

$$x = \int \left[\frac{a^2 c' e^{-At}}{1 - c' e^{-At}} \right]^{1/2} dt = \frac{a}{A} \int \left[\frac{c' u}{1 - c' u} \right]^{1/2} \frac{du}{u}$$

$$= -\frac{a\sqrt{c'}}{A} \int \frac{du}{\sqrt{-c'u^2 + u}} \qquad (7)$$

then $x = \frac{a}{A} \sin^{-1}(1 - 2c'u) + c''$ (8)

Again the constant c'' can be evaluated by setting $x = 0$ at $t = 0$, i.e, $x = 0$ at $u = 1$

$$c'' = -\frac{a}{A} \sin^{-1}(1 - 2c') \qquad (9)$$

Therefore, we have

$$x = \frac{a}{A} \left[\sin^{-1}(-2c' e^{-At} + 1) - \sin^{-1}(-2c' + 1) \right]$$

Using (4) and (5) we can write

$$x = \frac{1}{2aK}\left[\sin^{-1}\left[\frac{-v^2+a^2}{v^2+a^2}\right] - \sin^{-1}\left[\frac{-v_0^2+a^2}{v_0^2+a^2}\right]\right] \quad (10)$$

From (6) we see that $v \to 0$ as $t \to \infty$
Therefore

$$\lim_{t\to\infty} \sin^{-1}\left[\frac{-v^2+a^2}{v^2+a^2}\right] = \sin^{-1}(1) = \pi/2 \quad (11)$$

Also for very large initial velocities

$$\lim_{v_0\to\infty} \sin^{-1}\left[\frac{-v_0^2+a^2}{v_0^2+a^2}\right] = \sin^{-1}(-1) = -\pi/2 \quad (12)$$

Therefore using (10), (11), and (12)

$$x(t\to\infty) = \frac{\pi}{2ka}$$

and the particle can never move a distance greater than $\pi/2ka$ for any initial velocity.