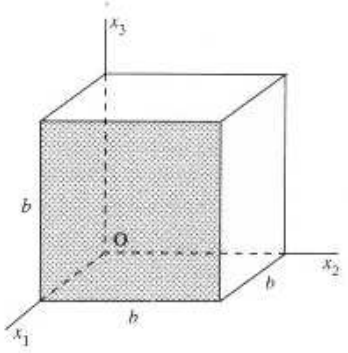
Prof. Anchordoqui

Problems set # 13 **Physics 411** December 6, 2007

1. Calculate the inertia tensor of a homogeneous cube of density ρ, mass M, and side length b. Let one corner be at the origin, and let three adjacent edges lie along the coordinate axes, as shown in the figure.



2. Find the principal moments of inertia and the principal axes for the cube in problem 1.

3. Find the height at which a billard ball should be struck so that it will roll with no initial slipping.

4. Two spheres are of the same diameter and same mass, but one is solid and the other one is a hollow shell. Describe in detail a non-destructive experiment to determine which is solid and which is hollow.

5. Imagine that the Earth is perfectly rigid, uniform and spherical and is spinning about its usual axis at its usual rate. A huge mountanin of mass $10^{-8}M_\oplus$ is now added at colatitude of 60°, causing the Earth to begin free precession. How long will it take the north pole (defined as the northern end of the diameter along ω to move 100 miles from its current position. Take $R_\oplus = 4000$ miles.

6. Consider a thin disk compose of two homogeneous halves connected along the diameter of the disk. If one half has density ρ and the other has density 2ρ, find the expression for the Lagrangian when the disk rolls without slipping along a horizontal surface. (The rotation takes place in the plane of the disk.)

7. A symmetric body moves without the influence of forces or torques. Let x_3 be the symmetry axis of the body and $\boldsymbol{L}$ be along x_3'. The angle between $bm\omega$ and x_3 is α. Let $\boldsymbol{\omega}$ and $\boldsymbol{L}$ initially be in the x_2-x_3 plane. What is the angular velocity of the symmetry axis about $\boldsymbol{L}$ in terms of I_1, I_3, ω, and α.