Prof. Anchordoqui

Problems set # 12 **Physics 411** November 29, 2007

1. Prof. Chad, a novice pool player, is faced with the corner pocket shot shown in the figure below. (Relative dimensions are also given.) Should Prof. Chad be worried about this being a "scratch shot" in which the cue ball will also fall into a pocket? Give details.

2. Calculate the differential cross section $\sigma(\theta)$ and the total cross section $\sigma_{\rm tot}$ for the elastic scattering of a particle from an impenetrable sphere of radius a; the potential is given by:

$$U(r) = \begin{cases} 0, & r > a \\ \infty, & r < a \end{cases}$$

3. Evaluate the prospects for neutrino detection at the IceCube facility. This telescope, which is currently being deployed near the Amundsen-Scott station, comprises a cubic-kilometer of ultra-clear ice about a mile below the South Pole surface, instrumented with long strings of sensitive photon detectors which record light produced when neutrinos interact in the ice. Estimate the total number of ultra-high energy events (10^8 GeV $< E_\nu < 10^{11}$ GeV) expected to be detected at IceCube during its lifetime of 15 years, if the cosmic neutrino flux is

$$\mathcal{F}_\nu(E_\nu) \simeq \ 6.0 \times 10^{-8} (E_\nu/{\rm GeV})^{-2} \ {\rm GeV}^{-1}\,{\rm cm}^{-2}\,{\rm s}^{-1}\,{\rm sr}^{-1} \ ,$$

and the total neutrino-nucleon cross section rises with energy as

$$\sigma_{\rm tot}(E_\nu) \simeq 6.04\,(E_\nu/{\rm GeV})^{0.358} \ {\rm pb} \ .$$

Assume that for this energy range the Earth becomes completely opaque to the propagation of neutrinos. Further details on the experiment are available at `http://icecube.wisc.edu/`.

4. Show that Rutherford scattering cross section (for the case $m_1 = m_2$) can be expressed in terms of the recoil angle as

$$\sigma_{\rm LAB}(\zeta) = \frac{k^2}{E}\,\frac{1}{\cos^3\zeta}.$$

5. Consider the case of Rutherford scattering in the event that $m_1 \gg m_2$. Obtain an approximate expression for the differential cross section in the lab coordinate system.

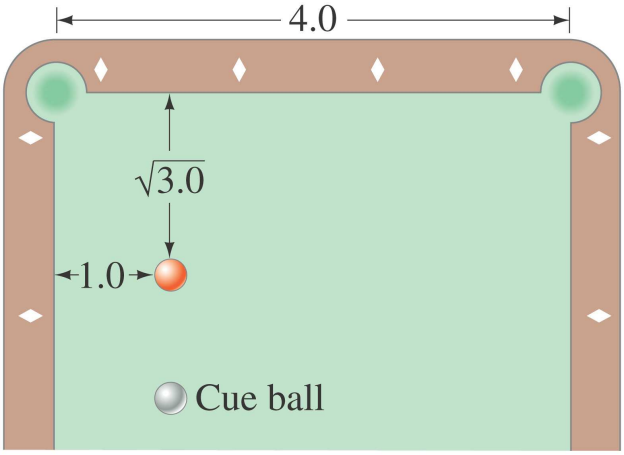
6. Consider the case of Rutherford scattering in the event that $m_2 \gg m_1$. Obtain an expression of the differential cross section in the CM system that is correct to first order in m_1/m_2.

7. A fixed force center scatters a particle of mass m according to the force law $F(r) = k/r^3$. If the initial velocity of the particle is u_0, show that the differential scattering cross section is

$$\sigma(\theta) = \frac{k\,\pi^2\,(\pi-\theta)}{m\,u_0^2\theta^2(2\pi-\theta)^2\,\sin\theta}.$$

8. It is found experimentally that in the elastic scattering of neutrons by protons ($m_n \approx m_p$) at relatively low energies, the energy distribution of the recoling protons in the LAB system is constant up to a maximum energy, which is the energy of the incident neutrons. What is the angular distribution of the scattering in the CM system?

9. Show that the energy distribution of particles recoiling from an elastic collision is always directly proportional to the differential elastic cross section in the CM system.


4.0
$\sqrt{3.0}$
1.0
Cue ball