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Problems set # 11 **Physics 411** November 15, 2007

1. Find the center of mass of a solid hemisphere of constant density.

2. Find the center of mass of a hemispherical shell of constant density and inner radius r_1 and outer radius r_2.

3. Find the center of mass of a uniformly solid cone of base $2a$ and height h.

4. The center of gravity of a system of particles is the point about which external gravitational forces exert no net torque. For a uniform gravitational force, show that the center of gravity is identical to the center of mass for a system of particles.

5. Pete (mass 80 kg) and Dave (mass 120 kg) are in a rowboat (mass 60 kg) on a calm lake. Dave is near the bow of the boat, rowing, and Pete is at the stern, 2 m from Dave (See figure below). Dave gets tired and stops rowing. Pete offers to row, so after the boat comes to rest they change places. How far does the boat moves as Pete and Dave change places. (Neglect any horizontal force exerted by the water).

6. A projectile of mass M explodes while in flight into 3 fragments. One mass ($m_1 = M/2$) travels in the original direction of the projectile, mass $m_2 = M/6$ travels in the opposite direction, and mass $m_3 = M/3$ comes to rest. The energy release in the explosion is 5 times the projectile's kinetic energy at explosion. What are the velocities?

7. In an elastic collision of two particles with masses m_1 and m_2, the initial velocities are u_1 and $u_2 = \alpha u_1$ ($\alpha > 0$). If the initial kinetic energies of the two particles are equal in the lab system, find the conditions on u_1/u_2 and m_1/m_2 so that m_1 will be at rest in the lab system after the collision.

8. A projectile is fired at an angle of 45° with initial kinetic energy E_0. At the top of its trajectory, the projectile explodes with additional energy E_0 into two fragments. One fragment of mass m_1 travels straight down. What is the velocity (magnitude and direction) of the second fragment of mass m_2 and the velocity of the first? What is the ratio m_1/m_2 when m_1 is a maximum?

9. A particle of mass m_1 and velocity u_1 collides with a particle of mass m_2 at rest. The two particles stick together. What fraction of the original kinetic energy is lost on the collision?

10. A billard ball of initial velocity u_1 collides with another billard ball (same mass) initially at rest. The first ball moves off at $\psi = 45°$. For an elastic collision, what are the velocities of both balls after the collision. At what lab angle does the second ball emerge?

11. A rubber ball is dropped from rest onto a linoleum floor a distance h_1 away. What is the coefficient of restitution? What fraction of the original kinetic energy is lost in terms of ϵ?

12. A steel ball of velocity 5 m/s strikes a smooth, heavy steel plate at an angle of 30° from the normal. If the coefficient of restitution is 0.8, at what angle and velocity does the steel ball bounce off the plane.

13. Two gravitating masses m_1 and m_2 are separated by a distance r_0 and released from rest. Show that when the separation is $r(< r_0)$, the speeds are

$$v_1 = m_2\sqrt{\frac{2G}{(m_1+m_2)}\left(\frac{1}{r}-\frac{1}{r_0}\right)}, \qquad v_2 = m_1\sqrt{\frac{2G}{(m_1+m_2)}\left(\frac{1}{r}-\frac{1}{r_0}\right)}.$$

14. Astronaut Sujeet Akula wanders too far away from the space shuttle orbiter while reparing a broken communications satellite. Sujeet realizes that the orbiter is moving away from him at 3 m/s. Sujeet and his maneuvering unit have a mass of 100 kg, including a pressurized tank of mass 10 kg. The tank includes only 2 kg of gas which is used to propel him in space. The gas escapes with a constant velocity of 100 m/s. (a) Will Sujeet run out of gas before he reaches the orbiter? (b) Though Sujeet always carries with him a pocket calculator, he is not so good in getting numerical values without using MAPLE. Can you help him to estimate with what velocity will he have to throw the empty tank away to reach the orbiter?