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Problems set # 10 **Physics 411** November 8, 2007

1. Write down the Lagrangian for a projectile (subject to no air resistance) in terms of its Cartesian coordinates $(x,\ y,\ z)$, with z measured vertically upward. Find the 3 Lagrange equations and show that they are exactly what you would expect for the equations of motion.

2. Write down the Lagrangian for a 1-dimensional particle moving along the x-axis and subject to a force $F = -kx$ (with k positive). Find the Lagrange equation of motion and solve it.

3. Consider a mass m moving in two dimensions with potential energy $U(x,\ y) = kr^2/2$, where $r^2 = x^2 + y^2$. Write down the Lagrangian, using coordinates x and y, and find the two Lagrange equations of motion. Describe their solutions. [This is the potential energy of an ion in an "ion trap," which can be used to study the properties of individual ions.]

4. Consider a bead that is threaded on a rigid circular hoop of radius R lying in the xy plane with its center at O, and use the angle ϕ of 2-dimensional polar coordinates as the one generalized coordinate to describe the bed's position. Write down the equations that give the Cartesian coordinates $(x,\ y)$ in terms of ϕ and the equation that gives the generalized coordinate ϕ in terms of $(x,\ y)$.

5. A particle is confined to move on the surface of a circular cone with its axis on the z axis, vertex at the origin (pointing down), and half-angle α. The particle's position can be specified by 2 generalized coordinates, which you can choose to be the coordinates $(\rho,\ \phi)$ of cylindrical polar coordinates. Write down the equations that give the 3 Cartesian coordinates of the particle in terms of the generalized coordinates $(\rho,\ \phi)$ and vice versa.

6. Use the Lagrangian method to find the acceleration of the Atwood machine, including the effect of the pulley's having moment of inertia I. [Hint: the kinetic energy of the pulley is $I\omega^2/2$, where ω is its angular velocity.

7. Using the usual angle ϕ as generalized coordinate, write down the Lagrangian for a simple pendulum of length l suspended from the ceiling of an elevator that is accelerating upward with constant acceleration a. (Be careful when writing T; it is probably safest to write the bob's velocity in component form.) Find the Lagrange equation of motion and show that it is the same as that for a normal, nonaccelerating pendulum, except that g has been replaced by $g + a$. In particular the angular frequency of small oscillations is $\sqrt{(g+a)/l}$. A simple pendulum (mass m and length l)

8. Prove that the potential energy of a central force $\vec{F} = -kr^n\hat{r}$ (with $n \neq -1$) is

$$U = \frac{kr^{n+1}}{n+1}.$$

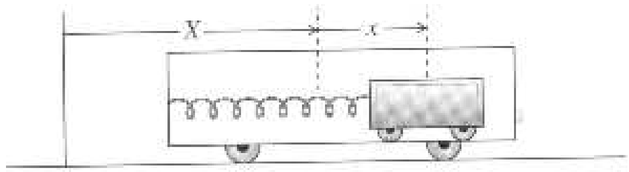
In particular if $n = 1$ then $\vec{F} = -k\vec{r}$ and $U = k\,r^2/2$

9. A small cart (mass m) is mounted on rails inside a large cart. The two are attached by a spring (force constant k) in such a way that the small cart is in equilibrium at the midpoint of the large. The distance from the small cart from its equilibrium is denoted x and that of the large one from a fixed point on the ground is X, as shown in the figure. The large cart is now force to

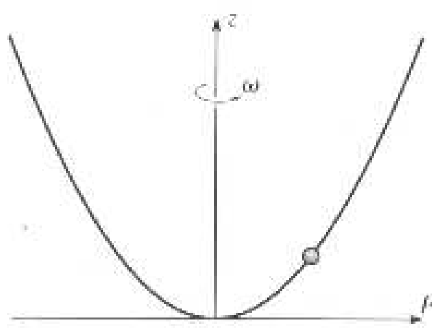
oscillate such that $X = A\cos\omega t$, with both A and ω fixed. Set up the Lagrangian for the motion of the small cart and show that the Lagrange equation has the form

$$\ddot{x} + \omega_0^2 x = B\cos\omega t\,,$$

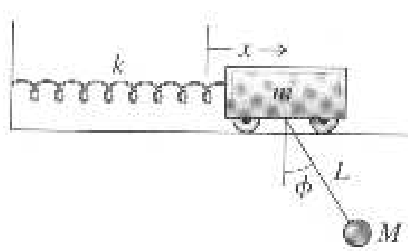
where ω_0 is the natural frequency $\omega_0 = \sqrt{k/m}$ and B is a constant.



10. Consider a bed of mass m sliding without friction on a wire that is bent in the shape of a parabola and is being spun with constant angular velocity ω about its vertical axis, as shown in the figure. Use cylindrical polar coordinates and let the equation of the parabola be $z = k\rho^2$. Write down the Lagrangian in terms of ρ as the generalized coordinate. find the equation of motion of the bed and determine whether there are positions of equilibrium, that is values of ρ at which the bed can remain fixed, whithout sliding up or down the spinning wire. Discuss the stability of any equilibrium positions yopu find.



11. A simple pendulum (mass M and length L) is supended from a cart (mass m) that can oscillate on the end of a spring of force constant k as shown in the figure. *(i)* Write the Lagrangian in terms of the 2 generalized coordinates x and ϕ, where x is the extension of the spring from its equilibrium length. [Hint: Be careful writing down the kinetic energy T. A safe way to get the velocity right is to write down the position of the bob at time t and then differentiate.] Find the 2 Lagrange equations. *(ii)* Simplify the equations to the case that both x and ϕ are small.



12. Consider a particle of mass m constrained to move on the surface of a paraboloid whose equation (in cylindrical coordinates) is $r^2 = 4az$. If the particle is subject to a gravitational force, show that the frequency of small oscillations about a circular orbit with radius $\rho = \sqrt{4az_0}$ is

$$\omega = \sqrt{\frac{2g}{a + z_0}}.$$