# Group Problems #4

Monday, August 29

## **Problem 1** *Consequences of Einstein's Postulates*

A railcar travels to the right with speed $v$, as shown in the Figure. The frame $S$ is at rest with respect to the earth, while $S'$ moves with the railcar. The "proper" length of the railcar is $L_0$: this is the length in its own rest frame, $S'$. At either end of the car, there are flashlamps $F_1$ and $F_2$, and at the exact center of the car there are two detectors $D_1$ and $D_2$, to record the arrival time of the light pulses emitted by the lamps. An observer at rest with respect to the car flips a switch, which triggers both lamps to flash at precisely the same moment in the $S'$ frame.

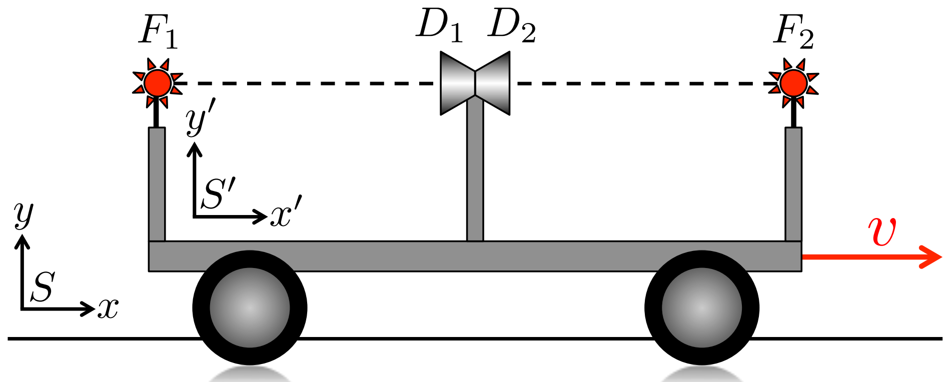


Figure 1: Rail car with flashlamps and detectors moving to the right.

(a) Make a space-time diagram ($t'$ vs. $x'$) of the situation in the $S'$ frame: draw the trajectories of both ends of the car, the center of the car, and the light pulses. In addition, mark the events corresponding to emission of the flashes from $F_1$ and $F_2$ and their detection at $D_1$ and $D_2$. Notice how the space-time diagram captures all the relevant information of the problem, at least according to an observer in the $S'$ frame. In particular, the "world lines" or trajectories of $F_{1,2}$ and $D_{1,2}$ are vertical since they are stationary in their own rest frame. In addition, the slope of the "light lines" for both pulses are the same (neglecting a negative sign), as

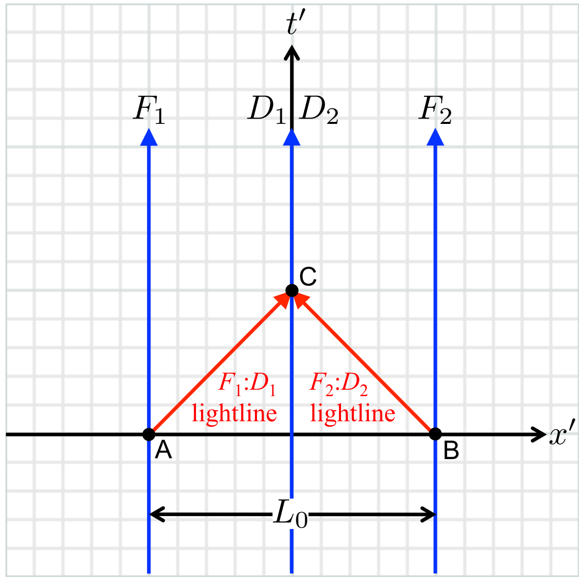


Figure 2: Space-time diagram in the $S'$ frame.

required by Einstein's second postulate. The events have been labeled as A: flash of $F_1$; B: flash of $F_2$; and $C$: detection of light pulses at $D_1$ and $D_2$. Note that $C$ is a single event since both light pulses arrive at a single point in space at a single point in time. The proper length of the car is also indicated.

(b) How much time does it take in the $S'$ frame for each light pulse to travel from the flashlamps to each detector, $\Delta t_1'$ and $\Delta t_2'$? As required by Einstein's second postulate, the speed of both light pulses is the same and equal to $c$. Thus the travel time for both light pulses is $\Delta t_1' = \Delta t_2' = (L_0/2)/c$.

(c) Is the length of the railcar larger, smaller, or equal to $L_0$ according to an observer in the $S$ frame? One of the consequences of Einstein's postulates is length contraction: "The length of an object in a reference frame through which the object moves is smaller than the length of the object in a frame in which the object is at rest." In this case, the object is the railcar and it is moving in the $S$ frame. Thus the length of the railcar in $S$ is smaller than $L_0$.

(d) An observer in the $S$ frame measures a travel time $\Delta t_1$ for the pulse going from $F_1$ to $D_1$ and $\Delta t_2$ for the pulse going from $F_2$ to $D_2$. Is $\Delta t_1$ longer, shorter, or equal to $\Delta t_2$? Use logic and Einstein's second postulate to explain your answer (i.e., don't use a formula). The arrival of the two light pulses at the detectors $D_{1,2}$ is a single event in $S'$, as discussed above. *All observers in all reference frames* **must** *agree that these arrivals constitute a single event*, so an observer in $S$ measures the simultaneous arrival of the two light pulses. However, the light pulse emitted from $F_1$ must travel a longer distance than the pulse emitted from $F_2$ since the railcar moves slightly to the right during the pulse's travel time. Since they arrive at the same time, and *since they both must travel at speed $c$ according to Einstein's second postulate* then we must have $\Delta t_1 > \Delta t_2$. The only way for this to happen and be consistent with Einstein's postulates is for $F_1$ to flash before $F_2$: events $A$ and $B$ are no longer simultaneous!

(e) Use the information in the parts above to make a space-time diagram in the $S$ frame.

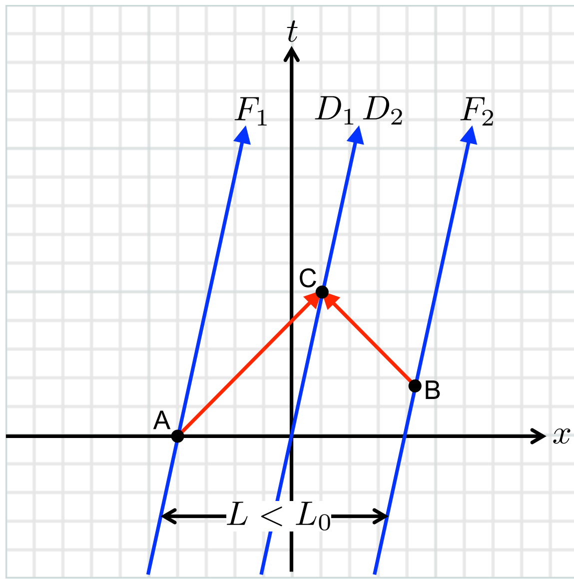


Figure 3: Space-time diagram in the $S$ frame.

Notice now that the world lines of $F_{1,2}$ and $D_{1,2}$ are sloped forward since the cart moves to the right in time: the slope of these world lines is $= 1/v$, where $v$ is the speed of the railcar. However, notice that the slope of the light lines are the same as in part (a) above, as required by Einstein's second postulate. (The slope of any worldline must be *larger* than that of a light line since nothing can travel faster than the speed of light.) Thus we see explicitly that event $B$ happens after event $A$ in frame $S$: the light pulses are no longer emitted simultaneously!