# Group Problems #3
# Solutions

Friday, August 26

## Problem 1 *2D Galilean Velocity Addition*

You are trying to swim directly East across a river which flows South. The river flows at 0.5 m/s and you can swim at 1 m/s in still water.

(a) If you point directly East while swimming, what angle will your velocity vector make with respect to the easterly direction in the reference frame of the bank? First, let's set up our coordinate system:

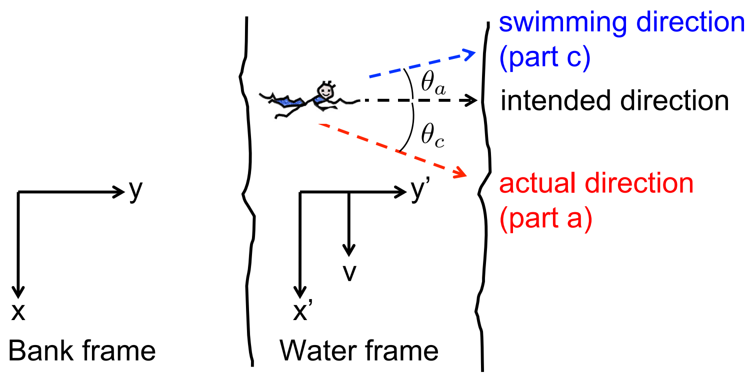


Figure 1: Coordinate system with the unprimed $S$ frame corresponding to the river bank, and the primed $S'$ frame corresponding to the moving water.

The Galilean velocity transformation for the coordinate system is:

$$u'_x = u_x - v \tag{1}$$
$$u'_y = u_y. \tag{2}$$

The swimmer swims relative to the water, so the most natural reference frame to work in is the $S'$ frame of the moving water. In part (a), the swimmer does not try to swim upstream, so $u'_x = 0 \Longrightarrow u_x = v$. So the swimmer's full effort is directed along the $y'$ direction: $u'_y = u_y = 1$ m/s. With $v = 0.5$ m/s:

$$\tan\theta_a = \frac{u_x}{u_y} = \frac{0.5}{1} \Longrightarrow \theta_a = \tan^{-1}(0.5) = 26.6°. \tag{3}$$

(b) What will be your speed in the reference frame of the bank?

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{(0.5)^2 + 1} = 1.12 \text{ m/s}. \tag{4}$$

(c) To go directly East relative to the bank, you must point upstream while swimming. That is, you must swim at an angle in the reference frame of the moving water. At what angle with respect to the easterly direction do you need to swim in this moving reference frame?

To swim directly east in the reference frame of the bank ($S$-frame), you must achieve $u_x = 0$. This implies that $u'_x = -v = -0.5$ m/s. Additionally, your *total* velocity relative to the water ($S'$-frame) must be 1 m/s:

$$(u')^2 = (u'_x)^2 + (u'_y)^2 = (1 \text{ m/s})^2 \tag{5}$$

$$\implies u'_y = \sqrt{(u')^2 - (u'_x)^2} = \frac{\sqrt{3}}{2} \text{ m/s}. \tag{6}$$

So now we can calculate the angle you need to swim relative to the water to travel straight Eastward in the bank's reference frame:

$$\tan\theta_c = \frac{u'_x}{u'_y} = -\frac{1/2 \text{ m/s}}{\sqrt{3}/2 \text{ m/s}} = -\frac{1}{\sqrt{3}} \tag{7}$$

$$\implies \theta_c = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30°. \tag{8}$$

(d) When you swim at this angle, what is your speed relative to the bank?

Transform back to the bank ($S$) frame: $u_x = u'_x + v = -0.5$ m/s $+0.5$ m/s $= 0$ (duh!). Also, $u_y = u'_y = \sqrt{3}/2$ m/s. So finally, $u = \sqrt{u_x^2 + u_y^2} = \sqrt{3}/2$ m/s.

(e) How is the situation described in this problem related to the Michelson-Morley experiment? Be as precise as possible.

This situation is analogous to the "fast" arm of the Michelson-Morley experiment, namely the one in which the light beam propagates in a direction perpendicular to the "ether wind". More particularly, the situation described in part (c), whereby the swimmer travels directly East in the rest frame of the water, is directly related to this "fast" arm if you imagine that the flowing water is the "ether wind", the swimmer is the beam of light, and the river bank is the Earth.