Group Problems #35 - Solutions

Wednesday, November 23

Problem 1 *Transition Selection Rules*

An electron is in the ground-state of a 1D infinite square well:

$$U(x) = \begin{cases} \infty, & \text{for } x \leq -L/2, \\ \infty, & \text{for } x \geq L/2, \\ 0, & \text{for } -L/2 < x < L/2. \end{cases} \tag{1}$$

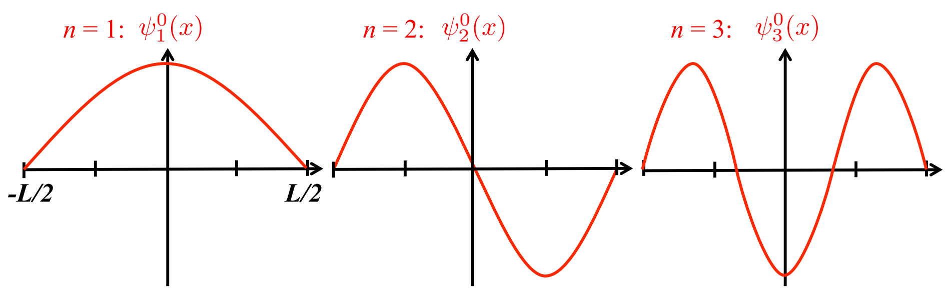
(a) What are the general solutions for the *unperturbed* Hamiltonian, $\psi_n^0(x)$? (Pay attention to the symmetry of the potential!!!)

The general solutions for this potential (note that the potential is centered around $x = 0$ in this case, $-L/2 < x < L/2$) are given by:

$$\psi_n^0(x) = \begin{cases} \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L}, & \text{for } n = 1, 3, 5, ... \\ \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, & \text{for } n = 2, 4, 6, ... \end{cases} \tag{2}$$

These solutions are equivalent to those we found when the well extended over $0 < x < L$.

(b) Draw the first three solutions of the unperturbed Hamiltonian, $\psi_1^0(x)$, $\psi_2^0(x)$, and $\psi_3^0(x)$.

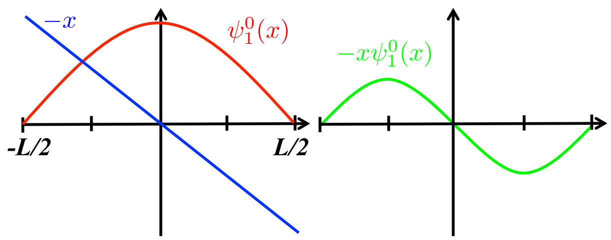


(c) At time $t = 0$ an electric field $\hat{\xi} = \xi\hat{x}$ is switched on for duration Δt. After the perturbation has been applied, the wavefunction can be approximated by,

$$\psi_{\mathrm{t=\Delta t}}(x) \simeq A\psi_1^0(x) + \psi_{\mathrm{pert}}(x,t) \quad (3)$$

$$= A\psi_1^0(x) - \frac{i\Delta t}{\hbar} W\psi_1^0(x), \quad (4)$$

where $W = e\xi x$ is the perturbation term caused by application of the electric field. Draw $\psi_{\mathrm{pert}}(x)$ for this case. (Ignore the "i" for now.)



(d) What can you deduce by graphical comparison of $\psi_{\mathrm{pert}}(x)$ and $\psi_2^0(x)$?

The similarity between $x\psi_1^0(x)$ and $\psi_2^0(x)$ indicates that the perturbation primarily induces a transition between the initial ground state ($n = 1$) and the 1st excited state ($n = 2$). Note that the amplitude of the perturbation part of the wavefunction depends on the duration it is applied, Δt, and also on the strength of the electric field, ξ.