Group Problems #2
Solutions

Wednesday, August 24

Problem 1 *Plotting Events on a Classical Space-Time Diagram*

You are walking at 2 m/s down a straight road. At a particular time you pass your friend Katrina, who is standing still. 5 s later a dog barks; at that moment he is 10 m ahead of you in the road. After another 5 s, a car backfires; at that moment it is 15 m behind you.

(a) Plot and label the events described above on a two-dimensional graph of time vs. position (space-time diagram) corresponding to your reference frame.

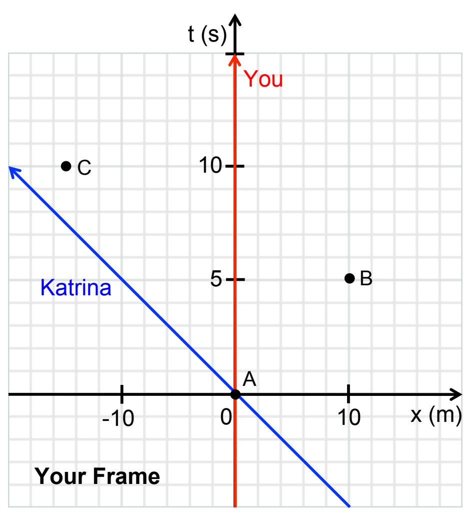

Figure 1: Space-time diagram in your reference frame.

(b) Plot and label the same events on a space-time diagram corresponding to Katrina's reference frame. (Assume your and Katrina's watches are synchronized.)

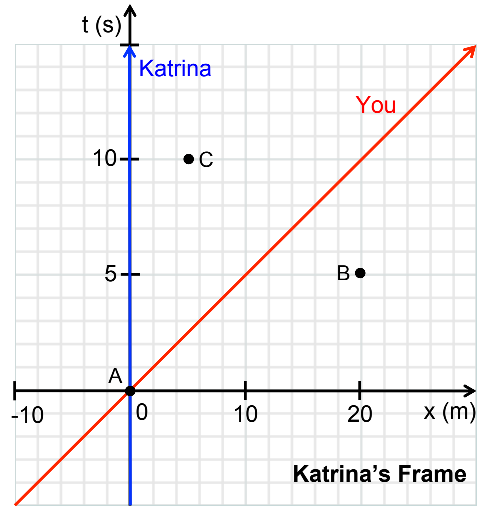

Figure 2: Space-time diagram in Katrina's reference frame.

Problem 2 *Relative Velocity*

If you throw a superball (perfectly elastic) with speed u at a stationary wall, it bounces back with the same speed in the opposite direction.

(a) What happens if you throw it at speed u towards a wall which is traveling towards you at speed w?

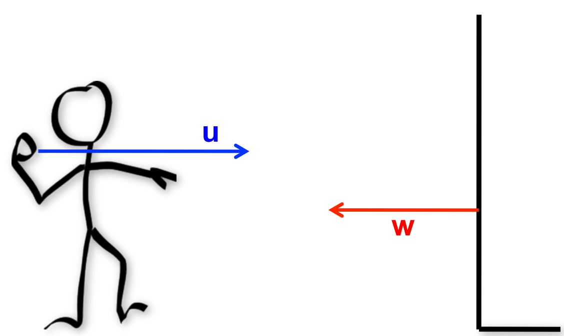

Figure 3: Superball approaching a moving wall.

Let's designate your reference frame as unprimed (S) and the wall's frame as primed (S'). So the ball moves with velocity $+u$ (to the right) in the S frame and the wall moves with velocity $v = -w$ (to the left) in the S frame. Obviously, the relative speed between the ball and wall is $u + w$ (classically). Formally, this can be obtained by doing a classical (Galilean) velocity transformation from the S to the S' frame: $u' = u - v = u + w$. Thus, in the S' frame, the ball approaches the wall with velocity $u' = u + w$ and will rebound with velocity $u'_{\text{reb}} = -(u + w)$.

Now transform the rebound velocity back to the unprimed S frame: $u_{\text{reb}} = u'_{\text{reb}} + v = -(u + w) + (-w) = -(u + 2w)$. Thus we see that the ball rebounds with speed $u + 2w$ in your reference frame.

(b) What is the answer in the limit in which w is much larger than u?

If $w \gg v$, then we can neglect u in the above equation, and $u_{\mathrm{reb}} = -2w$. So in this limit, the rebound velocity is independent of the ball's initial velocity.