Group Problems #21 - Solutions

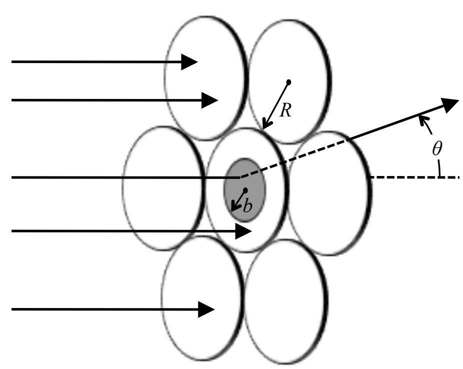
Wednesday, October 19

Problem 1 *Fraction of projectiles scattered at angles greater than θ*

(a) A single α particle projectile is incident on a single nucleus with an impact parameter b and scatters over an angle θ_b. If another α particle interacts with the nucleus with an impact parameter smaller than b, will it be scattered over an angle smaller or larger than θ_b.

A smaller impact parameter means that the α particle passes closer to the nucleus, and thus feels a larger Coulomb repulsion, so it will be scattered to larger angles.

(b) Now imagine that the target foil in the Rutherford experiment is only one atom thick, each with radius R, as shown in the Figure below. Find an expression as a function of b and R only that gives the fraction of projectiles scattered over angles greater than θ_b, assuming that the projectiles are spread uniformly over the area of the foil and all have the same energy (monochromatic). Neglect the interstitial areas between the "atomic discs" shown in the figure.



If the foil contains N atoms, its total area is $N\pi R^2$. However, since the incoming beam of projectiles is uniform and since we are neglecting the interstitial areas between the atomic discs, then the nucleus of each atom contributes equally. Thus, the fraction of projectiles scattered over angles larger than θ_b is just the number of projectiles with impact parameter smaller than b relative to number incident on the area of a single atomic disc: $f(\theta > \theta_b) = \pi b^2/\pi R^2 = (b/r)^2$.

(c) A real scattering foil may be many thousands of atoms thick. Let ρ and M be the density and molecular weight of the foil material. Find an expression for the number of target nuclei per unit volume n as a function of ρ, M, and Avogadro's constant N_A only.

N_A/M gives the number of atoms (and nuclei) per unit mass of the material, and ρ gives the mass per unit volume. So $n = \rho N_A/M$ is the number of nuclei per unit volume.

(d) Use the result from part (c) to find an expression for the fraction of projectiles scattered over angles greater than θ_b as a function of n, b, and the thickness of the foil (t) only. Neglect geometric screening of nuclei in successively deeper layers of the foil.

If we neglect the geometric screening of deeper nuclei, then the beam of projectiles "sees" all the nuclei in the material equally no matter their depth. Another way of saying this is that we can imagine that every nucleus in the material is in the same plane, at least as far as the projectile scattering is concerned. (By the way, this is not such a crazy assumption since nuclei are really small compared to atoms.) With this understanding, we see that nt is the number of nuclei per unit area "seen" by the beam of projectiles; that is, on average, each nucleus contributes an area $(nt)^{-1}$ to the field of view of the projectiles. Again, each nucleus contributes equally to the scattering since the projectile beam is uniform. Thus, $f(\theta > \theta_b) = \pi b^2/(nt)^{-1} = nt\pi b^2$. Plugging in for n gives:

$$f(\theta > \theta_b) = \pi b^2 \rho N_A t/M. \tag{1}$$

With the problem above, we have seen how you could predict the fraction of particles scattered from a beam into an arbitrary range of angles. In a real scattering problem, we would use the expression

$$b(\theta) = \frac{zZe^2}{8\pi\epsilon_0 K_0}\cot\frac{\theta}{2}, \tag{2}$$

to find a range of b corresponding to the desired range of θ and then use Eq. (1) above to calculate the corresponding fraction. Note that Eq. (2) was derived in the notes: z is the atomic number of the projectile, Z the atomic number of the nucleus, e the fundamental charge, ϵ_0 the permittivity of free space, and K_0 the projectile's initial kinetic energy.