Group Problems #17 - Solutions

Monday, October 3

Problem 1

Using the DeBroglie hypothesis, compute the wavelengths of the following:

(a) A 2200-lb car traveling at 80 mph.

The DeBroglie hypothesis relates an object's wavelength to its momentum:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m v}, \tag{1}$$

where h is Planck's constant, m is the object's mass, v is its velocity, γ is the Lorentz factor associated with the object, and λ is its DeBroglie wavelength. For a non-relativistic object ($v \ll c$), then $\lambda \sim 1$, so $p = mv$. For the mass, we find: $m = 2200$ lb.$= 1000$ kg, and for the velocity: $v = (80$ mile/h$)\times(1.61 \times 10^3$ meters/mile$)\times(1$ hour/60 minutes$)\times(1$ min/60 seconds$) = 35.8$ m/s. Putting it all together gives:

$$\begin{aligned} \lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \mathrm{kg\,m^2/s}}{(10^3\ \mathrm{kg})(35.8\ \mathrm{m/s})} &= \frac{6.626}{35.8} \times 10^{-37}\ \mathrm{m} && (2) \\ &= 1.85 \times 10^{-38}\ \mathrm{m}. && (3) \end{aligned}$$

(b) A 10-g bullet traveling at 500 m/s.

Here we have $m = 10^{-2}$ kg and $v = 500$ m/s, so

$$\begin{aligned} \lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \mathrm{kg\,m^2/s}}{(10^{-2}\ \mathrm{kg})(500\ \mathrm{m/s})} &= \frac{6.626}{5} \times 10^{-34}\ \mathrm{m} && (4) \\ &= 1.33 \times 10^{-34}\ \mathrm{m}. && (5) \end{aligned}$$

(c) A smoke-particle of mass 10^{-6} g moving at 1 cm/s.

Here we have $m = 10^{-9}$ kg and $v = 10^{-2}$ m/s, so

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \mathrm{kg\,m^2/s}}{(10^{-9}\ \mathrm{kg})(10^{-2}\ \mathrm{m/s})} = 6.626 \times 10^{-23}\ \mathrm{m}. \tag{6}$$

(d) An electron with kinetic energy 1 eV.

An electron can in principle travel at relativistic speeds, so we should first determine whether or not it is indeed relativistic. A simple way is to use the relation:

$$K = (\gamma - 1)mc^2 \longrightarrow \frac{K}{mc^2} = \gamma - 1. \tag{7}$$

We see that when $v \ll c$, then $\gamma \to 1$, which implies that the ratio K/mc^2 should be very small. Here, $K = 1$ eV and $mc^2 = 511$ keV, so indeed $K/mc^2 \ll 1$ and v must be much smaller than c; in other words, we are in the non-relativistic (classical) limit. In this limit, we have $K = p^2/2m \to p = \sqrt{2mK}$. We can make the math easier for ourselves by using $pc = \sqrt{2mc^2K}$. Now we can calculate the DeBoglie wavelength:

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(5.11 \times 10^5 \text{ eV})(1 \text{ eV})}} = 1.23 \text{ nm}. \tag{8}$$

Problem 2 *The Davisson-Germer Experiment*

If Davisson and Germer had used 100 volts to accelerate their electron beam instead of 54 volts, at which scattering angle ϕ would they have found a peak in the distribution of scattered electrons (the intensity)?

First calculate the wavelength of the electron. The kinetic energy K of an electron accelerated through a potential of 100 V is 100 eV. So, using the equation above we have:

$$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(5.11 \times 10^5 \text{ eV})(100 \text{ eV})}} = 0.123 \text{ nm}. \tag{9}$$

Now we use the Davisson-Germer formula for first-order ($n = 1$) diffraction, $2d \sin\theta = \lambda = 0.123$ nm, where $d = 0.091$ nm is the distance between adjacent crystal planes in Ni. Now we an solve for θ, the angle between the original direction of the electron beam and scattered direction:

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{0.123}{2 \cdot 0.091}\right) = 0.74 \text{ radians} = 42.52^\circ. \tag{10}$$

Now θ is not the angle measured by Davisson and Germer. Rather, they measured $\phi = 180^\circ - 2\theta$, so in this case they would have measured $\phi = 180^\circ - 85.04 \simeq 95^\circ$. Parenthetically, this would have been tough to measure given their geometry.