Group Problems #16 - Solutions

Friday, September 30

Problem 1

In lecture, we defined the intensity distribution for a double slit experiment:

$$I_{12}(x) = I_1(x) + I_2(x) + 2\sqrt{I_1 I_2}\cos\delta, \tag{1}$$

where $I_{1,2}(x)$ corresponds to only one slit being open at a time, and δ is the phase-angle difference between waves emanating from each slit when both are open. This intensity pattern can be rewritten as:

$$I_{12}(x) = I_0\cos^2(\delta/2), \tag{2}$$

where I_0 is the intensity when $\delta = 0$. A laser with wavelength $\lambda = 532$ nm (green) is directed at a double slit, producing an interference pattern described by Eq. 2. The intensity at the center of the pattern is measured to be 2.0 W/m^2.

(a) At what rate are photons detected at the center of the interference pattern?

First find the energy per photon:

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240 \text{ eV} - \text{nm}}{532 \text{ nm}} \cdot 1.602\times10^{-19} \text{ J/eV} \tag{3}$$

$$= 3.73\times10^{-19} \text{ J/photon}. \tag{4}$$

At the center of the interference pattern, $\delta = 0$, so $I_{12}(x=0) = I_0$. Using the fact that a Watt is 1 Joule/second/meter^2 (1 W = 1 J/s/m^2), we then have:

$$\frac{I_0}{E_\gamma} = \frac{2 \text{ J/s/m}^2}{3.73\times10^{-19} \text{ J/photon}} = 5.36\times10^{18} \text{ photons/s/m}^2. \tag{5}$$

(b) At what rate are photons detected when $\delta = \pi$?

Obviously, when $\delta = \pi$, then $\cos^2(\pi/2) = 0$, so $I_{12} = 0$.

(c) At what rate are photons detected at a point on the screen when the waves from the two sources are out of phase by 1/3 of an oscillation cycle?

A full oscillation cycle corresponds to an angular period of 2π. The problem states that δ is 1/3 of an oscillation cycle, so $\delta = 2\pi/3$. So,

$$I_{12} = I_0 \cos^2(\pi/3) = 0.25 I_0 = 1.34 \times 10^{18} \text{ photons/s/m}^2. \quad (6)$$