Errata

Chapter 1

- **Proposition 1.2.** $z_1 = x_1 + iy_1$.
- **Corollary 1.1.** The relation using De Moivre's theorem should read $s^n(\cos n\phi + i\sin n\phi) = r(\cos\theta + i\sin\theta)$.
- **Definition 1.6.** It should read "...(1.6) leads to..." instead of "...(1.5) leads to...", "Conversely, if the conditions (1.9) hold and the partial derivatives of $u(x,y)$ and $v(x,y)$ are continuous, the derivative $f'(z) = u_x + iv_x$ exists", and Eq. (1.11) should read $\delta f = (u_x + iv_x)\delta x + (u_y + iv_y)\delta y$.
- **Definition 1.15.** $\int_{z_1}^{z_2} f(z)\,dz = \int_{(x_1,y_1)}^{(x_2,y_2)} [u(x,y) + iv(x,y)][dx + idy]$ $= \int_{(x_1,y_1)}^{(x_2,y_2)} [u(x,y)dx - v(x,y)dy] + i\int_{(x_1,y_1)}^{(x_2,y_2)} [v(x,y)dx + u(x,y)dy]$.
- **Theorem 1.6** Equation (1.44) should read $\oint_C f(z)\,dz = -\int_A (v_x + u_y)\,dx\,dy + i\int_A (u_x - v_y)\,dx\,dy$.
- **Example 1.7** Equation (1.62) should read $\oint_C \frac{z^3+3}{z(z-i)^2} dz$.
- **Example 1.7.** There is a printing error in Eq. (1.65). In the denominator, $z(z-i)^2$, the minus sign is out of place.
- **Example 1.8.** An equal sign is missing in the definition of the exponential, i.e., $e^z = \sum_{n=0}^{\infty} z^n/n!$. In Eq. (1.81) the infinity signs got printed right on top of the summation signs instead of above them.
- **Theorem 1.14.** The sentence after Eq. (1.95) should read "where we have used Cauchy's formula to obtain the last line." Equation (1.103) should read $\frac{1}{2\pi i}\oint_{C_2} \frac{f(\zeta)}{z-\zeta}\,d\zeta = \sum_{j=1}^{\infty} \frac{b_j}{z^j}$.
- **Theorem 1.16.** The last equation of the proof should read $\lim_{z\to z_0}(z-z_0)f(z) = \lim_{z\to z_0}[c_{-1} + c_0(z-z_0) + c_1(z-z_0)^2 + \cdots] = c_{-1} = \mathrm{Res} f(z)|_{z=z_0}$
- **Example 1.15.** "... if we *had* reverse the sense..."
- **Exercise 1.4(i)** $f(z) = e^{iz^2}$.
- **Exercise 1.7. (iv)** $\int_C (z^3 + 3)\,dz$.

Chapter 2

- **Definition 2.10.** The definiteness property should read $\langle \boldsymbol{x}, \boldsymbol{x}\rangle = 0 \Leftrightarrow \boldsymbol{x} = 0$.
- **Proposition 2.1.** Equation 2.17 should read $(-2|\langle x,y\rangle|)^2 - 4||x||^2||y||^2 \leq 0$.
- **Exercise 2.5.(v)** Write down the matrix representation of T in the standard basis and use it to find $T(2,-1,-1)$.
- **Exercise 2.7** T is the projection onto the vector $(1,-5)$.

Chapter 3

- **Theorem 3.1** The first line of Eq. (3.46) should read $|u(t) - u_0| = \left|\int_{t_0}^{t} f(t', u(t'))dt'\right| \leq \left|\int_{t_0}^{t} |f(t', u(t'))|dt'\right|$. The sentence above Eq. (3.49) should read "... of successive approximations is now defined with ...".
- **Proposition 3.1** The sentence below Eq. (3.77) should read "In addition, if f satisfies the hypotheses of Picard's theorem, it is guaranteed the existence and uniqueness of a solution...".
- **Definition 3.21** The "dx" is missing in Eq. (3.121). It should read $\int_{-\infty}^{=\infty} \delta(x)\,f(x)\,dx = f(0)$. Equation (3.130) should read $I_f \equiv \lim_{\epsilon\to 0^+} \epsilon^{-1}\int_{-\infty}^{+\infty} f(x)g(x/\epsilon)dx = \lim_{\epsilon\to 0^+} \epsilon^{-1}\int_{-t}^{+t} f(x)\,g(x/\epsilon)\,dx$.

- **Corollary 3.3** Equation (3.145) should read $\int_{-\infty}^{+\infty} \Theta'(x) f(x) dx = -\int_{-\infty}^{+\infty} \Theta(x) f'(x) dx = -\int_{0}^{+\infty} f'(x) dx = f(0)$.
- **Definition 3.25** Since $\mathbb{K}(t,t')$ is a solution of the homogeneous equation, $\mathbb{G}(t,t')$ satisfies...
- **Lema 3.1.** Equation (3.187) should read $\int_a^b u_i(x)\, u_j(x)\, \rho(x) dx = 0 \quad \text{if} \quad \lambda_i \neq \lambda_j$
- **Theorem 3.6.** The first sentence in the proof should read: "We first show that *(ii)* holds. It is evident that if $G(x,x')$ exists then (3.196) is a solution of (3.194), because $L[u(x)] = \int_a^b L_x[G(x,x')]\, f(x')\, dx' = \int_a^b \delta(x-x')\, f(x')\, dx' = f(x)$; note that L_x acts on the first variable, which is unaffected by the integral." The sentence after Eq. (3.198) should read: "Integration of (3.194) over $[x'-\epsilon, x'+\epsilon]$ (with $\epsilon > 0$), $-\int_{x'-\epsilon}^{x'+\epsilon} \frac{d}{dx}[pG(x,x')]\, dx + \int_{x'-\epsilon}^{x'+\epsilon} q(x)\; G(x,x') dx = \int_{x'-\epsilon}^{x'+\epsilon} \delta(x-x')\; dx$, leads to $-[p(x)\; G'(x,x')]_{x=x'-\epsilon}^{x=x'+\epsilon} + \int_{x'-\epsilon}^{x'+\epsilon} q(x)\; G(x,x')\; dx = 1$.." Equation (3.200) should read: $\lim_{\epsilon\to 0} \int_{x'-\epsilon}^{x'+\epsilon} q(x)\; G(x,x') dx \to 0$.
- **Example 3.24** After Eq. (3.290) non-integral ν should read non-integer ν and after Eq. (3.291) integral n should read integer n.
- **Definition 3.40** Equation (3.371) should read $a_k = \frac{1}{L}\int_{-L}^{L} f(x)\; \cos\left(\frac{k\pi x}{L}\right)\; dx, \quad b_k = \frac{1}{L}\int_{-L}^{L} f(x)\; \sin\left(\frac{k\pi x}{L}\right)\; dx.$
- **Exercise 3.12** "Consider the boundary value problem $u'' + \lambda u = 0$, with $u(0) - u'(0) = 0$, $u(1) + u'(1) = 0$."
- **Exercise 3.14** "Let $L[u(x)] = -(x^2 u')'$, $x \in [1,2]$ be a Sturm-Liouville operator..."
- **Exercise 3.16** Show that: *(i)* Rodrigues formula is a solution of Legendre equation; *(ii)* $P_l^m(x) = (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$; *(iii)* $\int_0^a J_n^2(k_m^n r/a) r dr = \frac{1}{2}\, a^2\; J_{n+1}(k_m^n)$.

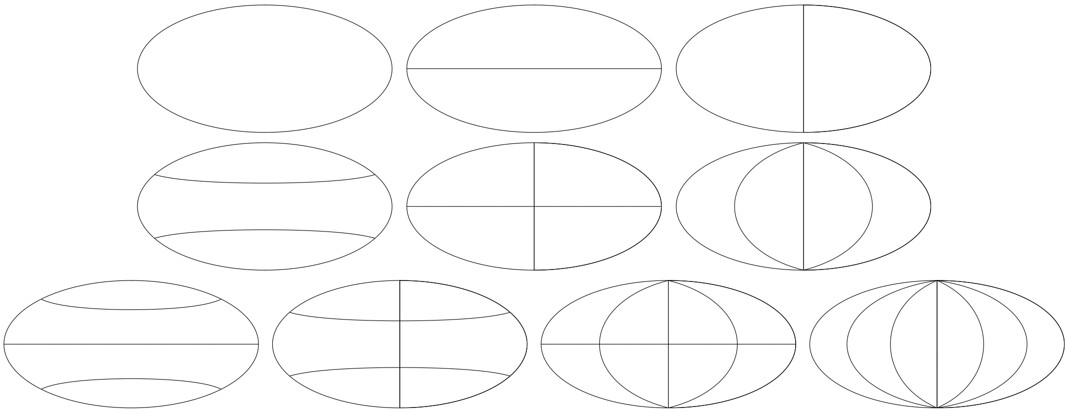
Chapter 4

- **Section 4.1** *Case II:* $b^2 - ac < 0$. The roots are conjugate complex: $\lambda_1 = \rho + i\sigma = \lambda_2^*$. Thus, $\xi = x + \lambda_1 y = x + \rho y + i\sigma y$ and $\eta = x + \lambda_2 y = x + \rho y - i\sigma y = \xi^*$. The standard form is

$$u_{\xi\xi^*} = 0\,, \tag{1}$$

  with general integral $u = \phi(\xi) + \psi(\xi^*)$.
- **Section 4.2.4** Equation (4.79) should read $G(\xi,\eta) = \frac{1}{2c}\Theta(\xi)\Theta(-\eta)$ and Eq. (4.80) becomes $G_{\xi\eta} = -\frac{1}{2c}\delta(\xi)\delta(\eta)$.
- **Section 4.3.2.** Equation (4.108) should read $K(x,t) = \frac{e^{-x^2/(4\alpha t)}}{2\pi} \int_{-\infty}^{+\infty} e^{-(ix/\sqrt{4\alpha t} - k\sqrt{\alpha t})^2} dk = \frac{e^{-x^2/(4\alpha t)}}{2\pi\sqrt{\alpha t}} \int_{-\infty}^{+\infty} e^{-z^2} dz$. The normalization condition before Eq. (4.111) should read $\int_{-\infty}^{+\infty} \delta(x-x') dx = 1$. In the line following Eq. (4.111) the second sentence should read "For a fixed $x \neq 0 \ldots$".
- **Section 4.3.3.** The title should read "Diffusion in a Finite Metal Bar." Equation (4.117) should read $T_n(t) = b_n e^{-(n\pi/L)^2 \alpha t}$ and Eq. (4.118) should read $u(x,t) = \sum_{n=1}^{\infty} b_n e^{-(n\pi/L)^2\alpha t} \sin(n\pi x/L)$. The expression between Eqs.(4.140) and (4.141) should read: $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left[i\omega + \alpha\left(\frac{n\pi}{L}\right)^2\right] \hat{g}_n(x',\omega) e^{i\omega(t-t')} d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{i\omega(t-t')} \frac{2}{L} \sin\left(\frac{n\pi x'}{L}\right)$.
- **Section 4.4.2.** The sentence before Eq. (4.165) should read: Assuming a solution of the form $u(r,\Omega) = R(r)\, Y(\Omega)$ we obtain.
- **Section 4.5.1.** In Eq. (4.208) $\mu = 0, 1, 2, 3$.
- **Section 4.5.2.** Equation (4.220) should read $(\Box^2 + m^2) G_F(x-x') = \delta^{(4)}(x-x')$. Equation (4.226) should read $S_F(p) = \frac{1}{p^2 - m^2 + (i\epsilon)^2}$.
- **Exercise 4.3.(v)** Determine the behavior of the solution for $t \to \infty$.

Coefficient	Earth	Moon	Mars
C_{20}	1.083×10^{-3}	$(0.200 \pm 0.002) \times 10^{-3}$	$(1.96 \pm 0.01) \times 10^{-3}$
C_{22}	0.16×10^{-5}	$(2.4 \pm 0.5) \times 10^{-5}$	$(-5 \pm 1) \times 10^{-5}$
S_{22}	-0.09×10^{-5}	$(0.5 \pm 0.6) \times 10^{-5}$	$(3 \pm 1) \times 10^{-5}$



- **Note added (Example 4.12.)** The gravity fields of the Earth, the Moon, and Mars have been described by a Laplace series with real eigenfunctions $U(r,\theta,\phi) = \frac{GM}{R}\left\{\frac{R}{r} - \sum_{n=2}^{\infty}\sum_{m=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\left[C_{nm}Y^l_{mn}(\theta,\phi) + S_{nm}Y^0_{mn}(\theta,\phi)\right]\right\}$, where M is the mass of the body and R is the equatorial radius. The real functions Y^l_{mn} and Y^0_{mn} are defined by $Y^l_{mn}(\theta,\phi) = P^m_n(\cos\theta)\cos(m\phi)$ and $Y^0_{mn}(\theta,\phi) = P^m_n(\cos\theta)\sin(m\phi)$. Satellites measurements have led to the numerical values given in the table. The nodal lines separating excess and deficit regions on the sphere for various (l,m) pairs are shown in the figure. The top row shows the $(0,0)$ monopole, and the partition of the sphere into two dipoles, $(1,0)$ and $(1,1)$. The middle row shows the quadrupoles $(2,0)$, $(2,1)$, and $(2,2)$. The bottom row shows the $l = 3$ partitions, $(3,0)$, $(3,1)$, $(3,2)$, and $(3,3)$.

Appendix C

- Following the sentence "By partial integration we obtain" the lower limit of integration after the first equality should be $-x + 1/2$.

Answers and comments on the excercises

- **Solution 1.3** $z^n = \alpha e^{i\beta}$ implies that $z = \alpha^{1/n}e^{i(\frac{\beta}{n}+\frac{2k\pi}{n})}$.
- **Solution 1.9** Letting ...
- **Solution 1.10** The dz is missing in the second relation of *(ii)*.
- **Solution 1.13(i)** $\lim_{z\to 1}\frac{\pi z(1-z^2)}{\sin(\pi z)} = \lim_{z\to 1}\frac{\pi z(1-z^2)}{\sin(\pi z+\pi-\pi)} = \lim_{z\to 1}\frac{\pi z(1-z)(1+z)}{\sin[\pi(z-1)+\pi]}$. Using the trigonometric identity, $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$, the equation becomes $\lim_{z\to 1}\frac{\pi z(1-z)(1+z)}{-\sin\pi(z-1)} = 2$. Duplicating this procedure $\lim_{z\to -1}\frac{\pi z(1-z^2)}{\sin(\pi z)} = -2$.
- **Solution 1.13(iii)** We know that around $z_0 = 0$, $\frac{1}{\sin(\pi z)} = \csc(\pi z) = \frac{1}{\pi z}\left[1 + \frac{\pi z^2}{3!} + O(\pi z^4)\right]$. This is so because csc (z) has a simple pole at $z = 0$. The residue is: $\lim_{z\to 0}\frac{z}{\sin z} = 1$. This implies that the first coeficient in the Laurent expansion of csc (z) is $c_{-1} = 1$. The Laurent series is then given by $\csc(z) = 1/z + c_0 + c_1 z + c_2 z^2 + \cdots$. To determine the other coefficients you can use the relation csc (z) $\sin(z) = 1$. Using the Taylor expansing for $\sin(z) = z - z^3/3! + z^5/5! + \cdots$ the previous relation can be rewritten as $(1/z + c_0 + c_1 z + ...)(z - z^2/3! + z^5/5! + ...) = 1$. By comparison of the coefficients it follows that $c_0 z = 0$ and $(-z^3/3!)(1/z) + c_1 z^2 = 0$, yielding $c_0 = 0$ and $c_1 = 1/6$.

- **Solution 1.16(i)** $f(z) = \frac{(z-k\pi+k\pi)^2}{\sin^2 z} = \sum_{-\infty}^{=\infty} c_n(z-k\pi)^n = \left[(z-k\pi)^2 + 2k\pi(z-k\pi) + k^2\pi^2\right] \frac{1}{(z-k\pi)^2}\left[1 + \frac{(z-k\pi)^2}{3!} + \cdots\right]^2$.
- **Solution 1.16(ii)** Res $f(z)\big|_{z=\pm i} = \frac{d}{dz}\left(\frac{z^2-1}{(z\pm i)^2}\right)\Big|_{z=\pm i} = \frac{\pm 2zi+2}{(z\pm i)^3}\Big|_{z=\pm i} = 0$.
- **Solution 1.17(i)** To calculate the residues of $\tan z$ apply the L'Hospital's rule.
- **Solution 2.1.** $T(c\boldsymbol{x}) = cT(\boldsymbol{x}) = c\lambda\boldsymbol{x} = \lambda(c\boldsymbol{x})$.
- **Solution 2.4. and 2.5.** The bars of $\neq$ are shifted 4 lines upward.
- **Solution 2.5.(ii)** The factors $(-2, 4, -1)$ are never applied to the transformations of the matrix vectors. The correct solution is $T(-2, 4, -1) = (0, 6, -8)$.
- **Solution 2.5.(iii)** $T(-4, 5, 1) = (2\times(-4)-5, 2\times(-5)-3\times(-4), -4-1) = (-13, 2, -5)$.
- **Solution 2.5.(vi)** The inverse matrix is $\begin{pmatrix} 0 & -4/3 & 1/3 \\ 0 & 1 & 0 \\ -1/2 & -2 & 1/2 \end{pmatrix}$.
- **Solution 2.6.(iii)** The rotation matrix is clockwise (it is counterclockwise in the original question), though it doesn't affect the answer. $T(2, 2) = \begin{pmatrix} -1-\sqrt{3} \\ \sqrt{3}-1 \end{pmatrix}$.
- **Solution 2.8** $U^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$
- **Solution 3.2(i)** The units of C should be minutes^{-1}.
- **Solution 3.2(ii)** $y = Ke^x - 3$, where $K = e^C$. Since $y(0) = -2$, then $K = 1$ and therefore $y = e^x - 3$.
- **Solution 3.3(i)** In general, at any given point the lines of force are tangent to the gradient $F \propto -\nabla\Phi = \partial_x\Phi\hat{\imath} + \partial_y\Phi\hat{\jmath}$. In the last sentence "equipotentail" should read "equipotential."
- **Solution 3.3(ii)** In the last sentence "equipotential lines" should read "streamlines."
- **Solution 3.4** The first sentence should read "In general, for $\frac{dy}{dx} = f(x, y)$: *(a)* if $f(x, y)$ is continuous $\Rightarrow$ existence of solution; *(b)* if $\partial_y f(x, y)$ is continuous $\Rightarrow$ uniqueness of solution."
- **Solution 3.5(i)** For $u(0) = 1$ and $f(t, u) = u^2$ Picard's iteration leads to $u^{(0)} = u(0) = 1$; $u^{(1)} = \int_0^t dt' + u^{(0)} = t + 1$; $u^{(2)} = u^{(0)} + \int_0^t (t'+1)^2 dt' = \frac{1}{3}t^3 + t^2 + t + 1$; $u^{(3)} = u^{(0)} + \int_0^t (\frac{1}{3}t'^3 + t'^2 + t' + 1)^2 dt' = \frac{1}{63}t^7 + \frac{2}{15}t^4 + \frac{5}{15}t^5 + \frac{8}{12}t^4 + t^3 + t^2 + t + 1$; $\cdots$; $u^{(n)} = \cdots + t^n + t^{n-1} + \cdots + t + 1$. You can now guess that the solution to the differential equation is of the form $u = \sum_{n=1}^{\infty} t^n = (1-t)^{-1}$, and you know this series converges for $|t| < 1$. If you determine the region where the solution exists using the hypotheses of Picard's theorem, you obtain $|t| < a$ and $|u-1| < b$, and so the region of convergence is bounded by $\min\{a, b/(b+1)^2\}$. Search for the position of the maximum, $[b/(b+1)^2]' = \frac{1}{(b+1)^2} - \frac{2b}{(b+1)^2} = 0 \to b = 1$, and hence for $a > 1/4$, $\min\{a, b/(b+1)^2\} = 1/4$. Then if you do not know the domain of convergence of the series, you can only ensure that the solution exists for $|t| < \frac{1}{4}$.
- **Solution 3.5(ii)** For $u(1) = 0$ and $f(t, u) = t - u^2$ Picard's iteraction leads to $u^{(0)} = 0$; $u^{(1)} = u^{(0)} + \int_1^t t'dt' = \frac{1}{2}t^2 - \frac{1}{2}$; $u^{(2)} = u^{(0)} + \int_1^t [t' - (\frac{1}{2}t'^2 - \frac{1}{2})^2]dt' = \frac{1}{2}(t^2-1) + \frac{1}{20}(1-t^5) + \frac{1}{6}(t^3-1) + \frac{1}{4}(1-t)$. The region of validity for the approximation in the rectangle $|t-1| < a$ and $|u| < b$ is $\min\{a, b/(a-b)^2)\}$.
- **Solution 3.8** The second *(iv)* should be *(v)* and *(v)* should be *(vi)*. The answer of *(vi)* should read $f(\pi) + f'(2\pi) + f''(b)$.
- **Solution 3.10** $M(\epsilon) \to -\langle T_2, f\rangle$
- **Solution 3.12** There is an extra open parenthesis in the third line. The boundary conditions imply that $-uu'|_0^1 = -u(1)u'(1) + u(0)u'(0) = |u(1)|^2 + |u(0)|^2 \geq 0$. *(iii)* Let $0 < \lambda_1 < \lambda_2 < \cdots$ be the eigenvalues. Graphically, you can establish that $1 < \sqrt{\lambda_1} < \pi/2$, $\pi/2 < \sqrt{\lambda_2} < 3\pi/2$, $\cdots$, $(2n-3)\pi/2 < \sqrt{\lambda_n} < (2n-1)\pi/2$, for $n = 3, 4, \cdots$; see Fig. 1. Furthermore, $\sqrt{\lambda_n} \approx (n-1)\pi$ for a large n.

- **Solution 3.13** "Likewise, a solution to $-u'' = 0$, satisfying $u_2'(1) = 0$, is ..."

- **Solution 3.14** The general solution of the homogeneous equation is $u(x) = c_1 + c_2 x^{-1}$.

- **Solution 3.16(i)** Let $y = (x^2-1)^l \Rightarrow \frac{dy}{dx} = l(x^2-1)^{l-1}2x$, and so $(x^2-1)\frac{dy}{dx} = l(x^2-1)^l 2x = 2xly$. Differentiate the previous equation $l+1$ times to obtain $\frac{d^{l+1}}{dx^{l+1}}[(x^2-1)\frac{dy}{dx}] = \frac{d^{l+1}}{dx^{l+1}}[2xly]$. Using the Leibnintz formula (3.258) you can rewrite the left-hand-side of the previous equation as $\underbrace{(x^2-1)\frac{d^{l+2}y}{dx^{l+2}}}_{s=0} + \underbrace{(l+1)2x\frac{d^{l+1}y}{dx^{l+1}}}_{s=1} + \underbrace{\frac{l(l+1)}{2}2\frac{d^l y}{dx^l}}_{s=2}$, whereas the right-hand-side becomes $\underbrace{2xl\frac{d^{l+1}y}{dx^{l+1}}}_{s=0} + \underbrace{2l(l+1)\frac{d^l y}{dx^l}}_{s=1}$. Substituting you obtain $(x^2-1)\frac{d^{l+2}y}{dx^{l+2}} + 2x\frac{d^{l+1}y}{dx^{l+1}} - l(l+1)\frac{d^l y}{dy^l} = 0$, which after manipulation becomes $(1-x^2)\frac{d^{l+2}y}{dx^{l+2}} - 2x\frac{d^{l+1}y}{dx^{l+1}} + l(l+1)\frac{d^l y}{dy^l} = 0$. It follows that $(1-x^2)\frac{d^2}{dx^2}[\frac{1}{2^l l!}\frac{d^l y}{dx^l}] - 2x\frac{d}{dx}[\frac{1}{2^l l!}\frac{d^l y}{dx^l}] + l(l+1)[\frac{1}{2^l l!}\frac{d^l y}{dx^l}]$. Therefore, Rodrigues formula (3.251) satisfies Legendre equation (3.241).

- **Solution 3.18** To integrate $a_0 = \frac{1}{L}\int_{-L}^{L}\sum_{k=-\infty}^{\infty}\delta(x-a+2kL)dx$ perform the change of variables $u = x-a+2kL \Rightarrow du = dx$; substituting $a_0 = \frac{1}{L}\int_{u_1}^{u_2}\sum_{k=-\infty}^{\infty}\delta(u)du = \frac{1}{L}\sum_{k=-\infty}^{\infty}\int_{u_1}^{u_2}\delta(u)\,du = \frac{1}{L}$, where $u_1 = -L-a+2kL$ and $u_2 = L-a+2kL$. For $a_m = \frac{1}{L}\int_{-L}^{L}\sum_{k=-\infty}^{\infty}\delta(x-a+2kh)\cos[m\pi(x-a)/L]\,dx$ perform the same change of variables, $a_m = \frac{1}{L}\int_{u_1}^{u_2}\sum_{k=-\infty}^{\infty}\delta(u)\cos[m\pi(u-2kL)/L]du = \frac{1}{L}\sum_{k=-\infty}^{\infty}\int_{u_1}^{u_2}\delta(u)\cos[m\pi(u-2kL)/L]du = \frac{1}{L}$. Note that $b_m = \frac{1}{L}\int_{-L}^{L}\sum_{k=-\infty}^{\infty}\delta(x-a+2kh)\sin[m\pi(x-a)/L]\,dx = 0$, therefore $\sum_{k=-\infty}^{\infty}\delta(x-a+2kL) = \frac{1}{L}\{\frac{1}{2} + \sum_{m=1}^{\infty}\cos[\frac{m\pi(x-a)}{L}]\}$.

- **Solution 3.19** In the second line, $e^{-k\sigma^2}$ should read $e^{-(k\sigma)^2}$.

- **Solution 4.4** In the second to last line where b_0 is defined, there is a 1/L before the integral that should be a 2/L instead; i.e., $b_0 = \frac{2}{L}\int_0^L f(x)\,dx$.

- **Solution 4.11** Hence $X''(x) = -\lambda X(x)$, $Y''(y) = -\mu Y(y)$, $\ddot{T}(t) = -c^2(\lambda+\mu)T(t)$.

- **Solution 4.12** The last part of this problem, which is in the missing page 202, should read "Therefore, $\phi(x) = \frac{1}{2}[f(x) + \frac{1}{c}\int_{\infty}^{x} q(\zeta)d\zeta]$ and $\psi(x) = \frac{1}{2}[f(x) - \frac{1}{c}\int_{\infty}^{x} q(\zeta)d\zeta]$, so $u(x,t) = \phi(x+ct) + \psi(x-ct) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} q(\zeta)\,d\zeta$, which is the d'Alembert solution."

- **Solution 4.13** The B_n are the Fourier coefficients of the initial displacement: $f(x) = \sum_{n=1}^{\infty} B_n \sin(\frac{n\pi x}{L})$. Using the trigonometric identity with $\alpha = \frac{n\pi x}{L}$ and $\beta = \frac{n\pi ct}{L}$, you have $u(x,t) = \frac{1}{2}\sum_{n=1}^{\infty} B_n[\sin(\frac{n\pi x}{L} + \frac{n\pi ct}{L}) + \sin(\frac{n\pi x}{L} - \frac{n\pi ct}{L})] = \frac{1}{2}\{\sum_{n=1}^{\infty} B_n \sin[\frac{n\pi}{L}(x+ct)] + \sum_{n=1}^{\infty} B_n \sin[\frac{n\pi}{L}(x-ct)]\} = \frac{1}{2}[f(x+ct) + f(x-ct)]$, which is the d'Alembert solution.

- **Solution 4.14** Setting $u(r,\theta) = R(r)\Theta(\theta)$ and applying the method of separation of variables you find $R''\Theta + R'\Theta/r + R\Theta''/r^2 = -k^2 R\Theta$, and so $r^2R''/R + rR'/R + r^2k^2 = -\Theta''/\Theta$. The r-dependent terms are now isolated on the left-hand side, while the θ-dependent term is isolated on the right hand side, and so by the usual separation of variables argument both sides must be equal to a constant, which you denote say by λ^2. This leads to $r^2R'' + rR' + (-\lambda^2 + r^2k^2)R = 0$ and $\Theta'' + \lambda^2\Theta = 0$. The general solution of the latter, $\Theta(\theta) = c_1\cos(\lambda\theta) + c_2\sin(\lambda\theta)$, does not satisfy $\Theta(\theta) = \Theta(\theta + 2\pi)$ unless $\lambda = n \in \mathbb{N}$. Next, you have to look for solutions of $r^2R'' + rR' + (-n^2 + r^2k^2)R = 0$ that remain bounded. If you make the change of variables $x = rk \Rightarrow R' = \widetilde{R}'/k$ it follows that $x^2\widetilde{R}'' + x\widetilde{R}' + (x^2 - n^2)\widetilde{R} = 0$, which is the differential equation of the Bessel function of order n. Actually, the equation has a pair of linearly independent solutions, but only the Bessel function $J_n(x)$ is regular as $x \to 0$. (The other solution is the Bessel function of order n of the second kind. It is unbounded as $x \to 0$.) Thus the solution is $R(r) = \widetilde{R}(x(r)) = J_n(x(r)) = J_n(kr)$. At this point you have an infinite set of solutions of the Helmholtz equation that are periodic in θ and regular at the origin, *viz* $\cos(n\theta)J_n(kx)$, with $n = 0,1,2,\cdots$ and $\sin(n\theta)J_n(kx)$, with $n = 1,2,3,\cdots$. You can now set $u(r,\theta) = \sum_{n=0}^{\infty} A_n \cos(n\theta)\, J_n(kr) + \sum_{n=1}^{\infty} B_n \sin(n\theta)J_n(kr)$ and impose the boundary condition $f(\theta) = u(1,\theta) = \sum_{n=0}^{\infty} A_n\cos(n\theta)J_n(k) + \sum_{n=1}^{\infty} B_n\sin(n\theta)J_n(k)$. However, $f(\theta)$ viewed as a function on the circle also has a unique Fourier expansion $f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(n\theta) + \sum_{n=1}^{\infty} b_n\sin(n\theta)$, for which the coefficients a_n and b_n are determined by $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$ and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(n\pi)dx$.

Compatibility of the two expressions for $f(\theta)$ requires $A_n = \frac{1}{J_n(k)}\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\cos(nx)\,dx$, with $n = 0, 1, 2, 3, \cdots$, and $B_n = \frac{1}{J_n(k)}\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)$, with $n = 1, 2, 3, \cdots$.

- **Solution 4.15** The first part in the missing page 202 reads as follows: *(i)* You must solve the equation $y_{tt} = \frac{g}{b(x)}\partial_x[A(x)y_x]$, where $b(x) = cx$, $A(x) = hcx$, with c constant, see Fig. 2. Without loss of generality you can use separable variables, $y(x,t) = X(x)T(t)$. The differential equation then becomes $X(x)T''(t) = \frac{g}{b(x)}T(t)[A'(x)X'(x) + A(x)X''(x)]$ and so $T''(T)/T(t) = \frac{g}{b(x)}[A'(x)X'(x) + A(x)X''(x)]/X(x) = -\lambda$. The solution of $T''(t) + \lambda T(t) = 0$ is $T(t) = \mathcal{A}\sin(\sqrt{\lambda}t) + \mathcal{B}\cos(\sqrt{\lambda}t)$, where $\mathcal{A}$ and $\mathcal{B}$ are constants. With the substitution $k^2 = \lambda/(gh)$, the X-equation, $\frac{g}{b(x)}[A'(x)X'(x) + A(x)X''(x)] + \lambda X(x) = 0$, becomes $X''(x) + X'(x)/x + k^2X(x) = 0$. The solution then takes the form $X(x) = CJ_0(kx) + \mathcal{D}Y_0(kx)$, where $J_0(kx)$ is the Bessel function of the first kind, Y_0 is the Bessel function of the second kind, and C, $\mathcal{D}$ are constants. Since $y(x,t)$ has to remain bounded as $x \to 0$ you should set $\mathcal{D} = 0$, because $\lim_{x\to 0} Y_0(kx) \to -\infty$. Then $y(x,t)[\mathcal{A}\sin(\sqrt{\lambda}t) + \mathcal{B}\cos(\sqrt{\lambda}t)]CJ_0(kx)$. In order to satisfy the boundary condition $y(L,t) = a\cos(\omega t) = [\mathcal{A}\sin(\sqrt{\lambda}t) + \mathcal{B}\cos(\sqrt{\lambda}t)]CJ_0(kL)$,
- **Solution 4.19** The 4th sentence should read "Writing $\varphi = R(r)\Theta(\theta)$ you can extract the following decouple differential equations: $\Theta'' = -n^2\Theta$ and $R'' + \frac{R'}{r} + (k^2 - \frac{n^2}{r^2})R = 0$."