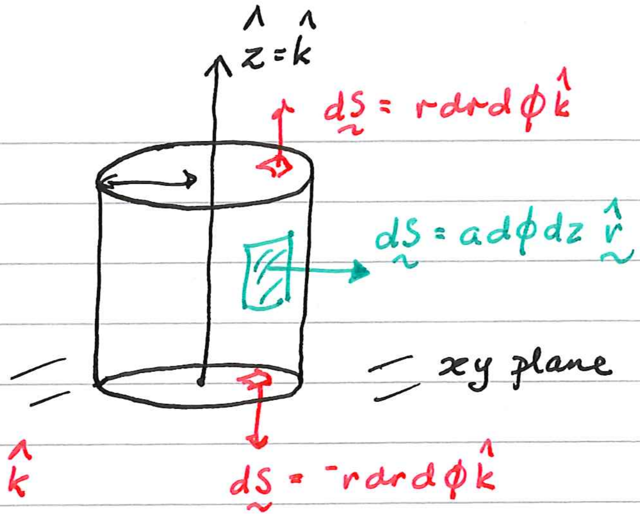
$$\underset{\sim}{v} = x\underset{\sim}{\hat{i}} + y\underset{\sim}{\hat{j}}$$




Top: $\underset{\sim}{v}\cdot\underset{\sim}{dS} = (x\hat{i} + y\hat{j})\cdot r\,dr\,d\phi\,\hat{k}$

$= 0$

Bottom: also zero, as $\underset{\sim}{v}$ has no $\hat{k}$ component.

Side wall: [Not $r$ ~ we are on the $r = a$ surface]

$$\underset{\sim}{v}\cdot\underset{\sim}{dS} = (a\cos\phi\,\underset{\sim}{\hat{i}} + a\sin\phi\,\underset{\sim}{\hat{j}})\cdot a\,d\phi\,dz(\cos\phi\,\underset{\sim}{\hat{i}} + \sin\phi\,\underset{\sim}{\hat{j}})$$

$$= a^2(\cos^2\phi + \sin^2\phi)\,d\phi\,dz$$

$$\Rightarrow \int_{\text{Surface}} \underset{\sim}{v}\cdot\underset{\sim}{dS} = \int_{z=0}^{h}\int_{\phi=0}^{2\pi} a^2\,d\phi\,dz = \boxed{a^2 2\pi h}$$

Notice the mix of polar coefficients and Cartesian vectors – all good.

But we could spot that $\underset{\sim}{v} = r\underset{\sim}{\hat{r}}$

So on the wall $\underset{\sim}{v} = a\hat{r}$

Then

$$\underset{\sim}{v}\cdot\underset{\sim}{dS} = a\underset{\sim}{\hat{r}}\cdot a\,d\phi\,dz\,\underset{\sim}{\hat{r}}$$

$$= a^2\,d\phi\,dz$$

$\underset{\sim}{\hat{r}}\cdot\underset{\sim}{\hat{r}} = 1$

$$\int_S \underset{\sim}{v}\cdot\underset{\sim}{dS} = \text{etc}\ldots$$