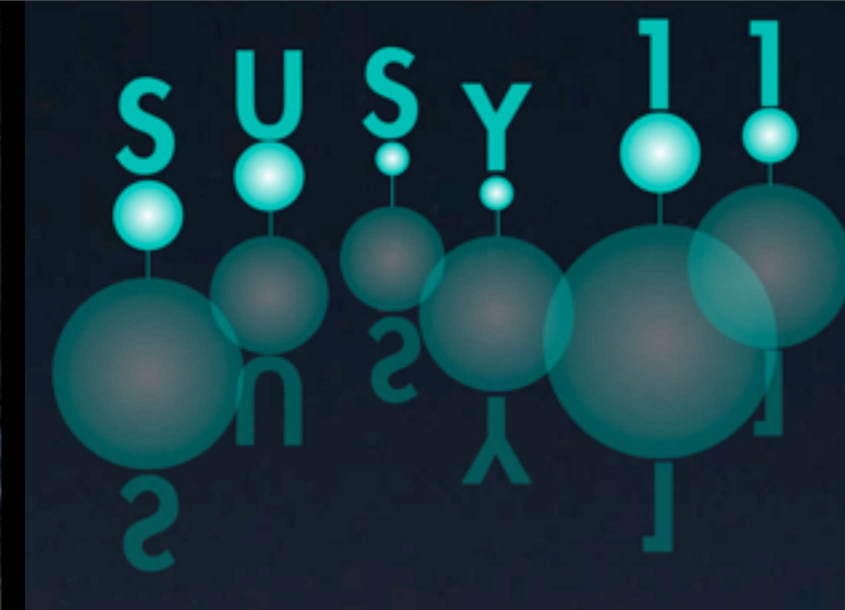

PARTICLE PHYSICS 2011

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Inelastic Scattering

In switching from a muon to a proton target

we replaced lepton current $j^\mu(\sim \overline{u}\gamma^\mu u)$ by proton current $J^\mu(\sim \overline{u}\Gamma^\mu u)$

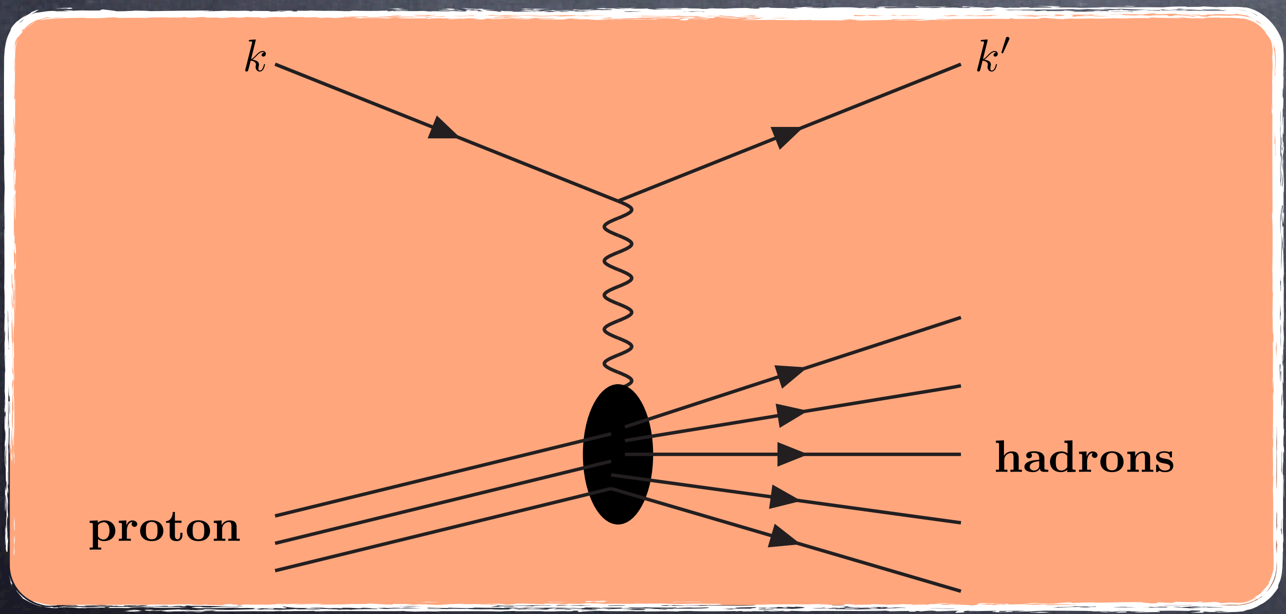
This is inadequate to describe inelastic events
because final state is not a single fermion
described by a Dirac $\bar{u}$ entry in matrix current

Therefore ☛ J^μ must have a more complex structure

Square of invariant amplitude $\overline{|\mathfrak{M}|^2} = \dfrac{e^4}{q^4} L^{\mu\nu}_{(e)} L^{(\mu)}_{\mu\nu}$ is generalized to

$$\overline{|\mathfrak{M}|^2} \propto L^{(e)}_{\mu\nu} \; W^{\mu\nu}$$

Leptonic part of diagram above photon propagator is left unchanged



☛ $L^{\mu\nu}_{(e)} = \dfrac{1}{2}\mathrm{Tr}[(\not{k}' + m_e)\,\gamma^\mu\,(\not{k} + m_e)\,\gamma^\nu]$

Hadronic Tensor

$W^{\mu\nu}$ parametrizes our ignorance of form of current at end of propagator

Most general form of tensor $W^{\mu\nu}$
is constructed out of $g^{\mu\nu}$ and independent momenta p and q
(with $p' = p + q$)

γ^μ is not included as we are parametrizing $|\mathfrak{M}|^2$
which is already summed and averaged over spins

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}(p^\mu q^\nu + q^\mu p^\nu)$$

We have omitted antisymmetric contributions to $W^{\mu\nu}$
since their contribution to cross section vanishes
because tensor $L^{(e)}_{\mu\nu}$ is symmetric

Note omission of W_3 in our notation
this spot is reserved for a parity violating structure function
when a neutrino beam is substituted for the electron beam
so that the virtual photon probe is replaced by a weak boson

Vertex Constraints

Current conservation at vertex requires $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

$$\begin{aligned} 0 &= q_\nu W^{\mu\nu} \\ &= -q_\nu W_1 g^{\mu\nu} + \frac{W_2}{M^2}(p\,.\,q)p^\mu + \frac{W_4}{M^2}q^2 q^\mu + \frac{W_5}{M^2}[q^2 p^\mu + (p\,.\,q)q^\mu] \end{aligned}$$

Setting coefficients of q^μ and p^μ to zero we find

$$-W_1 + \frac{W_4}{M^2}q^2 + \frac{W_5}{M^2}(p\,.\,q) = 0$$

$$\frac{W_2}{M^2}(p\,.\,q) + \frac{W_5}{M^2}q^2 = 0$$

which lead to

$$W_5 = -\frac{p\,.\,q}{q^2}W_2$$

$$W_4 = \left(\frac{p\,.\,q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2}W_1$$

New invariants

- Only 2 of 4 inelastic structure functions are independent and we can write without loss of generality

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

W_i are functions of Lorentz scalar variables that can be constructed from 4-momenta at hadronic vertex

- Unlike elastic scattering ☛ there are two independent variables and we choose

$$q^2 \quad \text{and} \quad \nu \equiv \frac{p \cdot q}{M}$$

- Invariant mass W of final hadronic system is related to ν and q^2 by

$$W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$$

Tensor Product

To evaluate cross section for $ep \to eX$

straightforward repetition of calculation for $e^-\mu^- \to e^-\mu^-$ scattering

Using
$$L^{\mu\nu}_{(e)} = \frac{1}{2}\mathrm{Tr}(\not{k}'\gamma^\mu \not{k}\gamma^\nu) + \frac{1}{2}m_e^2\mathrm{Tr}(\gamma^\mu\gamma^\nu)$$
$$= 2(k'^\mu k^\nu + k'^\nu k^\mu - (k'.k - m_e^2)g^{\mu\nu})$$

and noting $q^\mu L^{(e)}_{\mu\nu} = q^\nu L^{(e)}_{\mu\nu} = 0$ we find

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = 4W_1(k\,.\,k') + \frac{2W_2}{M^2}[2(p\,.\,k)(p\,.\,k') - M^2 k\,.\,k']$$

In laboratory frame this becomes

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = 4EE'\left\{W_2(\nu,q^2)\,\cos^2\frac{\theta}{2} + 2\,W_1(\nu,q^2)\,\sin^2\frac{\theta}{2}\right\}$$

recall ☛ $q^2 \simeq -2k\,.\,k' \simeq -2EE'(1-\cos\theta) = -4EE'\sin^2(\theta/2)$

Differential Cross Section

Including flux factor and phase space factor for outgoing electron

$$d\sigma = \frac{1}{4\left[(k\,.\,p)^2 - m^2M^2\right]^{1/2}}\left\{\frac{e^4}{q^4}L^{\mu\nu}_{(e)}W_{\mu\nu}4\pi M\right\}\frac{d^3k'}{2E'(2\pi)^3}$$

extra factor of $4\pi M$ arises because of $W^{\mu\nu}$ normalization

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{\rm lab} = \frac{1}{16\pi^2}\frac{E'}{E}\frac{\overline{|\mathfrak{M}|^2}}{4\pi M} \qquad \clubsuit$$

$$= \frac{(4\pi\alpha)^2}{16\pi^2q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}$$

$$= \frac{4\alpha^2E'^2}{q^4}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$

$$= \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$

to obtain final result we neglect mass of electron and used

$$q^2 \simeq -2k\,.\,k' \simeq -2EE'(1-\cos\theta) = -4EE'\sin^2(\theta/2)$$

Form Factor Summary

Convenient to express $d\sigma$ with respect to invariants ν and Q^2

$$\left.\frac{d\sigma}{dQ^2\, d\nu}\right|_{\text{lab}} = \frac{\pi}{EE'} \left.\frac{d\sigma}{dE' d\Omega}\right|_{\text{lab}}$$

$$= \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \left\{ W_2(Q^2, \nu) \cos^2 \frac{\theta}{2} + 2W_1(Q^2, \nu) \sin^2 \frac{\theta}{2} \right\}$$

It will be useful to make a compendium of our results on form factors

We keep to laboratory kinematic and neglect mass of electron

Differential cross section in energy (E) and angle (θ) of scattered e^-

can be written as

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{\text{lab}} = \frac{4\alpha^2 E'^2}{q^4} \left\{ \quad \right\}$$

where...

$$\left\{ \right\}$$

- For a muon target of mass m

  -- or quark target of mass m after substitutions $\alpha^2 \to \alpha^2 e_q^2$ --

$$\left\{ \quad \right\}_{e\mu \to e\mu} = \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2} \right) \delta \left(\nu + \frac{q^2}{2m} \right)$$

- For elastic scattering from a proton target

$$\left\{ \quad \right\}_{ep \to ep} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \delta \left(\nu + \frac{q^2}{2M} \right)$$

$\tau = -q^2/4M^2$ and M is mass of proton

- When proton target is broken up by bombarding electron

$$\left\{ \quad \right\}_{ep \to eX} = W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2}$$

Parton structure functions

Making use of delta function can be integrated over E'

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm lab} = \left(\frac{\alpha^2}{4E^2\,\sin^4(\theta/2)}\right)\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

Sign that there are structureless particles inside a complex system is that for small wavelengths proton described by suddenly starts behaving like a free Dirac particle and turns into

Proton structure functions thus become simply

$$2W_1^{\rm point} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right) \qquad W_2^{\rm point} = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

Point notation reminds us q is structureless particle

$Q^2 \equiv -q^2$ and m is quark mass

Using $\delta(x/a) = a\delta(x)$ parton structure functions can be rearranged to be dimensionless structure functions of ratio $Q^2/2m\nu$

(AND not Q^2 and ν independently)

$$2mW_1^{\rm point}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right),$$

$$\nu W_2^{\rm point}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

Elastic scattering form factor

Parton behavior can be contrasted with that for ep elastic scattering

for simplicity we set $\kappa = 0$ so that $G_E = G_M \equiv G$

then comparing and we have

$$W_1^{\text{elastic}} = \frac{Q^2}{4M^2} G^2(Q^2)\,\delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_2^{\text{elastic}} = G^2(Q^2)\,\delta\left(\nu - \frac{Q^2}{2M}\right)$$

A mass scale is explicitly present ☛ reflecting inverse size of proton

$G(Q^2)$ cannot be rearranged as function of single dimensionless variable

As Q^2 increases above $(0.71\ \text{GeV})^2$ form factor depresses elastic scattering

proton is more likely to break up

Point structure functions depend only on dimensionless variable $Q^2/2m\nu$

m merely serves as a scale for momenta

and no scale of mass is present

BJORKEN SCALING

In limit $Q \to \infty$ and $2M\nu \to \infty$ (such that $\omega = 2(q\,.\,p)/Q^2 = 2M\nu/Q^2$)

structure functions would have following property

$$MW_1(\nu, Q^2) \xrightarrow[\text{large } Q^2]{} F_1(\omega)\,,$$
$$\nu W_2(\nu, Q^2) \xrightarrow[\text{large } Q^2]{} F_2(\omega)$$

we have introduced proton mass instead of quark mass
to define dimensionless variable ω

Presence of free quarks is signaled by fact that:

inelastic structure functions are independent of Q^2 at given value of ω

IN LATE SIXTIES

deep inelastic scattering experiments conducted by SLAC-MIT Collaboration

showed that at sufficiently large $Q^2 \gg \Lambda^2_{\rm QCD}$

structure functions are approximately independent of Q^2

Parton Model

Basic idea: Represent inelastic scattering
as quasi-free scattering from point-like constituents within proton
when viewed from a frame in which proton has infinite momentum

Imagine reference frame in which target p has very large 3-momentum

$$\vec{p} \gg M$$

so-called infinite momentum frame

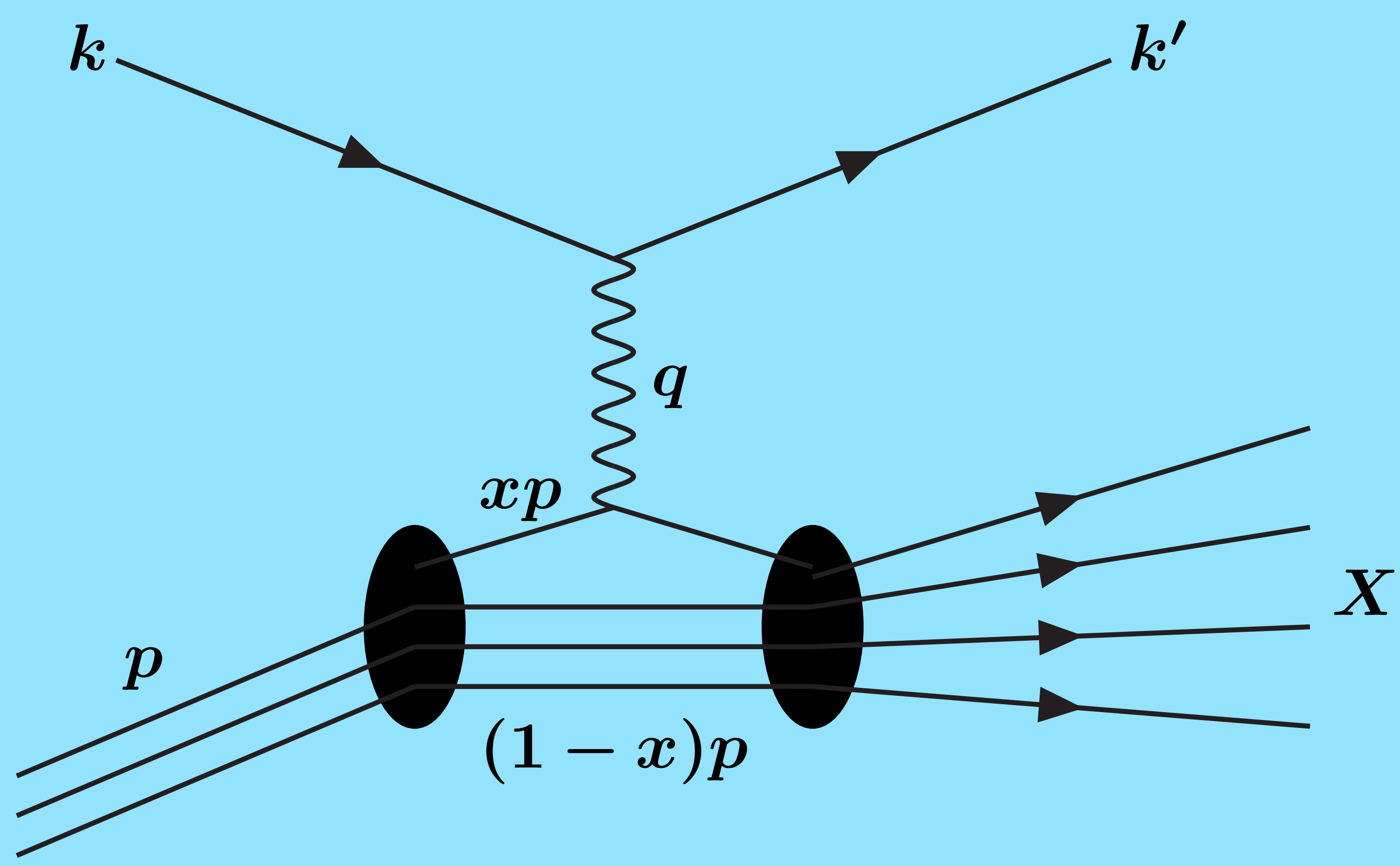
In this frame ☛ proton is Lorentz-contracted into a thin pancake
and lepton scatters instantaneously

Proper motion of constituents within p is slowed down by time dilation

We envisage proton momentum p as being made of partons
carrying longitudinal momentum $p_i = x_i p$
where momentum fractions x_i satisfy

$$0 \leq x_i \leq 1 \quad \text{and} \quad \sum_{\text{partons}\,(i)} x_i = 1$$

Kinematics of lepton-proton scattering in parton model



Infinite Momentum Frame

Assigning a variable mass xM to parton is of course OUT OF QUESTION!

If parton's momentum is xp

its energy can only be xE if we put $m = M = 0$

Equivalently ☛ proton can only emit parton moving parallel to it

$$p_\perp = 0$$

Because of the large momentum transfer ☛ $-q^2 \gg M$

interactions between partons can be neglected

and individual current-parton interactions may be treated incoherently

$$\left.\frac{d\sigma}{dtdu}\right|_{ep\to eX} = \sum_{\text{partons}(i)} \int dx\, f_i(x) \left.\frac{d\sigma}{dtdu}\right|_{eq_i\to eq_i}$$

$f_i(x)$ indicates probability of finding constituent i inside proton

and sum is over all contributing partons

Mandelstam variables carry hats

Assuming $s \gg M$ ☛ invariant variables of unpolarized scattering amplitude

$$\overline{|\mathfrak{M}|^2} = \frac{8e^4}{(k-k')^4}[(k' \,.\, p')(k \,.\, p) + (k' \,.\, p)(k \,.\, p')]$$
$$= 2e^4 \frac{s^2+u^2}{t^2}$$

become

$$\hat{s} = (k+xp)^2 \simeq x(2k\,.\,p) \simeq xs\,,$$
$$\hat{t} = (k-k')^2 = t = q^2\,,$$
$$\hat{u} = (k'-xp)^2 \simeq x(-2k'\,.\,p) \simeq xu$$

therefore

$$-\frac{t}{s+u} = -\frac{q^2}{2p\,.\,q} = \frac{Q^2}{2M\nu} = x$$

Consequently ☛ $x(s+u)+t=0$ or $\hat{s}+\hat{u}+\hat{t}=0$

Invariant scattering amplitude becomes

$$\overline{|\mathfrak{M}|^2} = 2\,(4\pi\alpha e_q)^2\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

Differential Cross Section

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{c.m.}} = \frac{|\mathfrak{M}|^2}{64\pi^2 s} \quad \text{with} \quad \overline{|\mathfrak{M}|^2} = \frac{1}{4}\sum_{\text{spins}} |\mathfrak{M}|^2 = 64\pi^2 \hat{s}\,\frac{d\sigma}{d\Omega} = 16\pi\hat{s}^2 \left.\frac{d\sigma}{d\hat{t}}\right|_{ij\to kl}$$

lead to an expression for differential cross section

$$\frac{d\sigma}{d\hat{t}} = \frac{2\pi\alpha^2 e_q^2}{\hat{s}^2}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$

Using invariant relations of Mandelstam variables

$$\begin{aligned}
\left.\frac{d\sigma}{dtdu}\right|_{eq_i\to eq_i} &= x\frac{d\sigma}{d\hat{t}d\hat{u}} \\
&= x\frac{d}{d\hat{u}}\int \frac{2\pi\alpha^2 e_q^2}{\hat{s}^2}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)\delta(\hat{s}+\hat{u}+\hat{t})d\hat{u} \\
&= x\frac{2\pi\alpha^2 e_q^2}{t^2}\left(\frac{s^2+u^2}{s^2}\right)\delta(x(s+u)+t))
\end{aligned}$$

Tensor Product

We can rewrite

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = 4W_1(k\,.\,k') + \frac{2W_2}{M^2}[2(p\,.\,k)(p\,.\,k') - M^2 k\,.\,k']$$

in terms of invariant variables

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = -2W_1 t + \frac{W_2}{M^2}\;[-su + M^2 t]$$

and because we assume $s \gg M^2$ we have

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = \frac{2}{M(s+u)}[x(s+u)^2 F_1 - suF_2]$$

where $t = -x(s+u), F_1 \equiv MW_1$ and $F_2 \equiv \nu W_2$

Tensor Product Substitution

Substituting ♖ into ✈

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{\rm lab} = \frac{1}{16\pi^2}\frac{E'}{E}\frac{\overline{|\mathfrak{M}|^2}}{4\pi M}$$

$$= \frac{(4\pi\alpha)^2}{16\pi^2 q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}$$

$$= \frac{4\alpha^2 E'^2}{q^4}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$

$$= \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$

Using kinematic relations in lab frame

$$s = 2ME, \qquad u = -2ME', \qquad t = -Q^2 = -4EE'\sin^2(\theta/2)$$

and

$$d\Omega dE' = 2\pi d(\cos\theta)dE' = \frac{4\pi M^2}{su}dt\left(-\frac{1}{2M}du\right)$$

we have ☛

$$\left.\frac{d\sigma}{dtdu}\right|_{ep\to eX} = \frac{4\pi\alpha^2}{t^2 s^2}\frac{1}{s+u}\left[(s+u)^2 xF_1 - usF_2\right]$$

Parton Model's Master Formula

Substituting ✈ and ✤ into ♫ and comparing coefficients of us & $s^2 + u^2$

we obtain master formula of parton model

$$2xF_1(x) = F_2(x) = \sum_i e_{q_i}^2 \, x \, f_i(x)$$ ⚑

We see that F_1 and F_2 are functions only of scaling variable x

-- here fixed by delta function in ✈ --

Using lab frame kinematic relation

$$q^2 \simeq -2k\,.\,k' \simeq -2EE'(1-\cos\theta) = -4EE'\sin^2(\theta/2)$$

we obtain ☛

$$\sin^2\frac{\theta}{2} = \frac{Q^2}{4EE'} = \frac{2M\nu x}{4E'\nu/y} = \frac{xyM}{2E'}$$

and

$$\cos^2\frac{\theta}{2} = \frac{E}{E'}\left(1 - y - \frac{Mxy}{2E}\right)$$

where

$$y = \frac{p\,.\,q}{p\,.\,k} \underbrace{=}_{\text{(lab)}} \frac{\nu}{E}$$

Callan-Gross Relation

Substituting previous trigonometric relations into ♣ we get

$$\frac{d\sigma}{dxdy} = \frac{8ME\pi\alpha^2}{Q^4}\left[xy^2 F_1 + \left(1 - y - \frac{Mxy}{2E}\right) F_2\right] \quad ♻$$

where we have used identity

$$dE' d\Omega = \frac{\pi}{EE'} dQ^2\, d\nu = \frac{2ME}{E'} \pi\; y\; dx\; dy$$

Substituting ⚑ into ♻ we obtain Callan-Gross relation

$$\frac{d\sigma}{dxdy} = \frac{2\pi\alpha^2}{Q^4} s\left[1 + (1-y)^2\right] \sum_i e_{q_i}^2\, x\, f_i(x)\,, \qquad (E \gg Mx)$$

Behavior $[1 + (1-y)^2]$ specific to scattering of e from massless fermions

This relation gave evidence that partons involved in DIS were fermions at a time when relation between partons and quarks was still unclear

Kinematic Region for DIS

Independent variables (E', θ, ϕ) (though dependence on latter is trivial)

Convenient to plot allowed kinematic region in $(Q^2/2ME) - (\nu/E)$ plane

Boundary of physical region is given by requirements that

$$0 \le \theta \le \pi, \quad 0 \le \nu \le E, \quad 0 \le x \le 1$$

Because $x = Q^2/2M\nu = (Q^2/2ME)/(\nu/E)$

contours of constant x are straight lines through origin with slope x

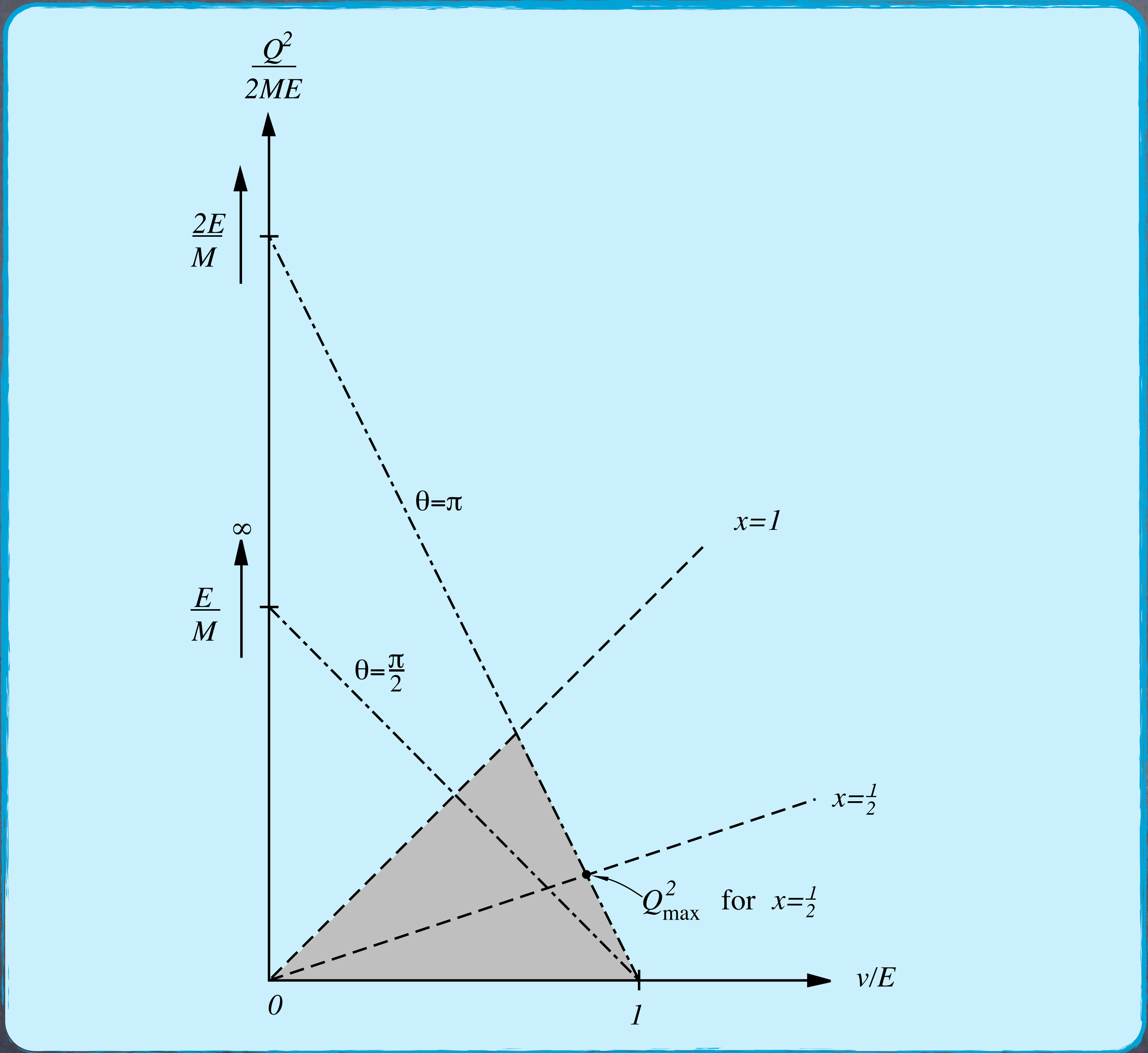
Relation between Q^2 and θ follows from

$$q^2 \simeq -2k\,.\,k' \simeq -2EE'(1-\cos\theta) = -4EE'\sin^2(\theta/2)$$

and is given by

$$\frac{Q^2}{2ME} = \frac{1}{M}(E-\nu)(1-\cos\theta)$$

Kinematic Region of DIS (cont'd)



Triangle is allowed kinematic region for deep inelastic scattering

Dot-dashed lines are curves of constant scattering angle θ

Dashed lines are lines of constant x

HERA

- Hadrons-Elektron-Ring-Anlage (HERA) at DESY
  First storage ring to collide positrons or electrons with protons
- It started operating at end of 1991 and ceased running in June 2007
- Two experiments (H1 and ZEUS) collected data from collisions of
  e^- or e^+ with an energy of $27.5\ \mathrm{GeV}$
  and p with an energy of $820\ \mathrm{GeV}$ until 1997
  $920\ \mathrm{GeV}$ starting from 1998 onwards
- This corresponds to $s = 4 \times 28 \times 820\,(920)\ (\mathrm{GeV})^2$
  allowing measurements of structure functions down to $x \approx 10^{-4}$
- Similar measurement in fixed target experiment requires $50\ \mathrm{TeV}$ beam
- One of first important results of H1 and ZEUS measurements:
  observation of steep rise of proton structure function F_2
  towards low values of Bjorken variable x
- This phenomenon has been successfully described by pQCD
  (perturbative QCD calculations)

Factorization

Simple parton model is not true in QCD
because properties we assumed for hadronic blob
are explicitly violated by certain classes of graphs in perturbation theory

Nevertheless

much of structure of parton model remains in perturbation theory
because of property of factorization

Factorization

permits scattering amplitudes with incoming high energy hadrons
to be written as a product of a hard scattering piece
and remainder factor which contains physics of low energy & momenta
Former contains only high energy and momentum components
and (because of asymptotic freedom) is calculable in perturbation theory

Latter piece describes non-perturbative physics
with single process independent function for each type of parton
known as parton distribution function (PDF)
Without property of factorization we would be unable to make prediction
for processes involving hadrons using perturbation theory

QCD Improved Parton Model

Assuming property of factorization holds
we can derive QCD improved parton model
Result for any process with a single incoming hadron leg is

$$\sigma(|q|,p) = \sum_i \int_0^1 dx\, \hat{\sigma}(|q|, xp, \alpha_s(\mu^2))\, f_i(x,\mu^2)$$

μ^2 is large momentum scale which characterizes hardness of interaction
sum i runs over all partons in incoming hadron
and $\hat{\sigma}$ is short distance cross section
calculable as a perturbation series in QCD coupling α_s

It is referred to as short distance cross section
because singularities corresponding to a long distance physics
have been factored out and abosorbed in structure functions f_i

Structure functions themselves are not calculable in perturbation theory

In order to perform factorization we have introduced a scale μ^2
which separates high and low momentum physics

DGLAP equation

No physical results can depend on particular value of factorization scale

This implies that any dependence on μ in σ has to vanish at least to order in α_s considered

$$\frac{d}{d\ln\mu^2}\sigma^{(n)} = \mathcal{O}(\alpha_s^{n+1})$$

Evolution of parton distributions with changes of scale μ predicted by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation

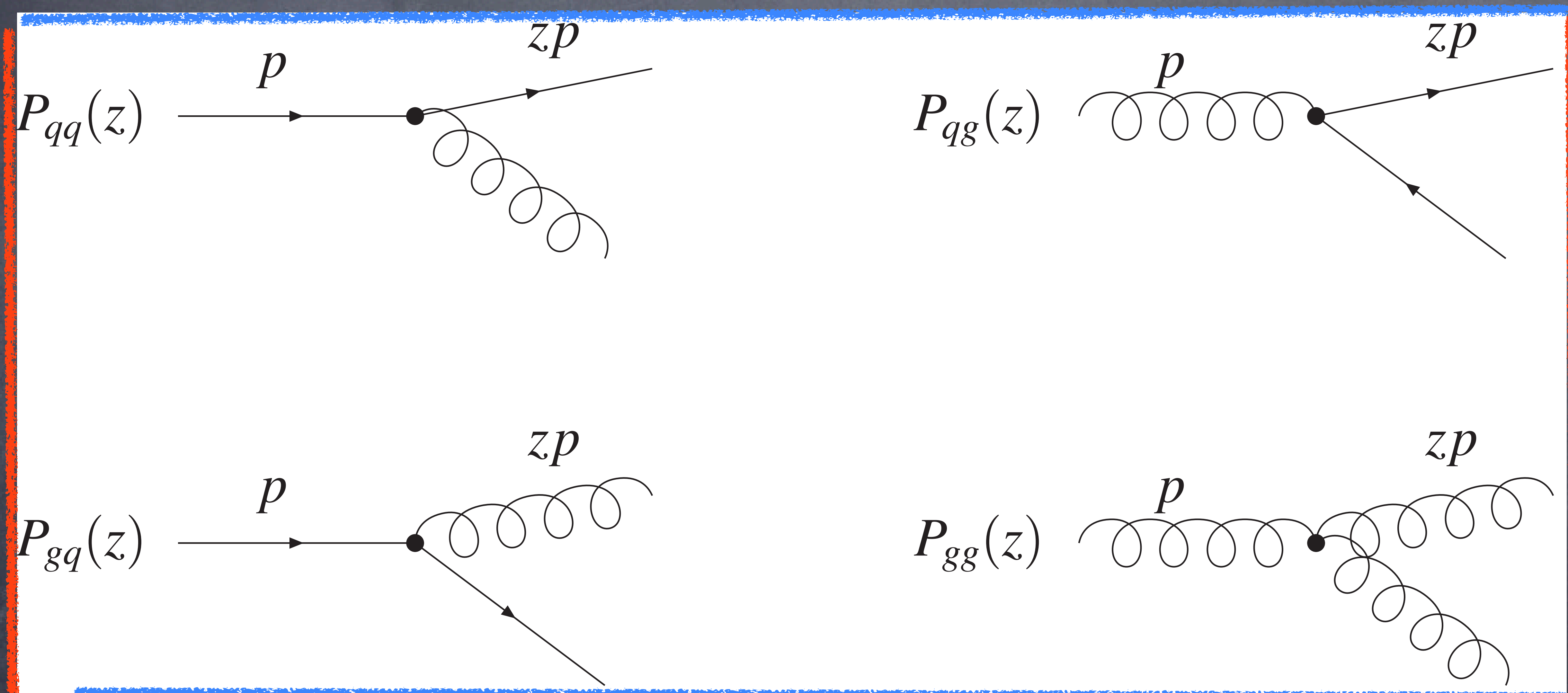
$$\frac{d}{d\ln\mu^2} f_i(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\sum_j \int_x^1 dz\, d\zeta\ \delta(x - z\zeta)\ P_{ij}(z,\alpha_s(\mu^2))\ f_j(\zeta,\mu^2)$$

matrix $\mathbf{P}$ is calculable as a perturbation series

$$P_{ij}(z,\alpha_s) = P_{ij}^{(0)}(z) + \frac{\alpha_s}{2\pi}P_{ij}^{(1)}(z) + \ldots$$

Illustration through Feynman diagrams

Examples of Feynman diagrams contributing to $\mathbf{P}$ in leading order QCD



We indicate collinear momentum flow (p incoming and zp outgoing) as it enters the calculation of corresponding splitting function

Splitting functions

First 2 terms of ♔ are needed for NLO predictions
which is standard approximation
(although often still with large uncertainties)

Splitting functions P_{ij} are currently known to NNLO

Performing ζ integration we obtain

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} q_i(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\sum_j \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{q_i q_j}(z) & P_{q_i g}(z) \\ P_{g q_j}(z) & P_{gg}(z) \end{pmatrix} \times \begin{pmatrix} q_j(x/z,\mu^2) \\ g(x/z,\mu^2) \end{pmatrix}$$

which is a system of coupled integro-differential equations
corresponding to different possible parton splittings

$$\frac{dq_i(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_x^1 \frac{dz}{z}\left[q_i(x/z,\mu^2)P_{qq}(z) + g(x/z,\mu^2)P_{qg}(z)\right] \quad ♘$$

$$\frac{dg(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\sum_j\int_x^1 \frac{dz}{z}\left[q_j(x/z,\mu^2)P_{gq}(z) + g(x/z,\mu^2)P_{gg}(z)\right] \quad ♗$$

$f_j(x, \mu^2)$

- Physical interpretation of PDFs $f_j(x, \mu^2)$
  again relies on infinite momentum frame
- In this frame $f_j(x, \mu^2)$ is number of partons of type j
  carrying fraction x of longitudinal momentum of incoming hadron
  and having a transverse dimension $r < 1/\mu$
- As we increase μ
  DGLAP equation predicts that number of partons will increase
- Viewed on smaller scale of transverse dimension r'

  such that $r' \ll 1/\mu$

  single parton of transverse dimension $1/\mu$
  is resolved into a greater number of partons

DGLAP kernels

DGLAP kernels P_{ij} have an attractive physical interpretation: probability of finding parton i in a parton of type j with a fraction z of longitudinal momentum of parent parton and transverse size less than $1/\mu$

Interpretation as probabilities implies that DGLAP kernels are positive definite for $z < 1$

They satisfy following relations:

$$\int_0^1 dz\, P_{qq}(z) = 0, \quad \int_0^1 dz\, x[P_{qq}(z) + P_{gq}(z)] = 0$$

and

$$\int_0^1 dz\, z[2\, n_f\, P_{qg} + P_{gg}] = 0$$

n_f is number of flavors

These equations correspond to quark number conservation and momentum conservation in splittings of quarks and gluons

DGLAP kernels at LO

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

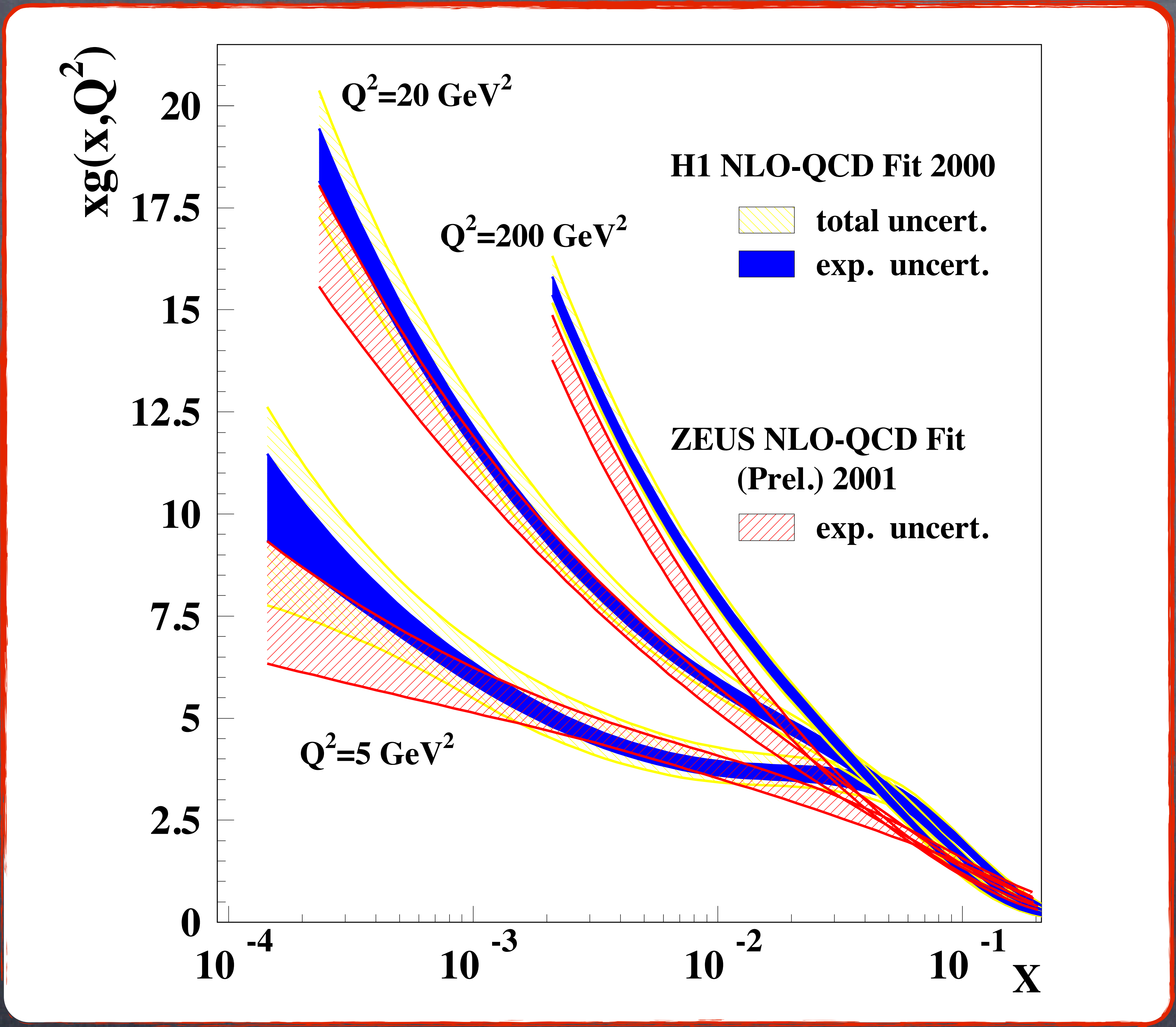
$$P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$P_{qg}(z) = \frac{z^2+(1-z)^2}{2}$$

and ☛

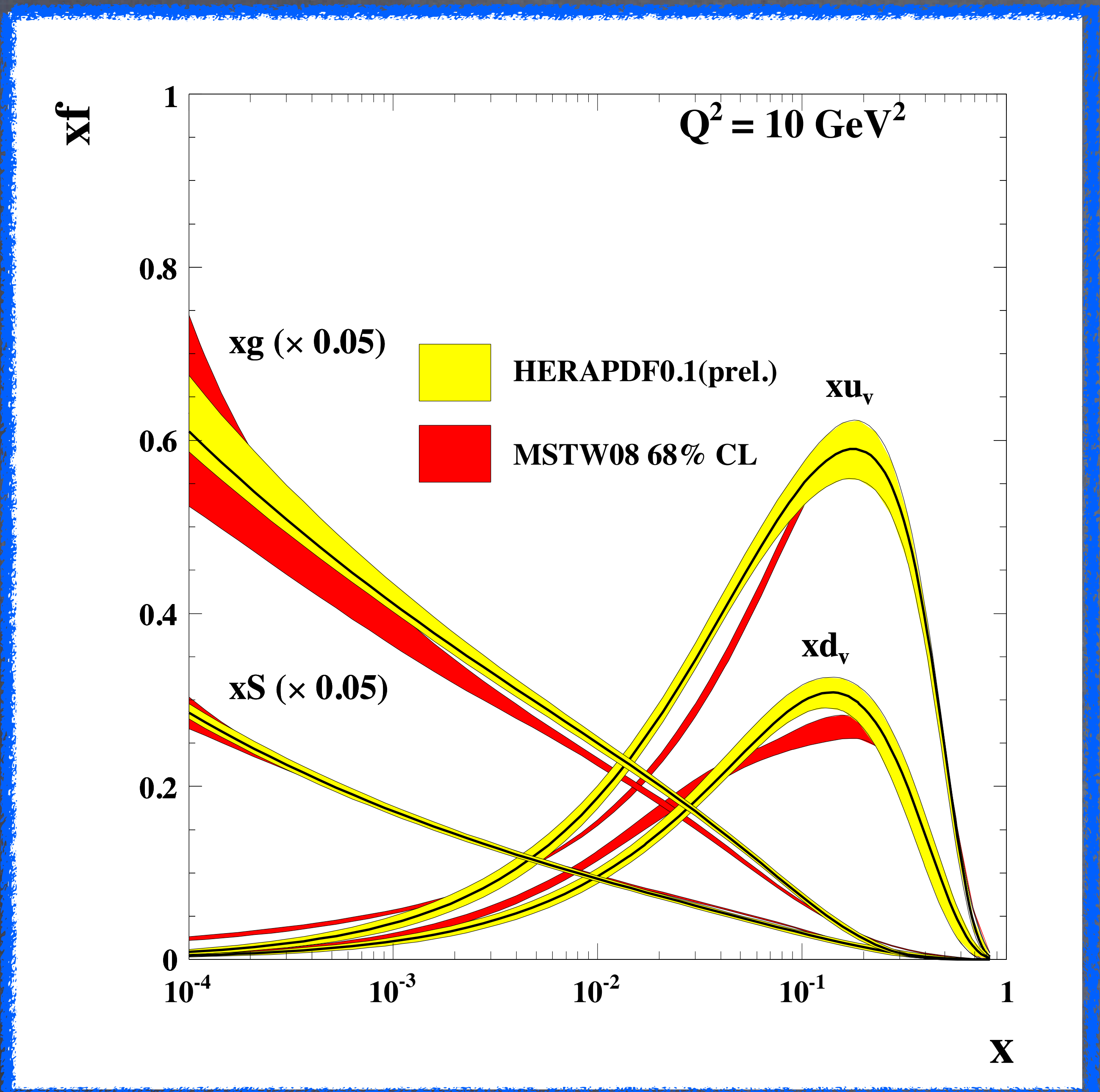
$$P_{gg}(z) = 6\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$

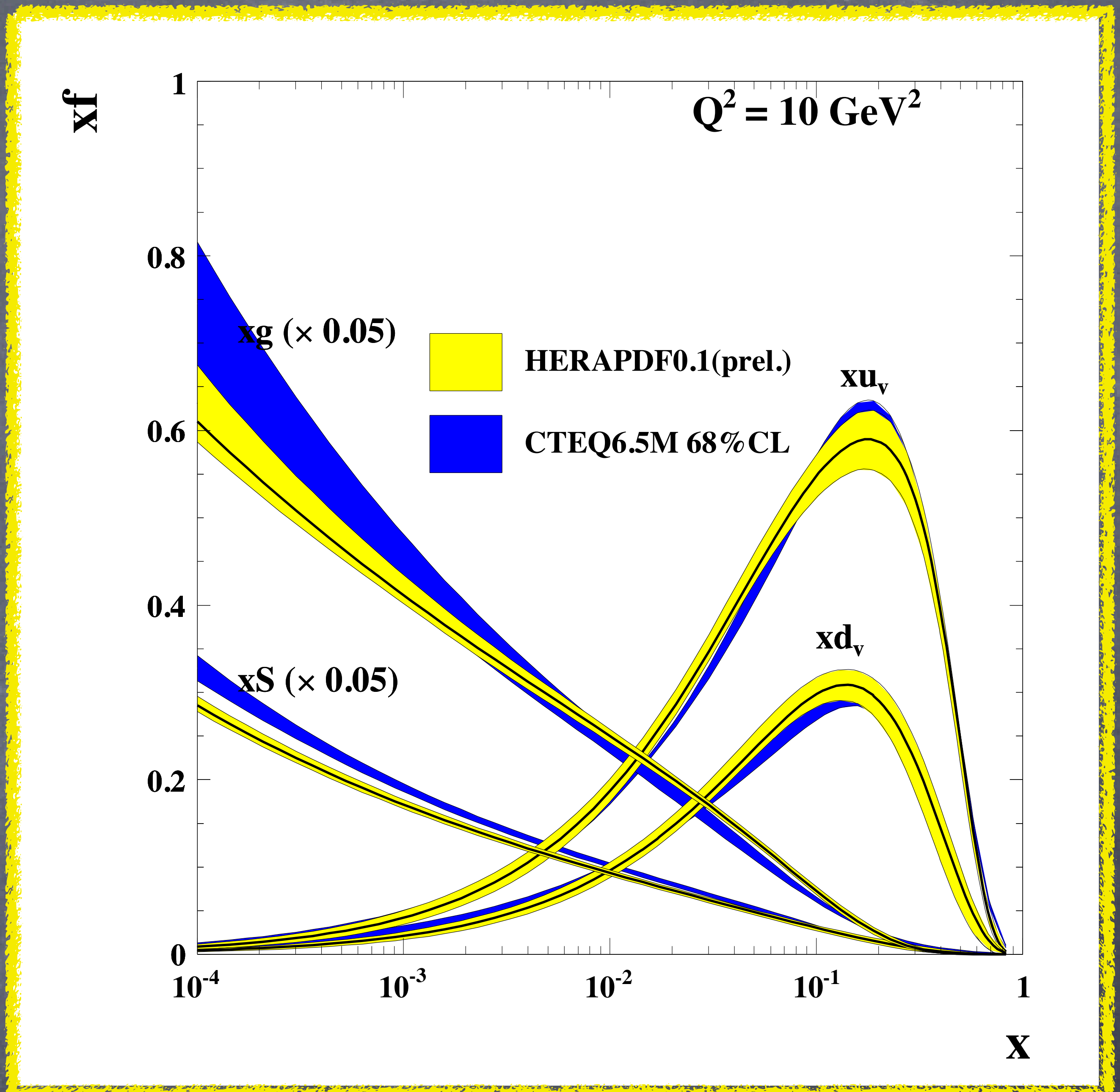
The rise of the gluon



Gluon momentum distributions $xf(x, Q^2)$ in proton as measured by ZEUS and H1 experiments at various Q^2

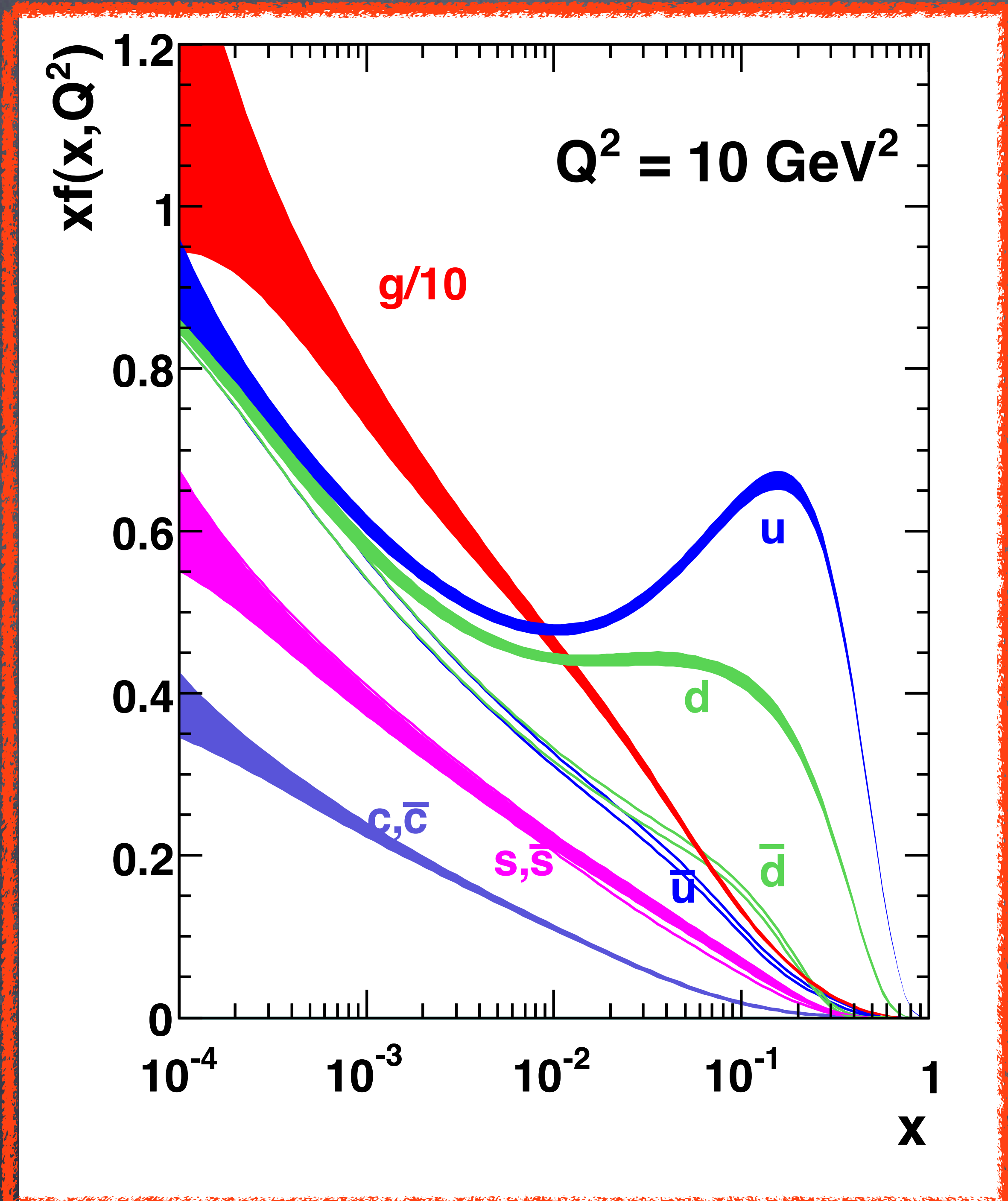
PDFs

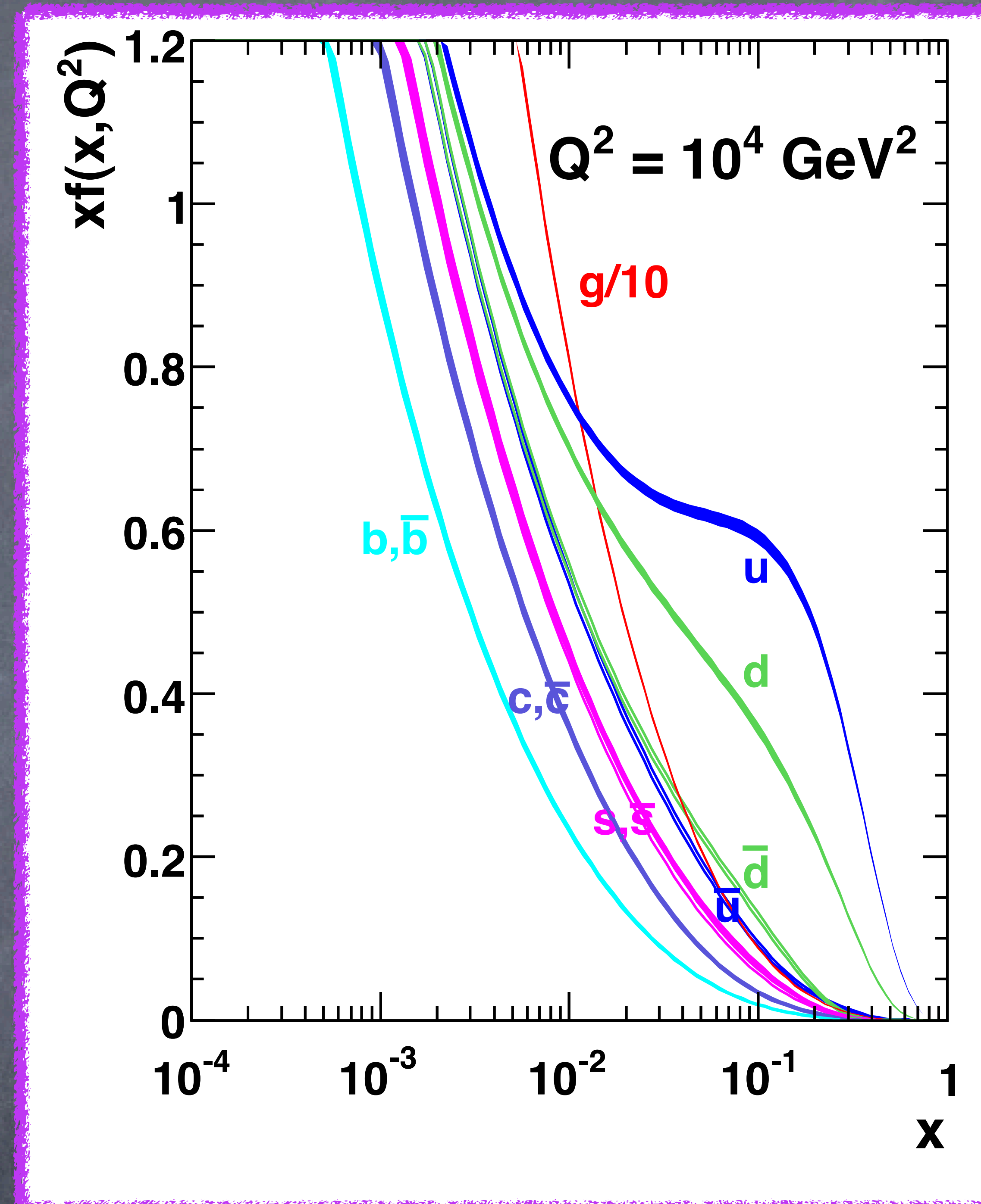




Valence, sea and gluon momentum distributions $xf(x, Q^2)$ in proton as measured by ZEUS and H1 experiments at $Q^2 = 10 \text{ GeV}^2$ are compared to MSTW (left) and CTEQ (right) parametrizations

Extrapolated PDFs to LHC energies





Evolution of gluon and quarks momentum distributions $xf(x,Q^2)$ in proton from a low scale at $Q^2 = 10$ GeV (left) to LHC energies at $Q^2 = 10^4$ GeV (right)

Jets

So far

we have not faced problem of how quarks turn into hadrons hitting detector

It was sufficient to state that

quarks must fragment into hadrons with unit probability

This gives

$$\sigma_{e^+e^-\to\text{hadrons}} = \sum_q \sigma_{e^+e^-\to q\bar{q}}$$
$$= 3\sum_q e_q^2\ \sigma_{e^+e^-\to\mu^-\mu^+}$$

For more detailed calculations ☛ this problem cannot be sidestepped

E.G. in $e^+\epsilon^- \to q\bar{q}$

produced $q\,\bar{q}$ separate with equal and opposite momentum in c.m. frame and materialize into back-to-back jets of hadrons which have momenta roughly collinear with original q and $\bar{q}$ directions

Hadrons may be misaligned by momentum transverse to q or $\bar{q}$ direction by an amount not exceeding about 300MeV

Fragmentation Functions

We can visualize jet formation as e.g. hadron bremsstrahlung

once q and $\bar{q}$ separate by a distance of around $1\ \mathrm{fm}$

α_s becomes large and strong color forces pull on separating $\bar{q}$ and q

To describe fragmentation of quarks into hadrons
we use an analogous formalism
to that introduced to describe quarks inside hadrons

For cross section $\sigma_{pp\to X}$ of some hadronic final state X we can write

$$\begin{aligned}\sigma_{pp\to X} &= \sum_{ijk}\int dx_1\, dx_2\, dz\, f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\\ &\times \hat{\sigma}_{ij\to k}(x_1,x_2,z,Q^2,\alpha_s(\mu^2),\mu^2) D_{k\to X}(z,\mu^2)\end{aligned}$$

$D_{k\to z}(z,\mu^2)$ is fragmentation function
and all other functions have a clear interpretation

Fragmentation Functions (cont'd)

Fragmentation function $D(z)$ describes transition $(\text{parton} \to \text{hadron})$

in same way

structure function $f(x)$ describes embedding $(\text{hadron} \to \text{parton})$

Like f functions D functions are subject to constraints imposed by momentum and probability conservation:

$$\sum_h \int_0^1 z D_q^h(z) dz = 1$$

$$\sum_q \int_{z_{\min}}^1 [D_q^h(z) + D_{\bar{q}}^h(z)] dz = n_h$$

Main features of jet fragmentation process can be derived from

$$dn_h/dz \approx (15/16)\, z^{-3/2}\, (1-z)^2$$

which provides a reasonable parametrization for $10^{-3} < z < 1$

Properties of jet hadronization

z_1	z_2	$\int_{z_1}^{z_2} (dn_h/dz)\, dz$	$\int_{z_1}^{z_2} z\,(dn_h/dz)\, dz$	$z_{\text{equivalent}}$
0.0750	1.0000	3	0.546	0.182
0.0350	0.0750	3	0.155	0.052
0.0100	0.0350	9	0.167	0.018
0.0047	0.0100	9	0.062	0.007
0.0010	0.0047	30	0.069	0.002


CU Next week