Particle Physics 2011

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Asymptotic Freedom & Infrared Slavery

- Hadrons are composite systems with many internal degrees of freedom
- Strongly interacting constituents of these systems so-called partons are described by QCD
- QCD is asymptotically free can be treated in perturbative way for very large values of $Q^2 \equiv -q^2$ (4-momentum transfer)
- Binding forces become increasingly strong if $Q^2 \lesssim 1\ \mathrm{GeV}^2$

  which is natural habitat of nucleons and pions

- Running of QCD coupling constant $\alpha_s(Q^2)$ diverges if Q^2 decreases to values near $\Lambda^2_{\mathrm{QCD}} \approx (250\ \mathrm{MeV})^2$

- Hadronization corresponds to a resolution of nucleon's size

  (somewhat below $1\ \mathrm{fm}$ or $10^{-15}\ \mathrm{m}$)

  and is referred to as onset of deep inelastic regime

Electron Scattering of Atom Cloud

When trying to deduce structure of composite objects like hadrons underlying idea is quite simple and straightforward

Suppose we want to determine charge distribution of atom cloud

Procedure to obtain this information is to scatter e^- on this cloud measure angular cross section and compare it with known σ for scattering of point distribution

As charge cloud certainly is not a point charge this would give us a form factor $F(q)$

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} |F(q)|^2$$

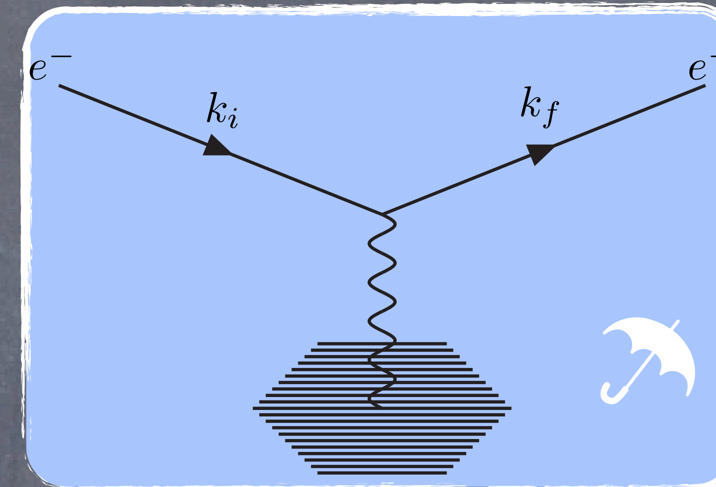
q is momentum transfer between incident electron and target $q = k_i - k_f$

We then attempt to deduce structure of target from $F(q)$ so determined

Form Factor and Reference Cross Section

Consider scattering of unpolarized electrons of energy E
from a static spinless charge distribution $-Ze\rho(\vec{x})$
normalized so that

$$\int \rho(\vec{x})\, d^3x = 1$$



For a static target
form factor in ☃ is just Fourier transform of charge distribution

$$F(\vec{q}) = \int \rho(\vec{x})\, e^{i\vec{q}\,\cdot\,\vec{x}}\, d^3x$$

while reference cross section for a structureless target is

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} \equiv \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} = \frac{(Z\alpha)^2 E^2}{4k^4\,\sin^4(\theta/2)}\,\left[1 - v^2\,\sin^2(\theta/2)\right]$$

$$k = |\vec{k}_i| = |\vec{k}_f|,\;\; v = k/E$$

and θ is angle through which electron is scattered

Soft Scattering Process

By virtue of normalization condition $F(0) = 1$

If $|\vec{q}\,|$ is not too large we can expand exponential in ☁ yielding

$$
\begin{aligned}
F(\vec{q}) &= \int \left(1 + i\vec{q}\,.\,\vec{x} - \frac{(\vec{q}\,.\vec{x})^2}{2} \ldots\right) \rho(\vec{x})\, d^3x \\
&= \int \left(1 + iq\,r\,\cos\theta - \frac{1}{2} q^2\, r^2\, \cos^2\theta \ldots\right) \rho(r) r^2\, d(\cos\theta)\, d\phi\, dr \\
&= 1 - \frac{1}{6}\,|\vec{q}|^2\,\langle r^2\rangle + \ldots
\end{aligned}
$$

where we have assumed that ρ is spherically symmetric

that is ☛ a function of $r \equiv |\vec{x}|$ alone

Then small-angle scattering just measures $$\langle r^2\rangle = \int r^2\,\rho(r)\,4\pi\,r^2\,dr$$

mean radius of charged cloud

This is because in small $|\vec{q}\,|$ limit ☛ photon in ☂ is soft

with its large wavelength can resolve only size of charge distribution $\rho(r)$

and is not sensitive to its detailed structure

Moving to the lab frame

Previous discussion cannot be applied directly to study proton structure

1st ☛ proton's magnetic moment is involved in scattering of electron not just its charge

2nd ☛ proton is not static but will recoil under electron's bombardment

If proton were a point charge e with Dirac magnetic moment $e/2M$ then we already know answer

We can take over result for electron-muon scattering and simply replace mass of muon by that of proton:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4(\theta/2)}\right)\frac{E'}{E}\left(\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$ ♪

where factor

$$\frac{E'}{E} = \left(1 + \frac{2E}{M}\sin^2\frac{\theta}{2}\right)^{-1}$$

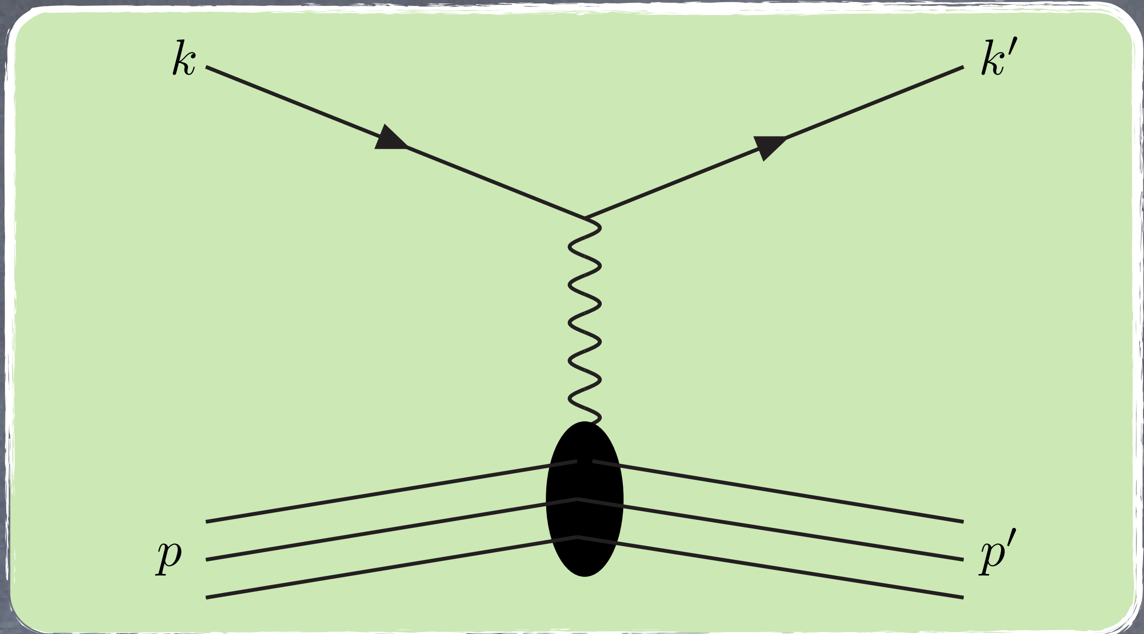
arises from recoil of target

See appendix E for hints on the lab kinematics calculations

Transition currents for elastic scattering

Copying calculation of electron muon cross section

lowest order amplitude for electron proton elastic scattering is



$$T_{fi} = -i \int e j_\mu \left(-\frac{1}{q^2} \right) (-e) J^\mu d^4 x$$

$$q = p - p'$$

electron and proton transition currents are

$$e j^\mu = e \overline{u}(k') \gamma^\mu u(k) e^{i(k'-k)\,.\,x}$$

$$-e J^\mu = -e \overline{u}(p') \left[\quad \right] u(p) e^{i(p'-p)\,.x}$$

J^μ

proton is extended structure ☛ we cannot replace square brackets γ^μ

--as we did for point spin-1/2 particles --

we know J^μ must be a Lorentz 4-vector

most general 4-vector constructed from $p,\ p',\ q,$ and Dirac γ matrices

$$[\quad] = \left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M}F_2(q^2)\, i\sigma^{\mu\nu}q_\nu\right]$$

♬

F_1 and F_2 are two independent form factors

κ is anomalous magnetic moment

(Terms involving γ^5 are ruled out by conservation of parity)

For $q^2 \to 0$ (that is when we probe with long-wavelength photons)
it does not make any difference that proton has structure at $\mathcal{O}(\mathrm{fm})$

We effectively see particle of charge e and magnetic moment $(1+\kappa)e/2M$

$$\kappa = 1.79$$

Form factors must be chosen so that in this limit $F_1(0) = 1$ & $F_2(0) = 1$

Corresponding values for neutron are $F_1(0) = 0,\ F_2(0) = 1$
and experimentally $\kappa_n = -1.91$

Rosenbluth formula

If we use ♬ to calculate differential cross section for elastic scattering we find an expression similar to ♪ known as Rosenbluth formula ⚑

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm lab} = \left(\frac{\alpha^2}{4E^2\sin^4(\theta/2)}\right)\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2(\theta/2) - \frac{q^2}{2M^2}(F_1+\kappa F_2)^2\sin^2(\theta/2)\right\}$$

Two form factors $F_{1,2}(q^2)$ parametrize our ignorance of detailed p structure represented by blob in ♨

These form factors can be determined experimentally by measuring $d\sigma/d\Omega$ as a function of θ and q^2

Note that if proton were point-like like muon then $\kappa = 0$ and $F_1(q^2) = 1$ for all q^2 ☛ and ⚑ would revert to ♪

Electric & Magnetic form factors

In practice ☛ it is better to use linear combinations of $F_{1,2}$

$$G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2, \qquad G_M \equiv F_1 + \kappa F_2$$

Defined so that no interference terms $G_E G_M$ occur in cross section

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm lab} = \left(\frac{\alpha^2}{4E^2 \sin^4(\theta/2)}\right) \frac{E'}{E} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right)$$

with $\tau \equiv -q^2/4M^2$

These proton form factors may be regarded as generalizations of non-relativistic form factor introduced in ☃

G_E and G_M are referred to as electric and magnetic form factors

Data on angular dependence of $ep \to ep$ scattering can be used to separate G_E, G_M at different values of q^2

Result for $G_E(q^2)$ is

$$G_E(q^2) \simeq \left(1 - \frac{q^2}{0.71}\right)^{-2}$$

in units of GeV^2

Proton radius

Small $-q^2$ is used to determine residual terms in expansion of ☏

mean square proton charge radius is

$$\langle r^2 \rangle = 6 \left(\frac{dG_E(q^2)}{dq^2} \right)_{q^2=0} = (0.81 \times 10^{-13} \text{ cm})^2$$

About same radius is obtained for magnetic moment distribution

Having measured size of proton
one might like to take a more detailed look at its structure
by increasing $-q^2$ of photon to give better spatial resolution

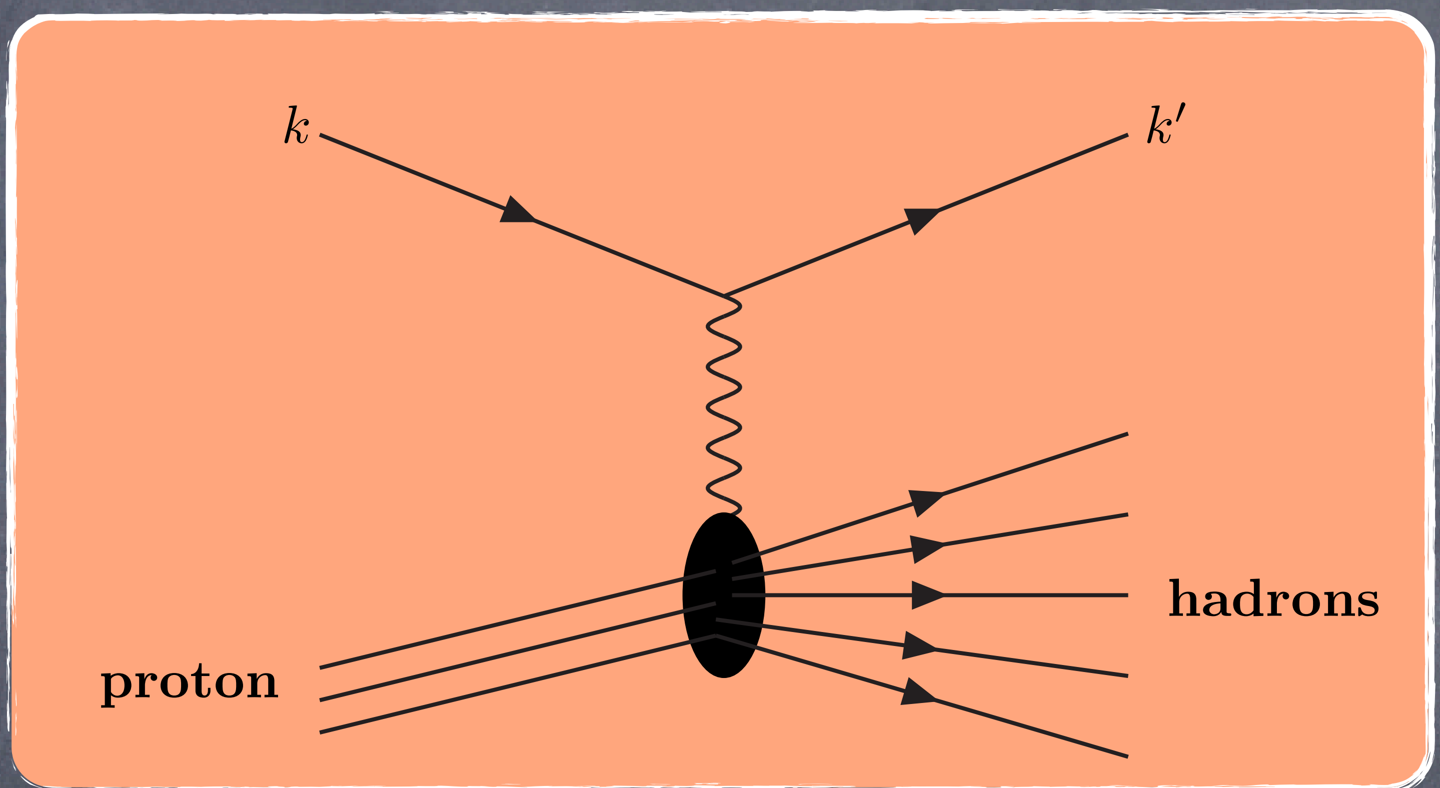
This is done by requiring large energy loss of bombarding electron

There is (however) a catch:
because of large transfer of energy ☛ proton will often break up

Δ-excitation

For modest $-q^2$ one might just excite proton into a Δ-state and hence produce an extra π-meson ☛ that is $ep \to e\Delta^+ \to ep\pi^0$

Picture of ♨ would therefore need to be generalized to



In this case, square of invariant mass is $W^2 \simeq M_\Delta^2$

When $-q^2$ is very large debris becomes so messy that initial state proton loses its identity completely
and new formalism must be devised to extract information from measurement

Inelastic Scattering

In switching from a muon to a proton target

we replaced lepton current $j^\mu(\sim \overline{u}\gamma^\mu u)$ by proton current $J^\mu(\sim \overline{u}\Gamma^\mu u)$

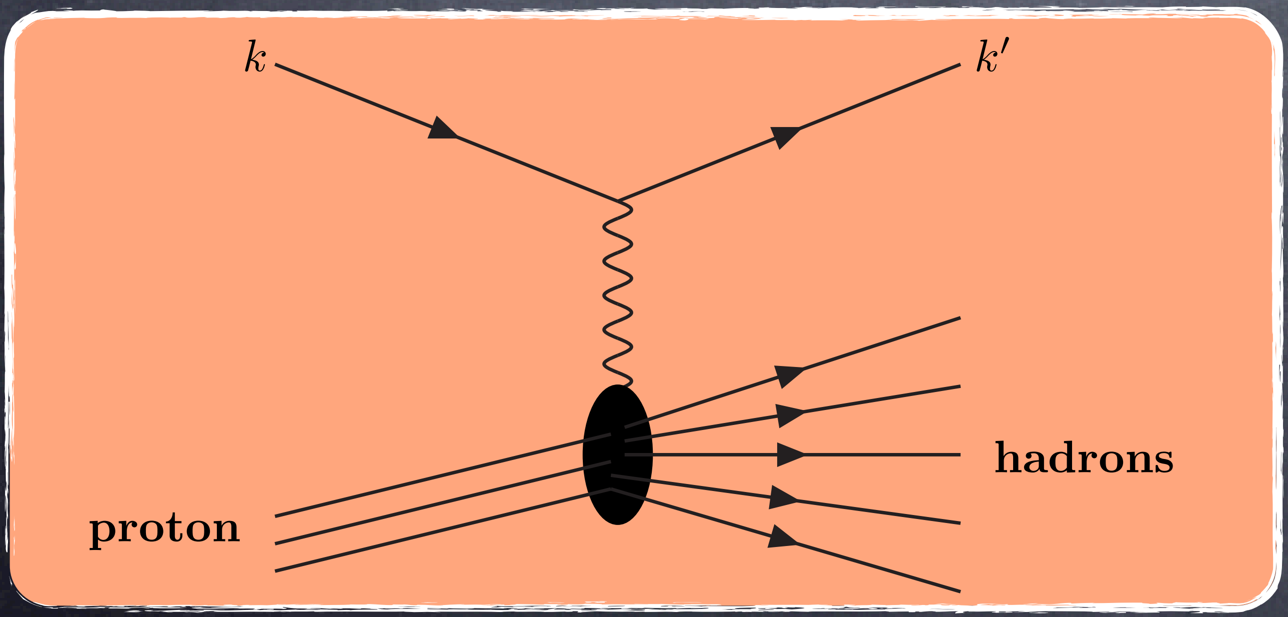
This is inadequate to describe inelastic events
because final state is not a single fermion
described by a Dirac $\bar{u}$ entry in matrix current

Therefore ☛ J^μ must have a more complex structure

Square of invariant amplitude $\overline{|\mathfrak{M}|^2} = \frac{e^4}{q^4} L^{\mu\nu}_{(e)} L^{(\mu)}_{\mu\nu}$ is generalized to

$$\overline{|\mathfrak{M}|^2} \propto L^{(e)}_{\mu\nu} \; W^{\mu\nu}$$

Leptonic part of diagram above photon propagator is left unchanged



☛ $L^{\mu\nu}_{(e)} = \frac{1}{2}\mathrm{Tr}[(\not{k}' + m_e)\,\gamma^\mu\,(\not{k} + m_e)\,\gamma^\nu]$

Hadronic Tensor

$W^{\mu\nu}$ parametrizes our ignorance of form of current at end of propagator

Most general form of tensor $W^{\mu\nu}$

is constructed out of $g^{\mu\nu}$ and independent momenta p and q

(with $p' = p + q$)

γ^μ is not included as we are parametrizing $|\mathfrak{M}|^2$

which is already summed and averaged over spins

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} (p^\mu q^\nu + q^\mu p^\nu)$$

We have omitted antisymmetric contributions to $W^{\mu\nu}$

since their contribution to cross section vanishes

because tensor $L^{(e)}_{\mu\nu}$ is symmetric

Note omission of W_3 in our notation

this spot is reserved for a parity violating structure function

when a neutrino beam is substituted for the electron beam

so that the virtual photon probe is replaced by a weak boson

Vertex Constraints

Current conservation at vertex requires $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

$$\begin{aligned} 0 &= q_\nu W^{\mu\nu} \\ &= -q_\nu W_1 g^{\mu\nu} + \frac{W_2}{M^2}(p\,.\,q)p^\mu + \frac{W_4}{M^2}q^2 q^\mu + \frac{W_5}{M^2}[q^2 p^\mu + (p\,.\,q)q^\mu] \end{aligned}$$

Setting coefficients of q^μ and p^μ to zero we find

$$-W_1 + \frac{W_4}{M^2}q^2 + \frac{W_5}{M^2}(p\,.\,q) = 0$$

$$\frac{W_2}{M^2}(p\,.\,q) + \frac{W_5}{M^2}q^2 = 0$$

which lead to

$$W_5 = -\frac{p\,.\,q}{q^2}W_2$$

$$W_4 = \left(\frac{p\,.\,q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2}W_1$$

New invariants

- Only 2 of 4 inelastic structure functions are independent and we can write without loss of generality

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\frac{1}{M^2}\left(p^\mu - \frac{p\,.\,q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\,.\,q}{q^2}q^\nu\right)$$

W_i are functions of Lorentz scalar variables that can be constructed from 4-momenta at hadronic vertex

- Unlike elastic scattering ☛ there are two independent variables and we choose

$$q^2 \quad \text{and} \quad \nu \equiv \frac{p\,.\,q}{M}$$

- Invariant mass W of final hadronic system is related to ν and q^2 by

$$W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$$

Tensor Product

To evaluate cross section for $ep \to eX$
straightforward repetition of calculation for $e^- \mu^- \to e^- \mu^-$ scattering

Using
$$L^{\mu\nu}_{(e)} = \frac{1}{2}\mathrm{Tr}(\not{k}'\gamma^\mu \not{k}\gamma^\nu) + \frac{1}{2}m_e^2\mathrm{Tr}(\gamma^\mu\gamma^\nu)$$
$$= 2(k'^\mu k^\nu + k'^\nu k^\mu - (k'.k - m_e^2)g^{\mu\nu})$$

and noting $q^\mu L^{(e)}_{\mu\nu} = q^\nu L^{(e)}_{\mu\nu} = 0$ we find

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = 4W_1(k\,.\,k') + \frac{2W_2}{M^2}[2(p\,.\,k)(p\,.\,k') - M^2 k\,.\,k']$$

In laboratory frame this becomes

$$L^{\mu\nu}_{(e)} W_{\mu\nu} = 4EE'\left\{W_2(\nu,q^2)\,\cos^2\frac{\theta}{2} + 2\,W_1(\nu,q^2)\,\sin^2\frac{\theta}{2}\right\}$$

recall ☛ $q^2 \simeq -2k\,.\,k' \simeq -2EE'(1-\cos\theta) = -4EE'\sin^2(\theta/2)$

Differential Cross Section

Including flux factor and phase space factor for outgoing electron

$$d\sigma = \frac{1}{4\left[(k\,.\,p)^2 - m^2M^2\right]^{1/2}}\left\{\frac{e^4}{q^4}L^{\mu\nu}_{(e)}W_{\mu\nu}4\pi M\right\}\frac{d^3k'}{2E'(2\pi)^3}$$

extra factor of $4\pi M$ arises because of $W^{\mu\nu}$ normalization

$$\begin{aligned}\left.\frac{d\sigma}{dE'd\Omega}\right|_{\text{lab}} &= \frac{1}{16\pi^2}\frac{E'}{E}\frac{\overline{|\mathfrak{M}|^2}}{4\pi M}\\ &= \frac{(4\pi\alpha)^2}{16\pi^2q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}\\ &= \frac{4\alpha^2E'^2}{q^4}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}\\ &= \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}\end{aligned}$$

to obtain final result we neglect mass of electron and used

$$q^2 \simeq -2k\,.\,k' \simeq -2EE'(1-\cos\theta) = -4EE'\sin^2(\theta/2)$$

Form Factor Summary

Convenient to express $d\sigma$ with respect to invariants ν and Q^2

$$\left.\frac{d\sigma}{dQ^2\,d\nu}\right|_{\text{lab}} = \frac{\pi}{EE'}\left.\frac{d\sigma}{dE'd\Omega}\right|_{\text{lab}}$$
$$= \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left\{W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right\}$$

It will be useful to make a compendium of our results on form factors

We keep to laboratory kinematic and neglect mass of electron

Differential cross section in energy (E) and angle (θ) of scattered e^-

can be written as

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{\text{lab}} = \frac{4\alpha^2 E'^2}{q^4}\left\{\quad\right\}$$

where...

$$\left\{ \right\}$$

- For a muon target of mass m
  -- or quark target of mass m after substitutions $\alpha^2 \to \alpha^2 e_q^2$ --

$$\left\{ \quad \right\}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2m} \right)$$

- For elastic scattering from a proton target

$$\left\{ \quad \right\}_{ep\to ep} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2M} \right)$$

$\tau = -q^2/4M^2$ and M is mass of proton

- When proton target is broken up by bombarding electron

$$\left\{ \quad \right\}_{ep\to eX} = W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2}$$

Parton structure functions

Making use of delta function can be integrated over E'

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\,\sin^4(\theta/2)}\right)\,\frac{E'}{E}\,\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

Sign that there are structureless particles inside a complex system is that for small wavelengths proton described by suddenly starts behaving like a free Dirac particle and turns into

Proton structure functions thus become simply

$$2W_1^{\text{point}} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right) \qquad W_2^{\text{point}} = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

Point notation reminds us q is structureless particle

$Q^2 \equiv -q^2$ and m is quark mass

Using $\delta(x/a) = a\delta(x)$ parton structure functions can be rearranged to be dimensionless structure functions of ratio $Q^2/2m\nu$

(AND not Q^2 and ν independently)

$$2mW_1^{\text{point}}(\nu, Q^2) = \frac{Q^2}{2m\nu}\,\delta\left(1 - \frac{Q^2}{2m\nu}\right),$$

$$\nu W_2^{\text{point}}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

Elastic scattering form factor

Parton behavior can be contrasted with that for ep elastic scattering

for simplicity we set $\kappa = 0$ so that $G_E = G_M \equiv G$

then comparing and we have

$$W_1^{\text{elastic}} = \frac{Q^2}{4M^2} G^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_2^{\text{elastic}} = G^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2M}\right)$$

A mass scale is explicitly present ☛ reflecting inverse size of proton

$G(Q^2)$ cannot be rearranged as function of single dimensionless variable

As Q^2 increases above $(0.71\ \text{GeV})^2$ form factor depresses elastic scattering

proton is more likely to break up

Point structure functions depend only on dimensionless variable $Q^2/2m\nu$

m merely serves as a scale for momenta

and no scale of mass is present

BJORKEN SCALING

In limit $Q \to \infty$ and $2M\nu \to \infty$ (such that $\omega = 2(q\,.\,p)/Q^2 = 2M\nu/Q^2$)

structure functions would have following property

$$MW_1(\nu, Q^2) \underset{\text{large } Q^2}{\longmapsto} F_1(\omega)\,,$$

$$\nu W_2(\nu, Q^2) \underset{\text{large } Q^2}{\longmapsto} F_2(\omega)$$

we have introduced proton mass instead of quark mass
to define dimensionless variable ω

Presence of free quarks is signaled by fact that:
inelastic structure functions are independent of Q^2 at given value of ω

IN LATE SIXTIES

deep inelastic scattering experiments conducted by SLAC-MIT Collaboration

showed that at sufficiently large $Q^2 \gg \Lambda^2_{\rm QCD}$

structure functions are approximately independent of Q^2

CU NEXT WEEK
study
u have a test