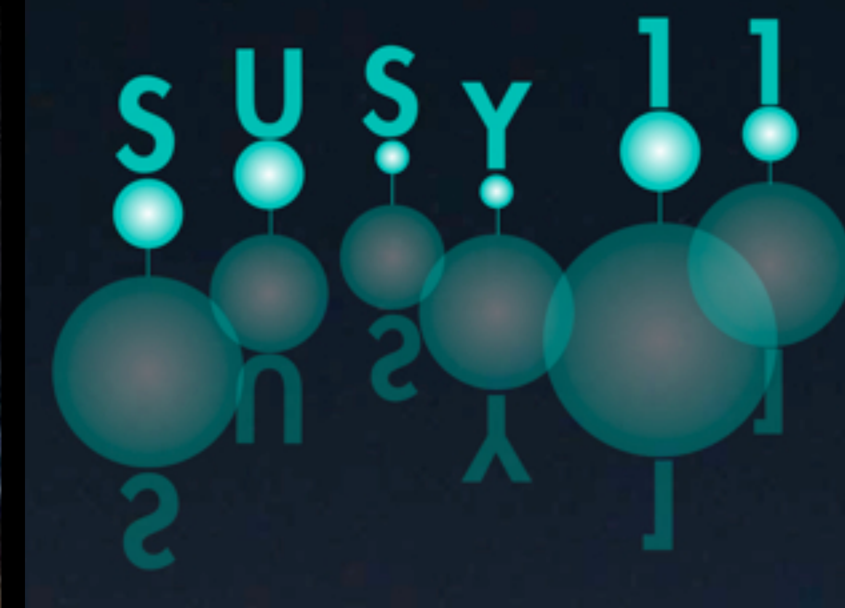


Particle Physics 2011

Luis Anchordoqui

Virtual Photons

- ✓ In Maxwell's theory of electromagnetism charged particles interact through their electromagnetic fields
- ✓ For many years it was difficult to conceive how such action-at-a-distance between charges came about
- ✓ In QFT ☛ we have such a tangible connection
- ✓ QFT approach visualizes force between electrons as interaction arising in exchange of "virtual" photons which can only travel distance allowed by uncertainty principle
- ✓ Virtual photons cannot live an existence independent of charges that emit or absorb them

Perturbative Approach

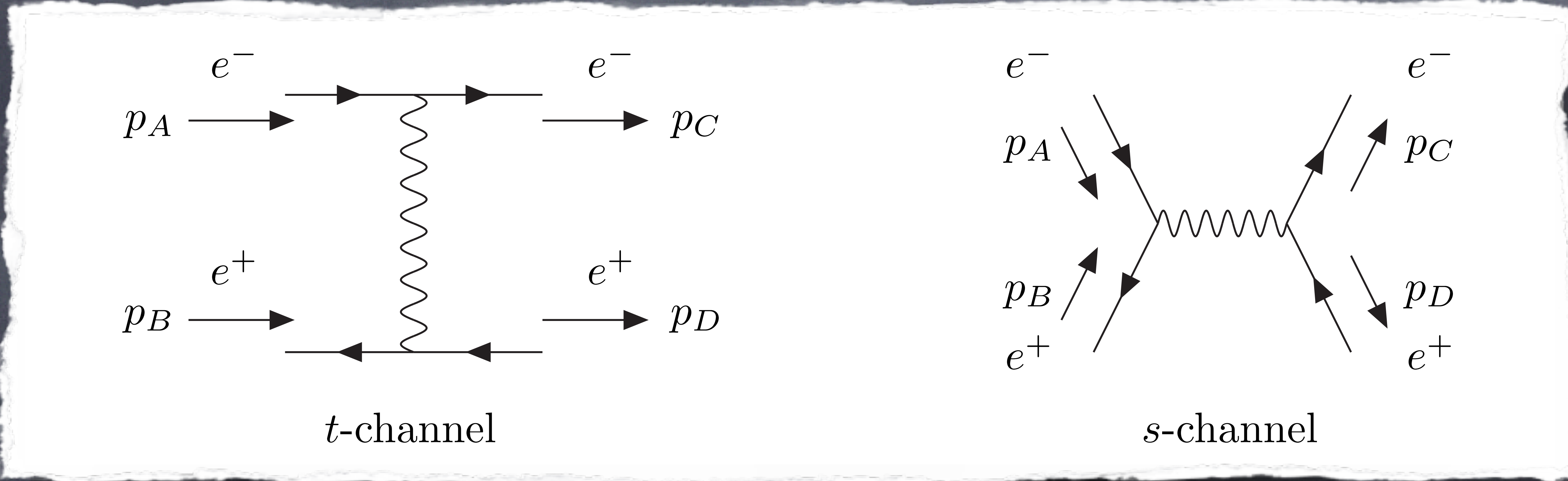
- When calculating scattering cross sections interaction between particles can be described by starting from a free field -- which describes incoming and outgoing particles -- and including an interaction Hamiltonian to describe how particles deflect one another

- Amplitude for scattering is sum of each possible interaction history over all possible intermediate particle states

- Number of times interaction Hamiltonian acts is order of perturbation expansion

- Perturbative series can be written as a sum over Feynman diagrams

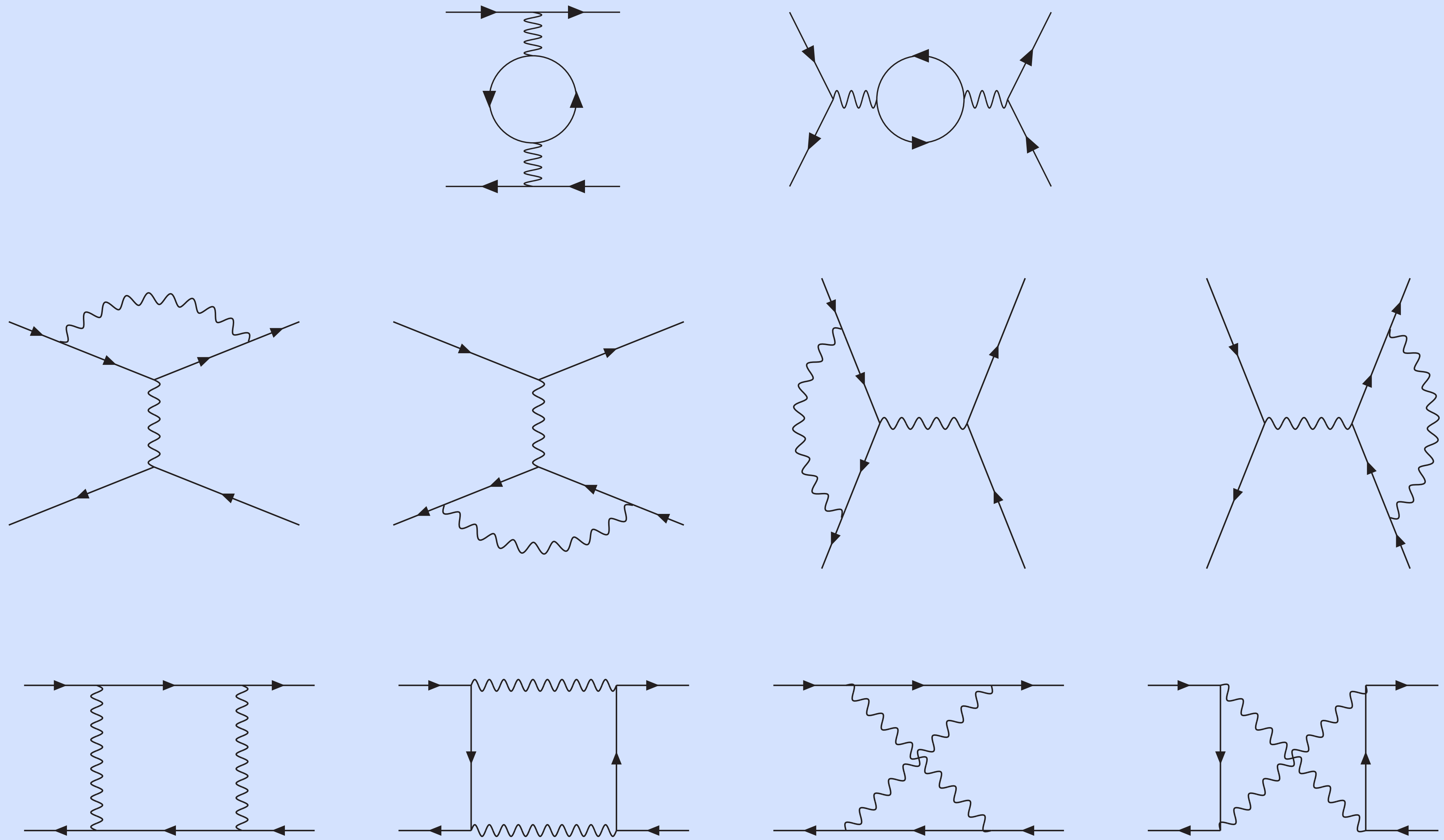
Bhabha scattering tree-level diagrams



Lowest order (tree level) diagrams for Bhabha scattering

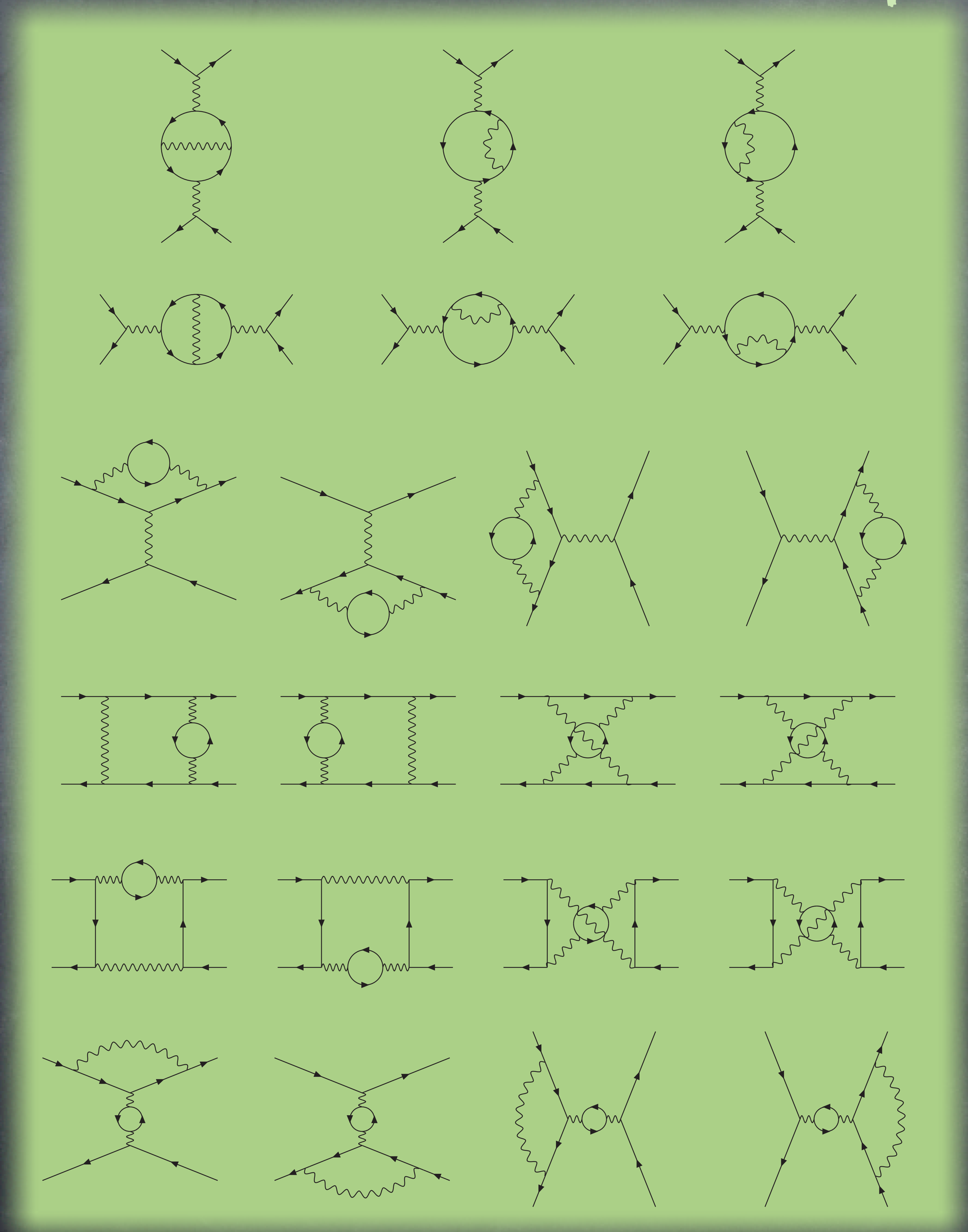
$$(e^+e^- \to e^+e^-)$$

Bhabha scattering one-loop diagrams



Various virtual contributions containing one-loop

Bhabha scattering two-loop diagrams



Spinless quarks & leptons

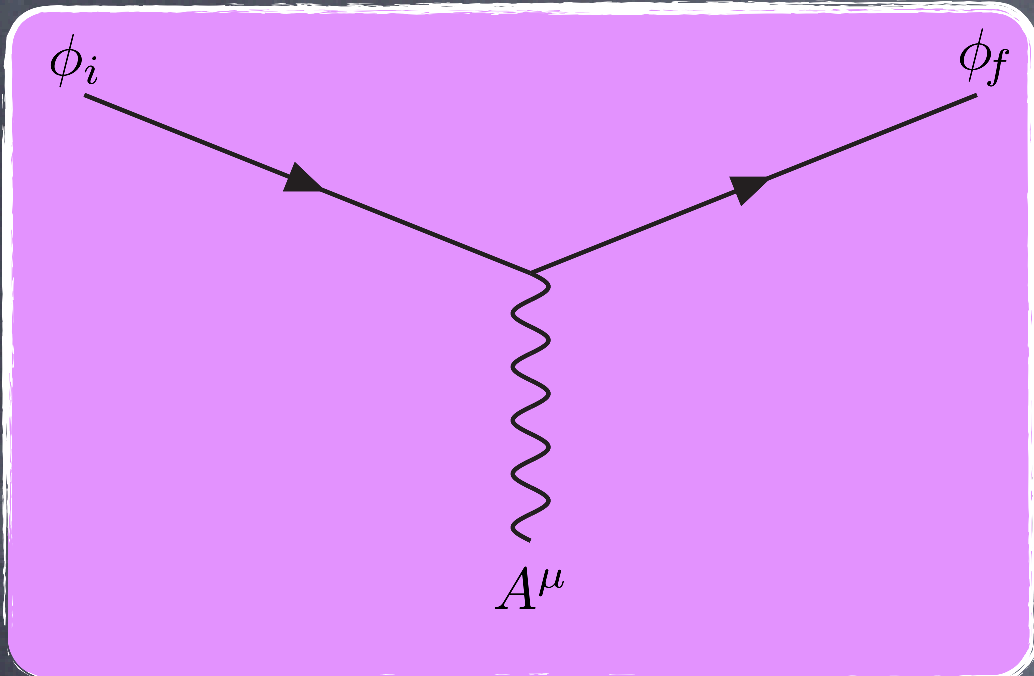
- In non-relativistic limit of perturbation theory
  we have introduced a factor like V_{ni} for each interaction vertex
  and for propagation of each intermediate state
  we have introduced a propagator factor like $1/(E_i - E_n)$
- Intermediate states are virtual in sense that energy is not conserved

$$E_n \neq E_i$$

  but there is energy conservation between initial and final states
  as indicated by delta function $\delta(E_f - E_i)$
- We'll generalize perturbative scheme to handle relativistic particles
- We illustrate how to use perturbation theory in a covariant way
  by choosing interacting particles to be spinless charged leptons
  as it is desirable to begin by avoiding complications of their spin
- No spinless quark or lepton has ever been observed in an experiment
- Spinless hadrons exist (e.g. π -meson)
  but are complicated composite structures of spin-1/2 q & spin-1 g
- Spin-0 leptons are completely fictitious objects
  → (that is leptons satisfying Klein-Gordon equation)

SPINLESS electron interacting with A^μ

Consider scattering of spinless electron in electromagnetic potential



In classical electrodynamics motion of a particle of charge e in electromagnetic potential $A^\mu = (\phi, \vec{A})$ is obtained by substitution $p^\mu \to p^\mu - eA^\mu$

Corresponding quantum mechanical substitution is therefore

$$i\partial^\mu \to i\partial^\mu - eA^\mu$$

Klein-Gordon equation becomes ☛ $$(\partial_\mu \partial^\mu + m^2)\phi = -V\phi$$

⧓ $$V = ie(\partial_\mu A^\mu + A^\mu \partial_\mu) - e^2 A^2$$ is (electromagnetic) perturbation

Working to lowest order we neglect $e^2 A^2$ term in ⧓

Transition Amplitude

e^- (from state ϕ_i to ϕ_f) from electromagnetic potential A_μ

$$
\begin{aligned}
T_{fi} &= -i \int \phi_f^*(x)\, V(x)\, \phi_i(x)\ d^4x \\
&= -i \int \phi_f^*\, ie(A^\mu \partial_\mu + \partial_\mu A^\mu)\, \phi_i\ d^4x
\end{aligned}
$$

Derivative in second term (which acts on both A^μ and ϕ_i)
can be turned around via integration by parts ☛ so that acts on ϕ_f^*

$$\int \phi_f^*\, \partial_\mu(A^\mu \phi_i)\, d^4x = -\int \partial_\mu(\phi_f^*)\ A^\mu\ \phi_i\ d^4x$$

surface term has been omitted

because potential is taken to vanish as $|\vec{x}|, t \to \pm\infty$

Electromagnetic current for $i \to f$ transition

We can now rewrite amplitude in a very suggestive form

$$T_{fi} = -i \int j^{fi}_{\mu} e A^{\mu} \, d^4 x$$

$$e j^{fi}_{\mu}(x) = ie[\phi^*_f (\partial_\mu \phi_i) - (\partial_\mu \phi^*_f)\phi_i]$$

by comparison with $e j^{\mu} = i\, e\, (\phi^* \, \partial^{\mu} \phi - \phi \, \partial^{\mu} \phi^*)$

can be regarded as electromagnetic current for $i \to f$ transition

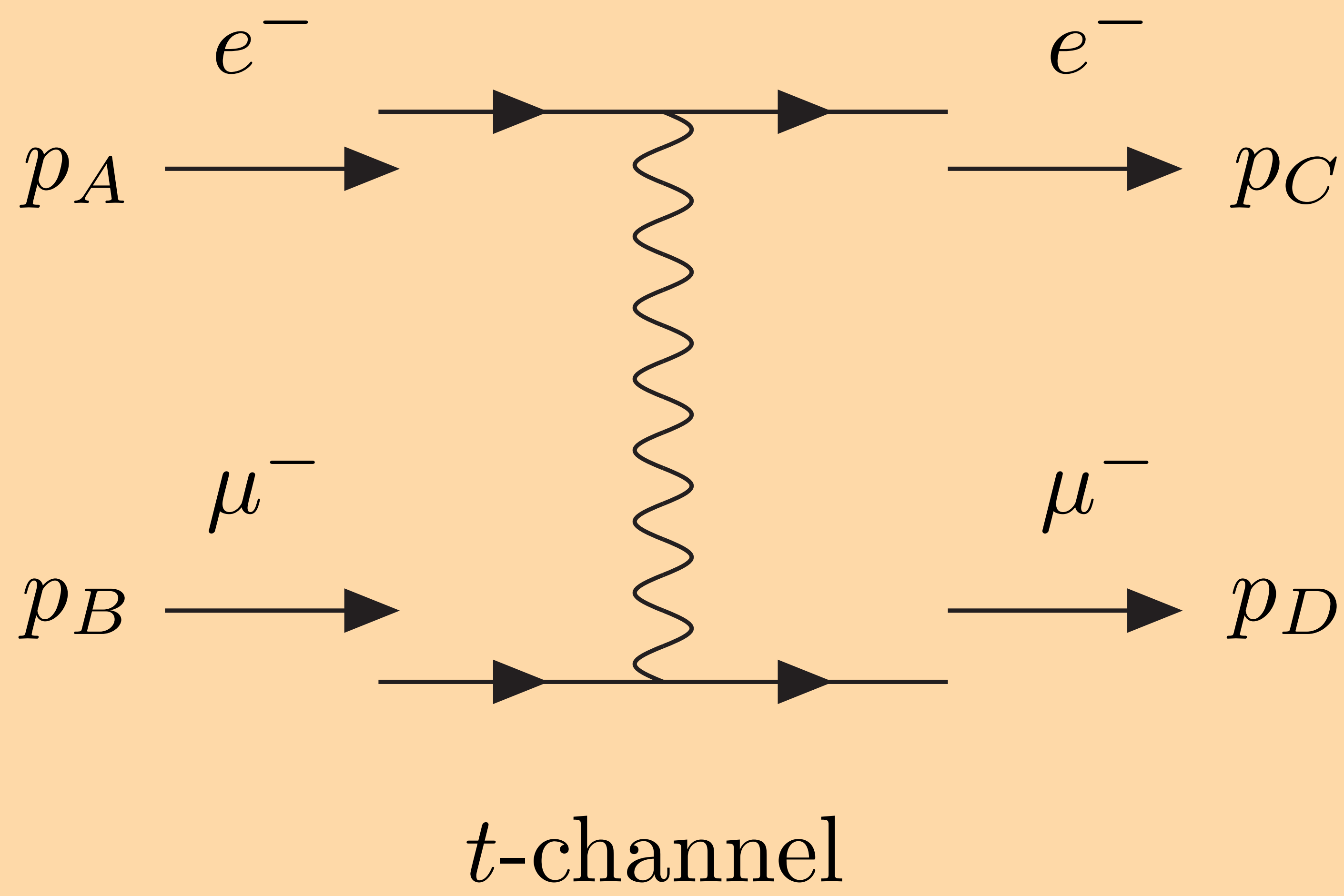
If incoming e^- has four momentum p_i we have $\phi_i(x) = N_i e^{-ip_i . x}$

-- N_i is normalization constant --

using analogous expression for ϕ_f it follows that

$$e j^{fi}_{\mu} = e N_i N_f (p_i + p_f)_{\mu} \, e^{i(p_f - p_i).x}$$

Spinless electron-muon scattering



tree level diagram for electron-muon scattering

Source Current

Using results for scattering of spinless electron off A^μ

we calculate scattering of same e off another charged particle

say a spinless muon

Calculation is a straightforward extension of previous one

we just have to identify electromagnetic potential A^μ

with its source charged spinless muon

This is done with help of Maxwell's equations $\square^2 A^\mu = j^\mu_{(2)}$

which determine A^μ associated with current

$$e j^\mu_{(2)} = e N_B N_D (p_D + p_B)^\mu \, e^{i(p_D - p_B).x}$$ ♜

where momenta are defined in Feynman diagram

Using $\square^2 e^{iq.x} = -q^2 \, e^{iq.x}$ we obtain

$$A^\mu = -\frac{1}{q^2} \, j^\mu_{(2)}$$

with $q = p_D - p_B$

Spinless electron-muon scattering @ tree level

Inserting this field due to muon into ♘

we find tree level amplitude for electron muon scattering

$$T_{fi} = -ie^2 \int j_\mu^{(1)}(x) \left(\frac{-1}{q^2} \right) j^\mu_{(2)} \, d^4x$$

Substituting ♜ and ♚ and carrying out x integration

$$T_{fi} = -i N_A \, N_B \, N_C \, N_D \, (2\pi)^4 \, \delta^{(4)}(p_D + p_C - p_B - p_A) \, \mathfrak{M}$$ ◒

where

$$-i\mathfrak{M} = [-ie(p_A + p_C)^\mu] \left(-i \frac{g_{\mu\nu}}{q^2} \right) [-ie(p_B + p_D)^\nu]$$ ☬

Invariant Amplitude

- A consistency check on shows that we would have obtained same amplitude considering muon scattering off A^μ produced by electron
- Consequently $\mathfrak{M}$ (as defined by) is called invariant amplitude
- delta function expresses energy-momentum conservation for process
- Photon propagator carries Lorentz indices because is spin-1 particle
- Four-momentum q of photon is determined by four-momentum conservation at vertices
- We see that $q^2 \neq 0$ and we say photon is virtual or off-mass shell
- Each vertex factor contains electromagnetic coupling e and a 4-vector index to connect with photon index
- Particular distribution of minus signs and factors i has been made to give correct result for higher order diagrams
- Note that multiplicative of three factors gives $-i\mathfrak{M}$
- Whenever same vertex or internal line occurs in Feynman diagram corresponding factor will contribute multiplicatively to amplitude $-i\mathfrak{M}$ for that diagram

Boundary Conditions

- ✗ To relate these calculations to experimental observables we must set normalization N of free particle wave functions

$$\phi = N\, e^{i(\vec{p}.\vec{x}-Et)}$$

- ✗ Recall that probability density of particles described by ϕ is

$$\rho = 2E|N|^2$$

- ✗ Proportionality of ρ to E was just what we needed to compensate for Lorentz contraction of volume element d^3x and to keep number of particles ρd^3x unchanged

- ✗ We then work with a volume V and normalize to $2E$ particles within that volume

$$\int_V \rho\, dV = 2E$$

$$N = \frac{1}{\sqrt{V}}$$

Cross-Section

Transition rate per unit volume of process $A + B \to C + D$ is

$$W_{fi} = \frac{|T_{fi}|^2}{TV}$$

T is interval of interaction and transition amplitude is given by ◒

Upon squaring one delta function remains

and $(2\pi)^4$ times other gives TV

Making use of ✤ we obtain

$$W_{fi} = (2\pi)^4 \frac{\delta^{(4)}(p_A + p_B - p_C - p_D)|\mathfrak{M}|^2}{V^4}$$ ❦

Experimental results on $AB \to CD$ scattering

quoted as cross section ☛ related to transition rate according to

$$\text{cross section} = \frac{W_{fi}}{(\text{initial flux})}(\text{number of final states})$$ ✻

factors in brackets ☛ density of incoming and outgoing states

Normalization

For single particle ☛ quantum physics restricts # of final states in V with momenta in element d^3p to be $V\,d^3p/(2\pi)^3$ but we have $2E$ particles in V

$$\text{No. of final states/particle} = \frac{V\,d^3p}{(2\pi)^3\,2E}$$

For particles C, D scattered into momentum elements d^3p_C, d^3p_D

$$\text{No. of available final states} = \frac{V\,d^3p_C}{(2\pi)^3 2E_C}\,\frac{V\,d^3p_D}{(2\pi)^3 2E_D} \quad ♣$$

It is easiest to calculate initial flux in lab frame

# of beam particles passing per unit area per unit time ☛ $|\vec{v}_A| 2E_A/V$

# of target particles per unit volume ☛ $2E_B/V$

normalization-independent measure of ingoing density by taking

☛ $$\text{Initial flux} = |\vec{v}_A|\,\frac{2E_A}{V}\,\frac{2E_B}{V} \quad ✤$$

Differential Cross Section

All in all ☛ $d\sigma$ for scattering into $d^3p_C\, d^3p_D$

$$d\sigma = \frac{V^4}{|\vec{v}_A| 2E_A\, 2E_B}\frac{1}{V^4}|\mathfrak{M}|^2\frac{(2\pi)^4}{(2\pi)^6}\delta^{(4)}(p_A + p_B - p_C - p_D)\,\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}$$

Arbitrary normalization volume cancels

drop V and work in unit volume

Normalize to $2E$ particles/unit volume

Normalization factor of wave function is $N = 1$

Lorentz invariant phase space

$$dQ = (2\pi)^4\,\delta^{(4)}(p_A + p_B - p_C - p_D)\frac{d^3p_C}{(2\pi)^3\,2E_C}\frac{d^3p_D}{(2\pi)^3\,2E_D}$$

δ integration over p_D enforcing 3-momentum conservation sets $\vec{p}_C = -\vec{p}_D$

$$\begin{aligned} dQ &= \frac{1}{4\pi^2}\frac{d^3p_C}{2E_C}\frac{1}{2E_D}\delta(E_A + E_B - E_C - E_D) \\ &= \frac{1}{4\pi^2}\frac{p_C^2\,dp_C\,d\Omega}{4E_C E_D}\delta(W - E_C - E_D) \qquad ♟ \end{aligned}$$

$d\Omega$ is element of solid angle about $\vec{p}_C$ & $\sqrt{s} \equiv W = E_A + E_B$

Using $W = E_C + E_D = (p_f^2 + m_C^2)^{1/2} + (p_f^2 + m_D^2)^{1/2}$

$$\frac{dW}{dp_f} = p_f\left(\frac{1}{E_C} + \frac{1}{E_D}\right)$$

and rewrite ♟ as

$$\begin{aligned} dQ &= \frac{1}{4\pi^2}\frac{p_f}{4}\left(\frac{1}{E_C + E_D}\right) dW\; d\Omega\; \delta(W - E_C - E_D) \\ &= \frac{1}{4\pi^2}\frac{p_f}{4\sqrt{s}}\,d\Omega \qquad \text{with } |\vec{p}_C| = |\vec{p}_D| = p_f \end{aligned}$$

Differential cross section in c.m. frame

Incident flux for a general collinear collision between A and B reads

$$\begin{aligned} F &= |\vec{v}_A - \vec{v}_B|\, 2E_A\, 2E_B \\ &= 4(|\vec{p}_A|E_B + |\vec{p}_B|E_A) \\ &= 4[(p_A.p_B)^2 - m_A^2 m_B^2)]^{1/2} \end{aligned}$$

and hence differential cross section in center-of-mass is

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{c.m.}} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathfrak{M}|^2$$

♨ where ☛ $|\vec{p}_A| = |\vec{p}_B| = p_i$

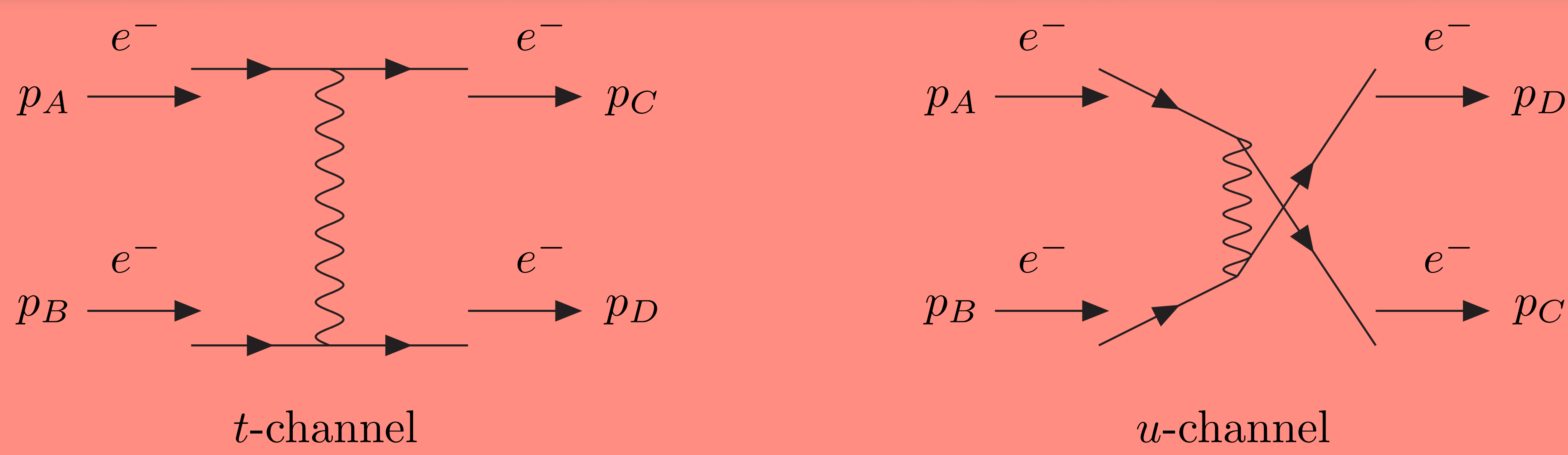
In special case where all four particles have identical masses (including commonly seen limit $m \to 0$) ♨ reduces to

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{c.m.}} = \frac{|\mathfrak{M}|^2}{64\pi^2 s}$$

$e^-e^- \to e^-e^-$ invariant amplitude

For electron-electron scattering we need
account for identical particles in the initial and final states,
amplitude should be symmetric under interchange of particle labels

$C \leftrightarrow D$ and $A \leftrightarrow B$



Lowest-order Feynman diagrams for Møller scattering

tree level invariant amplitude for scattering of a spinless electron

$$-i\mathfrak{M} = -i\left(-\frac{e^2(p_A+p_C)_\mu(p_B+p_D)^\mu}{(p_D-p_B)^2} - \frac{e^2(p_A+p_D)_\mu(p_B+p_C)^\mu}{(p_C-p_B)^2} \right)$$

symmetry under $p_C \leftrightarrow p_D$
ensures that $\mathfrak{M}$ is also symmetric under $p_A \leftrightarrow p_B$

Scattering of spin-1/2 particles

Free electron of 4-momentum p^μ is described by $\psi = u(p)e^{-ip.x}$

satisfying $(\gamma_\mu p^\mu - m)\psi = 0$

Electron in an electromagnetic field A^μ

obtained by substitution $p^\mu \to p^\mu - eA^\mu$

where we have again taken e to be charge of the electron

We find ☛ $$(\gamma_\mu p^\mu - m)\psi = \gamma^0 V \psi$$ ◉

perturbation is given by $$\gamma^0 V = e\gamma_\mu A^\mu$$

Introduction of γ^0 is to make ◉ of form $(E + \dots)\psi = V\psi$

so that potential energy enters in same way as in Schrödinger eq.

Spin-1/2 transition current

Using first-order perturbation theory $T_{fi} = -i \int d^4x \, \phi^*(x) \, V(x) \, \phi_i(x)$

amplitude for scattering of an electron from state ψ_i to state ψ_f is

$$
\begin{aligned}
T_{fi} &= -i \int \psi_f^\dagger(x) \, V(x) \, \psi_i(x) \, d^4x \\
&= -ie \int \overline{\psi}_f \, \gamma_\mu \, A^\mu \, \psi_i \, d^4x \\
&= ie \int j_\mu^{fi} \, A^\mu \, d^4x
\end{aligned}
$$

where ☛

$$
\begin{aligned}
e \, j_\mu^{fi} &\equiv e \, \overline{\psi}_f \, \gamma_\mu \, \psi_i \\
&= e \, \overline{u}_f \, \gamma_\mu \, u_i \, e^{i(p_f - p_i).x}
\end{aligned}
$$

is electromagnetic transition current between states i and f

Tree level transition amplitude for electron-muon scattering

$$
\begin{aligned}
T_{fi} &= -ie^2 \int j_\mu^{(1)}(x) \left(\frac{-1}{q^2}\right) j^\mu_{(2)}\, d^4x \\
&= -i(e\overline{u}_C\gamma_\mu u_A)\left(\frac{-1}{q^2}\right)(e\overline{u}_D\gamma^\mu u_B)\ (2\pi)^4\ \delta^{(4)}(p_A + p_B - p_C - p_D)
\end{aligned}
$$

$$q = p_A - p_C$$

Recall that the invariant amplitude $\mathfrak{M}$ is defined by

$$T_{fi} = -i(2\pi)^4\delta^{(4)}(p_A + p_B - p_C - p_D)\,\mathfrak{M}$$

and so we have

$$-i\mathfrak{M} = (-ie\overline{u}_C\gamma^\mu u_A)\left(\frac{-ig_{\mu\nu}}{q^2}\right)(-ie\overline{u}_D\gamma^\nu u_B)$$

Spin average

To calculate unpolarized cross section
we must amend cross section formulae of spinless particles

By unpolarized we mean that
no information about electron spins is recorded in experiment

To allow for scattering in all possible spin configurations
we therefore have to make replacement

$$|\mathfrak{M}|^2 \rightarrow \overline{|\mathfrak{M}|^2} \equiv \frac{1}{(2s_A+1)(2s_B+1)} \sum_{\text{spins}} |\mathfrak{M}|^2$$

where s_A, s_B are spins of incoming particles

We average over spins of incoming particles
and sum over spins of particles in final state

How to obtain unpolarized cross section

To obtain (unpolarized) cross section take square of modulus of

$$\mathfrak{M} = -e^2 \overline{u}(k')\,\gamma^\mu\, u(k)\left(\frac{1}{q^2}\right)\overline{u}(p')\,\gamma^\nu\, u(p)$$

and carry out spin sums

with momenta $p_A = k, p_B = p, p_C = k', p_D = p'$ and $q = k - k'$

Convenient to separate sums over electron and muon spins

$$\overline{|\mathfrak{M}|^2} = \frac{e^4}{q^4} L^{\mu\nu}_{(e)} L^{(\mu)}_{\mu\nu}$$

tensor associated with electron vertex

$$L^{\mu\nu}_{(e)} \equiv \frac{1}{2}\sum_{e-\mathrm{spins}} [\overline{u}(k')\gamma^\mu u(k)][\overline{u}(k')\gamma^\nu u(k)]^*$$

and with a similar expression for $L^{(\mu)}_{\mu\nu}$

A little bit of algebra

$$L^{\mu\nu}_{(e)} \equiv \frac{1}{2} \sum_{e-\text{spins}} [\overline{u}(k')\gamma^\mu u(k)][\overline{u}(k')\gamma^\nu u(k)]^* \quad ✷$$

using $\gamma^{\nu\dagger}\gamma^0 = \gamma^0\gamma^\nu$

$$\begin{aligned}[u^\dagger(k')\,\gamma^0\,\gamma^\nu\,u(k)]^\dagger &= [u^\dagger(k)\,\gamma^{\nu\dagger}\,\gamma^0\,u(k')] \\ &= [\overline{u}(k)\gamma^\nu u(k')]\end{aligned}$$

✷ becomes

$$L^{\mu\nu}_{(e)} = \frac{1}{2}\sum_{s'} \overline{u}^{(s')}_\alpha(k')\,\gamma^\mu_{\alpha\beta}\,\underbrace{\sum_s u^{(s)}_\beta(k)\,\overline{u}^{(s)}_\gamma(k)}_{(\not{k}+m_e)_{\beta\gamma}}\;\gamma^\nu_{\gamma\delta}\,u^{(s')}_\delta(k')$$

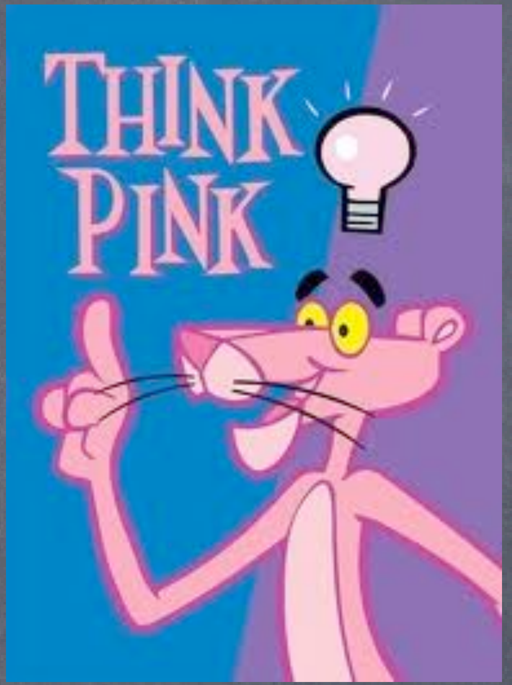
mass of electron

spin summations look like a forbidding task

Think Pink

$L^{\mu\nu}_{(e)}$ becomes trace of product of 4×4 matrices

$$L^{\mu\nu}_{(e)} = \frac{1}{2}\mathrm{Tr}[(\not{k}' + m_e)\,\gamma^\mu\,(\not{k} + m_e)\,\gamma^\nu]$$



Using trace theorems

evaluation of tensor associated with electron vertex

$$\begin{aligned} L^{\mu\nu}_{(e)} &= \frac{1}{2}\mathrm{Tr}(\not{k}'\gamma^\mu\not{k}\gamma^\nu) + \frac{1}{2}m_e^2\mathrm{Tr}(\gamma^\mu\gamma^\nu) \\ &= 2(k'^\mu k^\nu + k'^\nu k^\mu - (k'.k - m_e^2)g^{\mu\nu}) \end{aligned}$$

Evaluation of $L^{\mu\nu}_{(\mu)}$ is identical

$$L^{(\mu)}_{\mu\nu} = 2(p'_\mu p_\nu + p'_\nu p_\mu - (p'.p - m_\mu^2)g_{\mu\nu})$$

m_μ is mass of muon

Hints 4 Calculation: Trace Theorems

$$\text{Tr}\ \mathbb{I} = 4$$

trace of an odd number of γ^μ vanishes

$$\text{Tr}(\not a\ \not b) = 4\, a\ .\ b$$

$$\text{Tr}(\not a\ \not b\ \not c\ \not d) = 4[(a\ .\ b)(c\ .\ d) - (a\ .\ c)(b\ .\ d) + (a\ .\ d)(b\ .\ c)]$$

$$\text{Tr}(\gamma_5) = 0$$

$$\text{Tr}(\gamma_5\ \not a\ \not b) = 0$$

$$\text{Tr}(\gamma_5\ \not a\ \not b\ \not c\ \not d) = 4i\, \epsilon_{\mu\nu\lambda\sigma}\ a^\mu\ b^\nu\ c^\lambda\ d^\sigma$$

Spin average $e^- \mu^- \rightarrow e^- \mu^-$ amplitude

Forming product of ♆ and ▣

obtain exact form for spin average amplitude

$$
\begin{aligned}
\overline{|\mathfrak{M}|^2} &= \frac{8e^4}{q^4}[(k' \,.\, p')(k \,.\, p) + (k' \,.\, p)(k \,.\, p') \\
&+ m_e^2 p' \,.\, p - m_\mu^2 k' \,.\, k + 2m_e^2 m_\mu^2]
\end{aligned}
$$

In extreme relativistic limit neglect terms containing m_μ^2 and m_e^2

Mandelstam Variables

Cross sections and decay rates can be written using kinematic variables that are relativistic invariants

For any two particle to two particle process (m_μ^2)

we have at our disposal 4-momenta associated with each particle

invariant variables are scalar products $p_A \cdot p_B, p_A \cdot p_C, p_A \cdot p_D$

conventional to use related (Mandelstam) variables

$$\begin{aligned} s &= (p_A + p_B)^2 = (p_C + p_D)^2 \\ t &= (p_A - p_C)^2 = (p_B - p_D)^2 \\ u &= (p_A - p_D)^2 = (p_B - p_C)^2 \end{aligned}$$

Because $p_i^2 = m_i^2$ (with $i = A, B, C, D$) and $p_A + p_B = p_C + p_D$ due to energy momentum conservation

$$\begin{aligned} s + t + u &= \sum_i m_i^2 + 2p_A^2 + 2p_A.(p_B - p_C - p_D) \\ &= \sum_i m_i^2 \end{aligned}$$

i.e. only two of the three variables are independent

Forward and Backward Scattering

To get a better feel for $s, t,$ and u let us evaluate them explicitly

Taking

$$p_A = (E, \vec{k}_i),\ p_B = (E, -\vec{k}_i),\ p_C = (E, \vec{k}_f),\ p_D = (E, -\vec{k}_f), E = (k^2 + m^2)^{1/2}$$

in center-of-mass frame for particles all of mass m

$$\begin{aligned} s &= (p_A + p_B)^2 = 4(k^2 + m^2), \\ t &= (p_A - p_C)^2 = -(\vec{k}_i - \vec{k}_f)^2 = -2k^2(1 - \cos\theta) \\ u &= (p_A - p_D)^2 = -(\vec{k}_i + \vec{k}_f)^2 = -2k^2(1 + \cos\theta) \end{aligned}$$

θ is center-of-mass scattering angle

i.e. $\vec{k}_i \,.\, \vec{k}_f = k^2 \cos\theta$

As $k^2 \geq 0$ we have $s \geq 4m^2$

and since $-1 \leq \cos\theta \leq 1$ we have $t \leq 0$ and $u \leq 0$

Note that $t = 0 (u = 0)$ corresponds to forward (backward) scattering

Hints for Calculation

In $(p_A - p_C)^2$ energy component cancels so

$$(p_A - p_C)^2 = -(\vec{k}_i - \vec{k}_f)^2$$

Unpolarized scattering amplitude

In extreme relativistic limit Mandelstam variables become

$$
\begin{aligned}
s &\equiv (k+p)^2 &&\simeq 2k\,.\,p \simeq 2k'\,.\,p' \simeq 4k^2\,,\\
t &\equiv (k-k')^2 &&\simeq -2k\,.\,k' \simeq -2p\,.\,p' \simeq -2k^2(1-\cos\theta)\,,\\
u &\equiv (k-p')^2 &&\simeq -2k\,.\,p' \simeq -2k'\,.\,p \simeq -2k^2(1+\cos\theta)
\end{aligned}
$$

where $p_A \equiv k, p_B \equiv p, p_C \equiv k'$ and $p_D \equiv p'$

At high energies unpolarized $e^-\mu^- \to e^-\mu^-$ scattering amplitude

$$
\begin{aligned}
\overline{|\mathfrak{M}|^2} &= \frac{8e^4}{(k-k')^4}[(k'\,.\,p')(k\,.\,p)+(k'\,.\,p)(k\,.\,p')]\\
&= 2e^4\,\frac{s^2+u^2}{t^2}
\end{aligned}
$$

Hints for calculation

In massless limit relativistic energy relation becomes $E^2 = p^2$

so

$$(k^\mu - k'_\mu)^2 = -2k^\mu k'_\mu = -2k\,.\,k' = 2(|\vec{k}||\vec{k}'| - \vec{k}\,.\,\vec{k}')$$

Crossing

Amplitude for $e^- e^+ \to \mu^+ \mu^-$

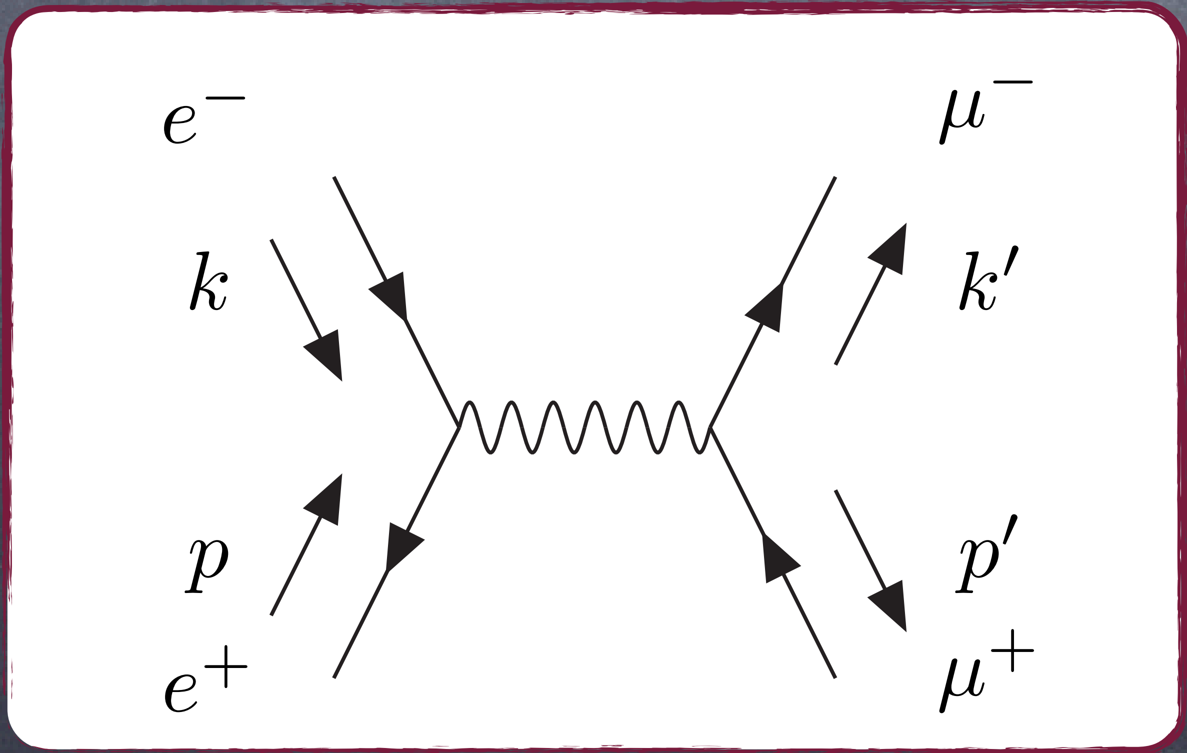
by crossing result for $e^- \mu^- \to e^- \mu^-$

required interchange is $k' \leftrightarrow -p$ in ❖

that is $s \leftrightarrow t$ ☛ we obtain

$$\overline{|\mathfrak{M}|^2} = 2e^4 \frac{t^2 + u^2}{s^2}$$

$e^- e^+ \to \mu^+ \mu^-$ ☛ s-channel process



Feynman tree level diagram

Hints for Calculation

$k' \leftrightarrow -p$ means $k' \to -k'$ and $p \to -p$ yielding

$$s = (k-p)^2$$

$$t = (k+k')^2$$

Differential cross section for $e^- e^+ \to \mu^+ \mu^-$ scattering

using $$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{c.m.}} = \frac{|\mathfrak{M}|^2}{64\pi^2 s}$$

In center-of-mass frame we have $$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{cm}} = \frac{1}{64\pi^2 s} 2e^4 [\tfrac{1}{2}(1+\cos^2\theta)]$$

quantity in square brackets is $(t^2+u^2)/s^2$

Using $\alpha = e^2/4\pi$ this becomes $$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{cm}} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

To obtain total cross section we integrate over θ and ϕ

$$\sigma_{e^+e^-\to\mu^+\mu^-} = \frac{4\pi\alpha^2}{3s}$$

Positron-Electron Tandem Ring Accelerator

PETRA accelerator

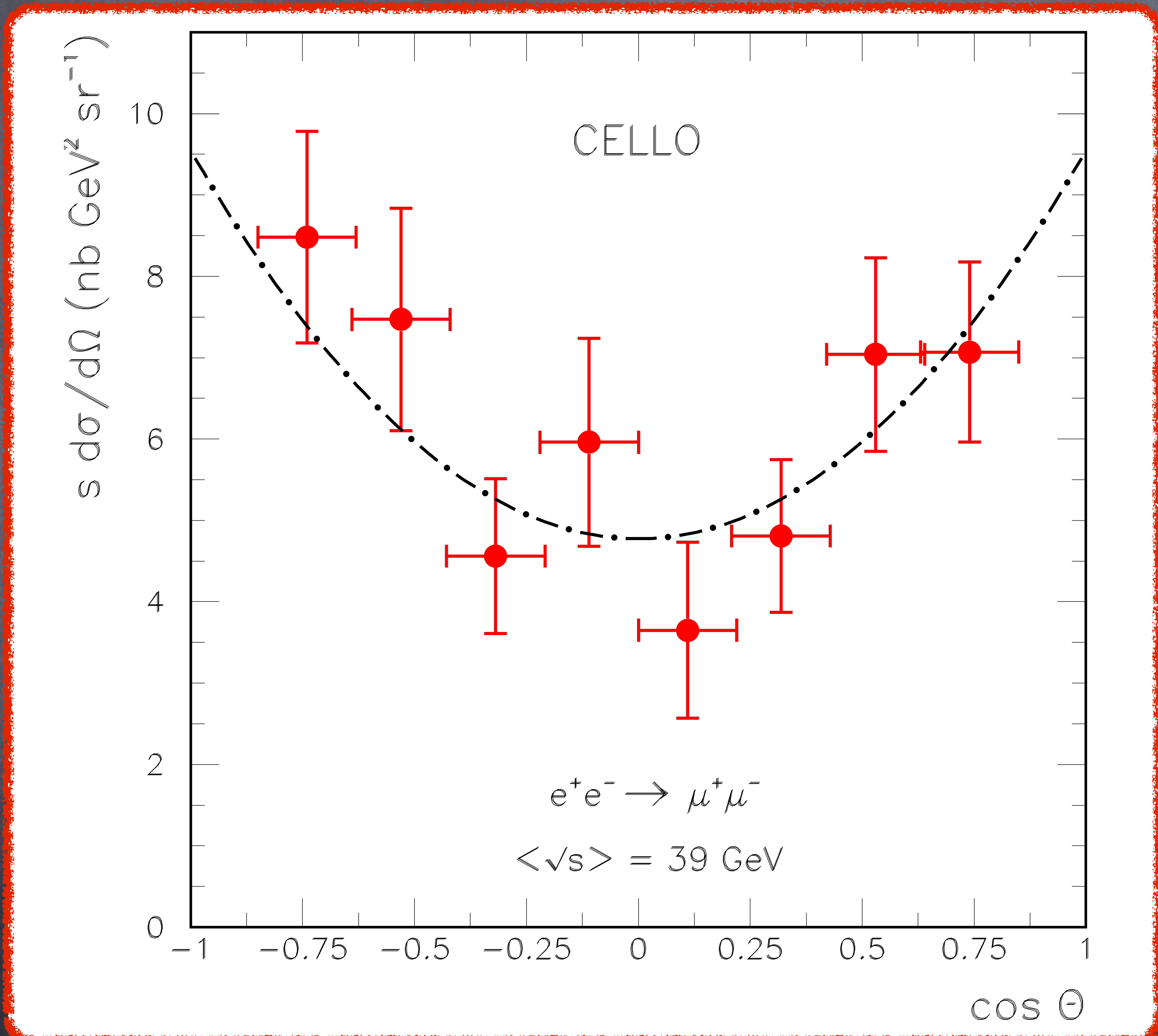
e^+e^- beams

collisions at $\sqrt{s} = 2E_{\mathrm{beam}}$

$$\sigma_{e^+e^-\to\mu^+\mu^-} = \frac{20(\mathrm{nb})}{E^2_{\mathrm{beam}}/\mathrm{GeV}^2}$$ ⌬

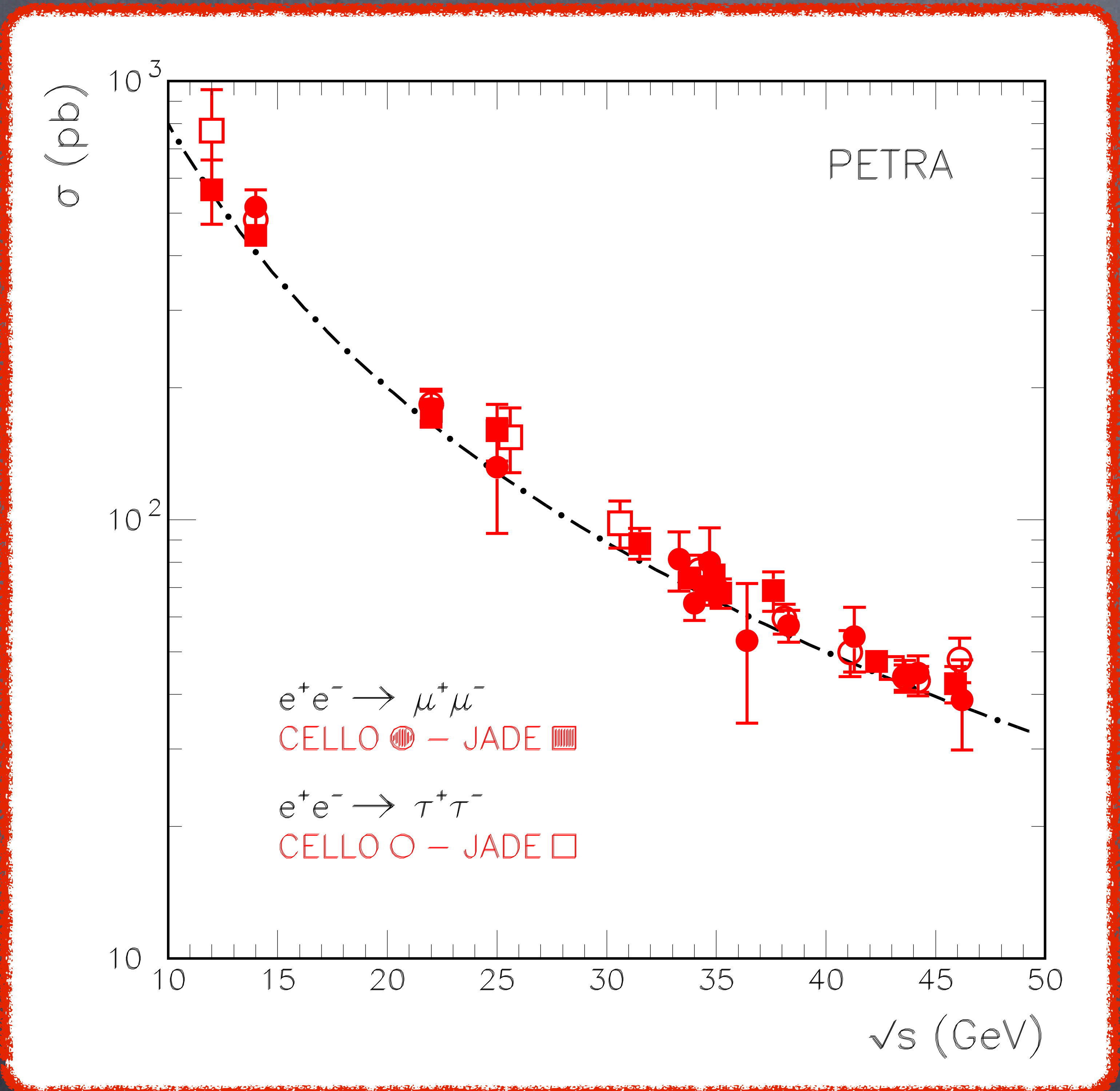
Corrections to ⌬ of order $\alpha^3,\ \alpha^4, \ldots$ from higher order diagrams

$e^+e^- \to \mu^+\mu^-$ angular distribution for $(\sqrt{s}) = 39$ GeV



$- \cdot -$ relativistic limit of lowest order QED prediction

Cross section



cross section measured at PETRA versus center-of-mass energy

solid (open) symbols $e^+e^- \to \mu^+\mu^- \; (e^+e^- \to \tau^+\tau^-)$

– · – relativistic limit of lowest order QED prediction

CU on Tuesday