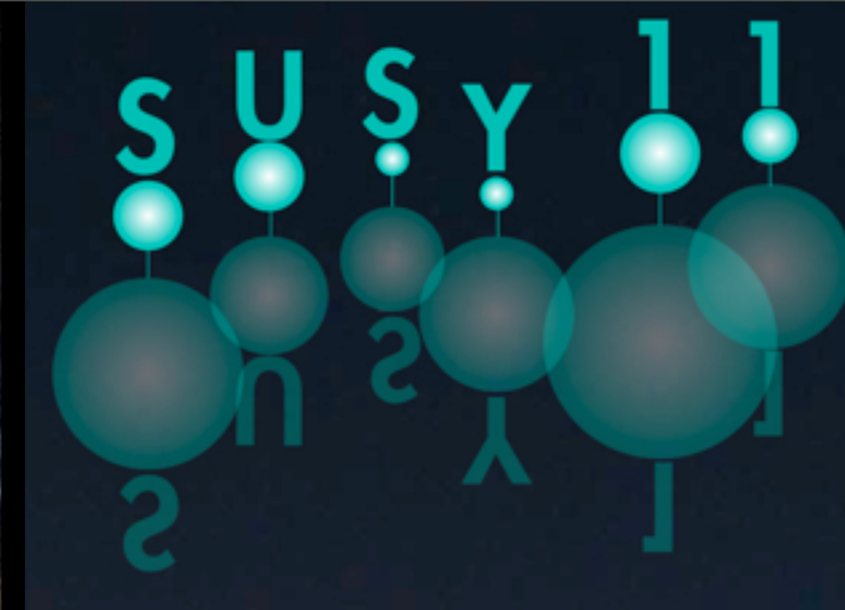
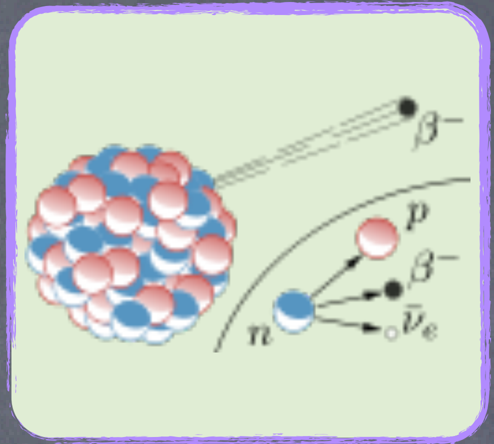


Particle Physics 2011

Luis Anchordoqui

Weak Interactions



- Oldest and best-known examples of weak processes are: β -decay of atomic nuclei

  and more fundamental neutron decay $n \to p\bar{\nu}e^-$

- By analogy to emission of photons in nuclear γ -decay

  Fermi considered neutrino-electron pair to be created and emitted in nuclear transition of a neutron to a proton

Inspired by current-current form of electromagnetic interaction he proposed that invariant amplitude for β-decay process be given by

$$\mathfrak{M} = G_F \left(\overline{u}_n \gamma^\mu u_p\right) \left(\overline{\nu}_e \gamma_\mu u_e\right)$$

effective coupling G_F needs to be determined by experiment (known as Fermi constant)

- Amplitude explained properties of some features of β-decay but not others

- Attempts to unravel true form of weak interaction in following 25 yr lead to a whole series of ingenious β-decay experiments reaching climax with discovery of parity violation in 1956

Charge-raising weak current

- Only essential change required in Fermi's original proposal

  was replacement of γ^μ by $\gamma^\mu(\mathbb{I} - \gamma^5)$

- Fermi had not forseen parity violation
  and had no reason to include a $\gamma^5\gamma^\mu$ contribution

  a mixture of γ^μ and $\gamma^5\gamma^\mu$ automatically violates parity conservation
  e.g. charge-raising weak current

$$J^\mu = \overline{u}_\nu \gamma^\mu \frac{1}{2}(\mathbb{I} - \gamma^5) u_e$$

  couples an ingoing negative helicity electron e_L
  to an outgoing negative helicity neutrino

- Besides configuration (e_L^-, ν_L) ☛ charge-raising weak current

  also couples following (ingoing, outgoing) lepton pair configurations:

$$(\overline{\nu}_R, e_R^+),\ (0, \nu_L e_R^+),\ (e_L^- \overline{\nu}_R, 0)$$

Charge lowering weak current

Charge-lowering weak current $J^\mu = \bar{u}_e \gamma^\mu \frac{1}{2}(\mathbb{I} - \gamma^5) u_\nu$
is hermitian conjugate of charge-raising weak current

$$
\begin{aligned}
J^{\mu\dagger} &= [\overline{u}_\nu \, \gamma^\mu \tfrac{1}{2}(\mathbb{I} - \gamma^{\mathbf{5}})\mathbf{u_e}]^\dagger \\
&= [u_\nu^\dagger \, \gamma^0 \, \gamma^\mu \tfrac{1}{2}(\mathbb{I} - \gamma^{\mathbf{5}})\mathbf{u_e}]^\dagger \\
&= u_e^\dagger \gamma^0 \gamma^0 \tfrac{1}{2}(\mathbb{I} - \gamma^{\mathbf{5}})\gamma^{\mu\dagger}\gamma^{\mathbf{0}}\mathbf{u}_\nu \\
&= \overline{u}_e \gamma^0 \tfrac{1}{2}(\mathbb{I} - \gamma^{\mathbf{5}})\gamma^{\mathbf{0}}\gamma^\mu \mathbf{u}_\nu \\
&= \overline{u}_e \, \gamma^\mu \tfrac{1}{2}(\mathbb{I} - \gamma^{\mathbf{5}})\mathbf{u}_\nu
\end{aligned}
$$

Weak interaction amplitudes are of form ☛ $\mathfrak{M} = \dfrac{4G_F}{\sqrt{2}} J^\mu J_\mu^\dagger$

Charge conservation requires that

$\mathfrak{M}$ be product of charge-raising and charge-lowering current

Factor of 4 arises because

currents are defined with normalized with projector operator $\frac{1}{2}(\mathbb{I} - \gamma^{\mathbf{5}})$

rather than old-fashioned $(\mathbb{I} - \gamma^{\mathbf{5}})$

$\dfrac{1}{\sqrt{2}}$ is pure convention (to keep original definition of G_F which did not include γ^5)

Parity Violation

Cumulative evidence of many experiments is that indeed only ν_L (and $\overline{\nu}_R$) are involved in weak interactions

absence of mirror image states $\overline{\nu}_L$ and ν_R
clear violation of parity invariance

Charge conjugation C transforms a ν_L state into a $\overline{\nu}_L$ state
C is violated

$(\mathbb{I} - \gamma^5)$ form leaves weak interaction invariant under combined CP operation

E.G.

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^+ \to \mu^+ \nu_R) = 0 \qquad P \text{ violation},$$
$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^- \to \mu^- \bar{\nu}_L) = 0 \qquad C \text{ violation}$$

but

$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \bar{\nu}_R) \qquad CP \text{ invariance}$$

ν denotes a muon neutrino

Fermi constant

Values of G_F obtained from measurements of

neutron lifetime

$$G_F = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$$ *

and muon lifetime

$$G_F = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$$ ※

are found to be within a few percent

Comparison of these results supports assertion that Fermi constant is same for all leptons and nucleons and hence universal

Nuclear β-decay and decay of muon have same physical origin

We'll see reason for small difference is important

Neutrino Probe

Although experiments exposing violation of parity in weak interactions
(polarized $^{60}\mathrm{Co}$ decay, K decay, π decay, etc)

are some of highlights in development of particle physics

parity violation and its $V - A$ structure

can now be demonstrated experimentally more directly with neutrinos

This is analogous to study of electromagnetic lepton-quark interaction

To predict neutrino-quark cross sections
we clearly need to know form of quark weak currents

Quarks interact electromagnetically just like leptons
(apart from their fractional charge)

Therefore we construct quark weak current just as we did for leptons

Invariant amplitude of CC interaction

We model charge-raising quark current

$$J_q^\mu = \overline{u}_u \gamma^\mu \frac{1}{2}(\mathbb{I} - \gamma^5) u_d$$ ✇

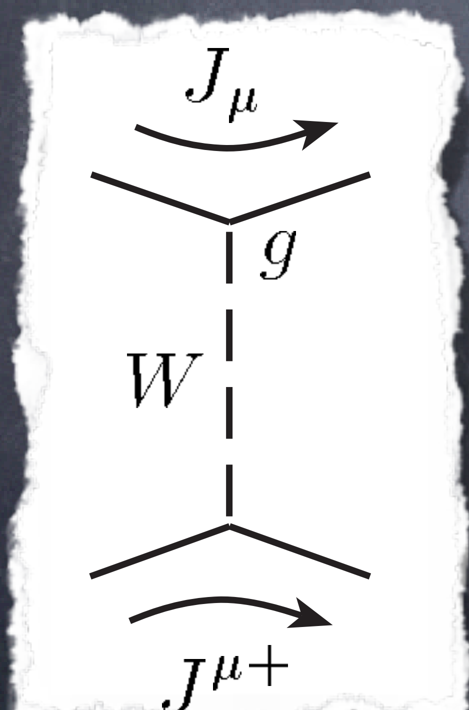
on weak current

$$J_e^\mu = \overline{u}_\nu \gamma^\mu \frac{1}{2}(\mathbb{I} - \gamma^5) u_e$$ ☢

hermitian conjugates give charge-lowering weak currents

Short range of weak interaction

results from exchange of a heavy gauge boson of mass m_W



$$= \left(\frac{g}{\sqrt{2}} J_\mu\right) \frac{1}{m_W^2} \left(\frac{g}{\sqrt{2}} J_\mu^\dagger\right)$$ ⚡

$$= \frac{4G_F}{\sqrt{2}} J_\mu J^{\mu\dagger}$$ ✈

Upon inserting currents ✇ and ☢ into ✈ ☛ we obtain invariant amplitude for charged current (CC) neutrino-quark scattering

Isoscalar Nucleons

To confront pQCD predictions with experiment

it is simplest to consider isoscalar nucleon targets

in which nuclei contain equal numbers of protons and neutrons

$$N = (p+n)/2$$

Procedure to embed constituent cross section is familiar from last class

$$\sigma = \int_0^1 dx \int_0^{xs} dQ^2 \, \frac{d^2\sigma_{\nu N}^{\rm CC}}{dx dQ^2}$$

where

$$\frac{d^2\sigma_{\nu N}^{\rm CC}}{dx dQ^2} = \frac{G_F^2}{4\pi x}\left(\frac{m_W^2}{Q^2+m_W^2}\right)^2 \left[Y_+ \, F_2^\nu(x,Q^2) - y\, F_{\rm L}^\nu(x,Q^2) + Y_- \, xF_3^\nu(x,Q^2)\right]$$

is differential cross-section given in terms of structure functions

$Y_+ = 1 + (1-y)^2, \; Y_- = 1-(1-y)^2, \; y = Q^2/sx$ and $s = 2E_\nu m_N$

νN @ LO

At LO in pQCD ☛ structure functions are given in terms of PDFs as

$$F_2^\nu = x(u + d + 2s + 2b + \bar{u} + \bar{d} + 2\bar{c} + 2\bar{t}),$$
$$xF_3^\nu = x(u + d + 2s + 2b - \bar{u} - \bar{d} - 2\bar{c} - 2\bar{t}),$$
and $F_L^\nu = 0$

and hence ♻ can be written in an old hat form

☎

$$\frac{d^2\sigma_{\nu N}^{CC}}{dxdy} = \frac{G_F^2\, s}{\pi}\left(\frac{m_W^2}{Q^2 + m_W^2}\right)^2 \left[xq_\nu^{CC}(x,Q^2) + (1-y)^2 x\overline{q}_\nu^{CC}(x,Q^2)\right]$$

where

$$q_\nu^{CC}(x,Q^2) = \frac{u_v(x,Q^2) + d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2) + d_s(x,Q^2)}{2} + s_s(x,Q^2) + b_s(x,Q^2)$$

$$\overline{q}_\nu^{CC}(x,Q^2) = \frac{\bar{u}_s(x,Q^2) + \bar{d}_s(x,Q^2)}{2} + \bar{c}_s(x,Q^2) + \bar{t}_s(x,Q^2)$$

subscripts v and s label valence and sea contributions
u, d, c, s, t and b
denote distributions for various quark flavors in a proton

$\bar{\nu}N$ @ LO

Calculation of $\bar{\nu}N$ scattering proceeds along lines of νN scattering with replacement of $F_2^\nu, xF_3^\nu, F_L^\nu \to F_2^{\bar{\nu}}, xF_3^{\bar{\nu}}, F_L^{\bar{\nu}}$

At leading order $F_2^{\bar{\nu}} = x(u + d + 2c + 2t + \bar{u} + \bar{d} + 2\bar{s} + 2\bar{b})$,

$xF_3^{\bar{\nu}} = x(u + d + 2c + 2t - \bar{u} - \bar{d} - 2\bar{s} - 2\bar{b})$

Going through same steps, we obtain

$$\frac{d^2\sigma_{\bar{\nu}N}^{\rm CC}}{dxdy} = \frac{G_F^2\, s}{\pi}\left(\frac{m_W^2}{Q^2+m_W^2}\right)^2 \left[x\bar{q}_{\bar{\nu}}^{\rm CC}(x,Q^2) + (1-y)^2 x q_{\bar{\nu}}^{\rm CC}(x,Q^2)\right]$$

If there were just three valence quarks in a nucleon ☛ $\bar{q}^{\rm CC}(x,Q^2) = 0$

neutrino-nucleon and antineutrino-nucleon scattering data would exhibit dramatic $V - A$ properties of weak interaction

$$\frac{d\sigma_{\nu N}^{\rm CC}}{dy} = c\,, \qquad \frac{d\sigma_{\bar{\nu}N}^{\rm CC}}{dy} = c(1-y)^2$$

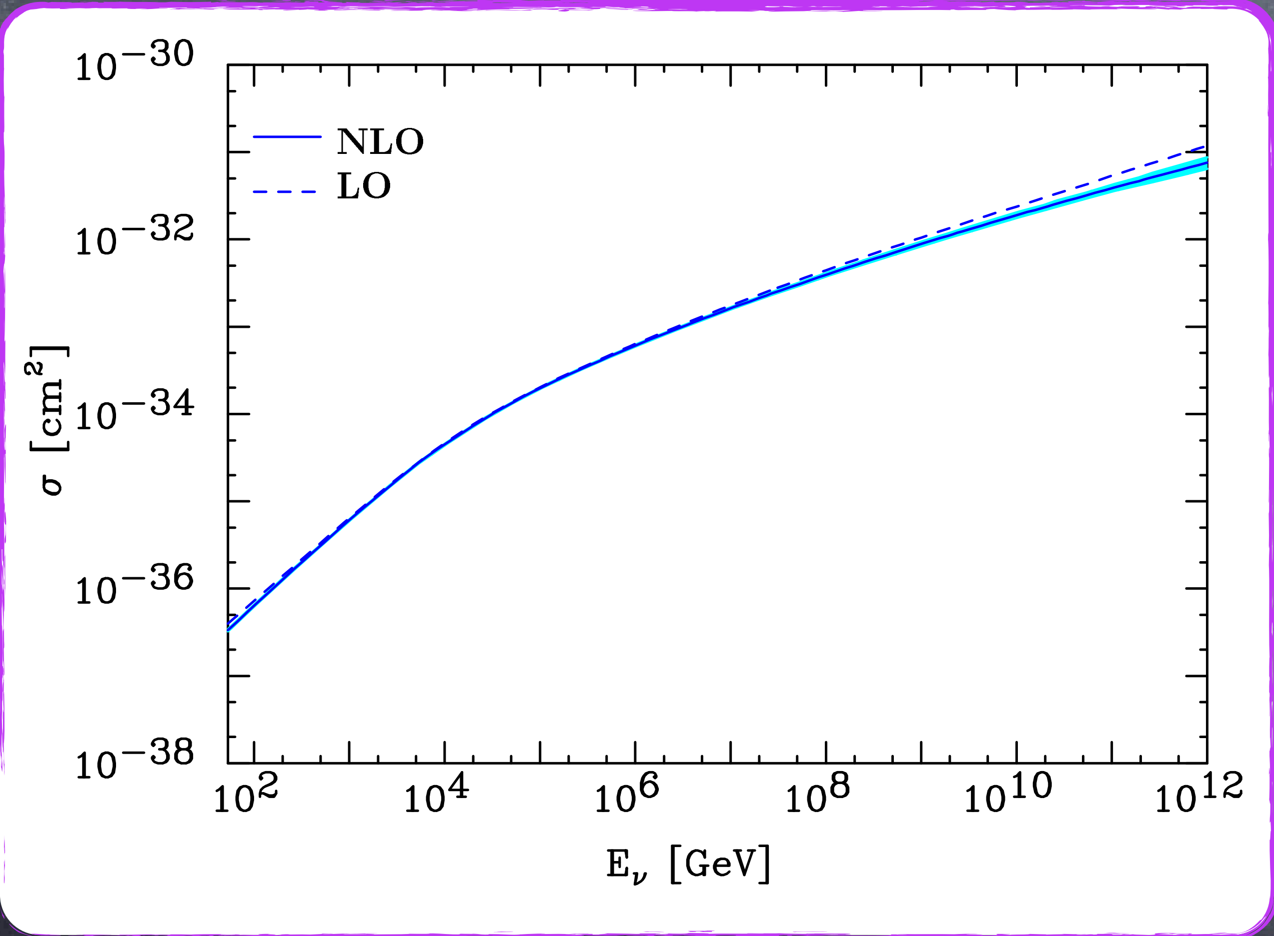
c can be found from ☎

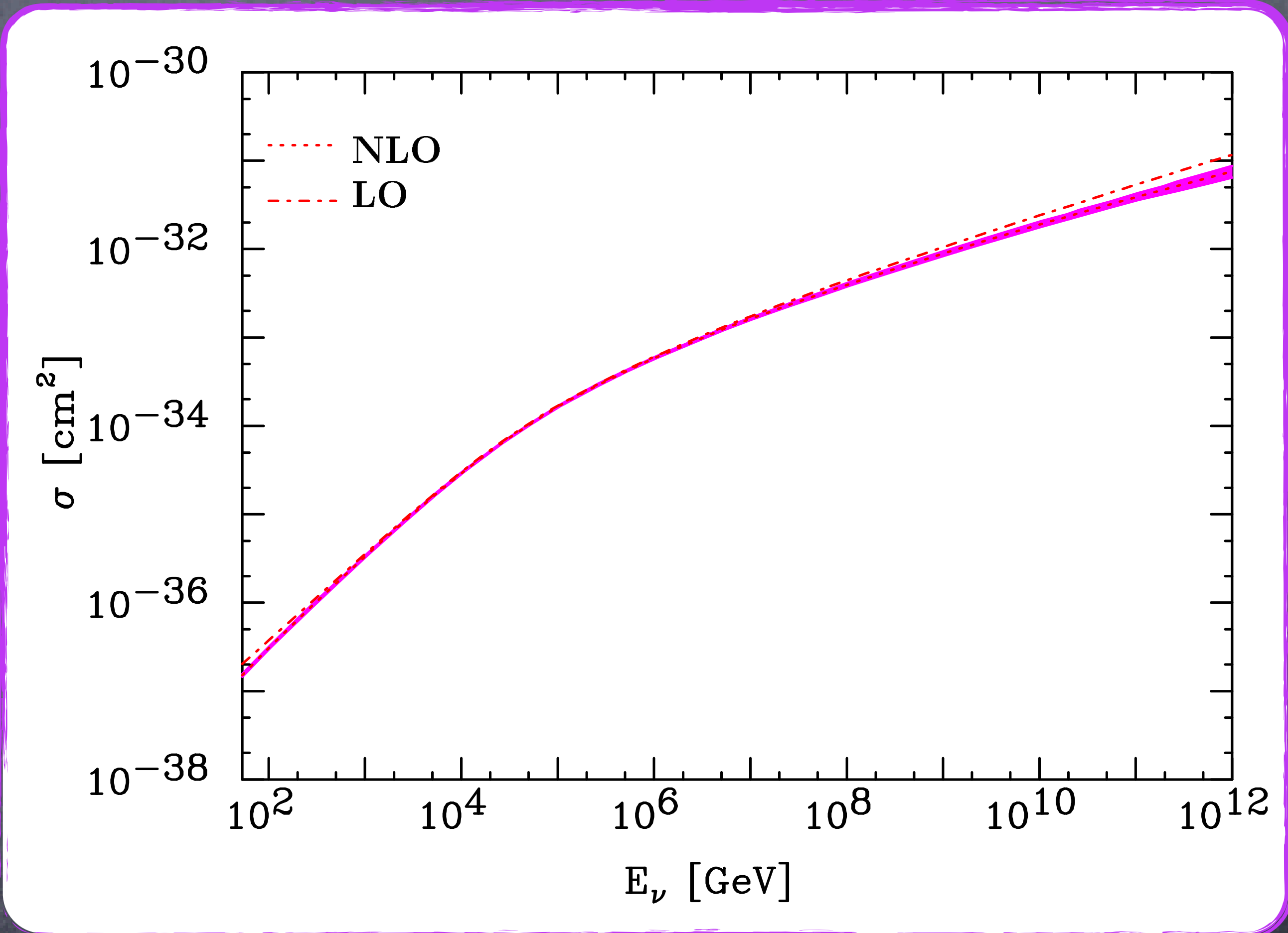
for integrated cross sections $\dfrac{\sigma_{\bar{\nu}N}^{\rm CC}}{\sigma_{\nu N}^{\rm CC}} = \dfrac{1}{3}$

NLO

At NLO ☛ relation between structure functions & quark momentum distributions involve further QCD calculable coefficient functions and contributions from $F_{\rm L}$ can no longer be neglected

QCD predictions for structure functions are obtained by solving DGLAP evolution equations at NLO





NLO inclusive νN (left) and $\bar{\nu} N$ (right) cross section with $\pm 1\sigma$ uncertainties (shaded band) compared with LO calculation

Weak Neutral Current Interactions

Discovery of neutrino-induced muonless events in 1973

heralded a new era in particle physics

These events ☛ most readily interpretable as

$$\nu_\mu(\overline{\nu})N \to \nu_\mu(\overline{\nu}) + \text{ hadrons}$$

are evidence of a weak neutral current

$$J_\mu^{\rm NC}(\nu) = \frac{1}{2}\left(\overline{u}_\nu \gamma^\mu \frac{1}{2}(\mathbb{I} - \gamma^5) u_\nu\right)$$

$$J_\mu^{\rm NC}(q) = \left(\overline{u}_q \gamma^\mu \frac{1}{2}(c_V^q \mathbb{I} - c_A^q \gamma^5) u_q\right)$$

with vector and axial-vector couplings given by

$$c_V^f = T_f^3 - 2\sin^2\theta_w Q_f \qquad c_A^f = T_f^3$$

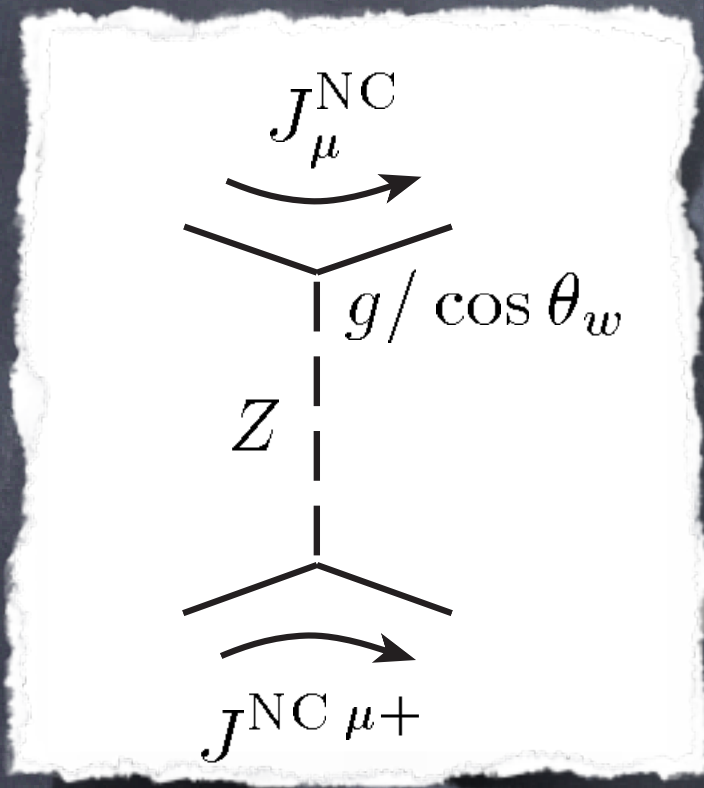
T_f^3 & Q_f are third component of weak isospin & charge of fermion f

Lepton	T	T^3	Q	Y	Quark	T	T^3	Q	Y
ν_e	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
e_L^-	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
					u_R	0	0	$\frac{2}{3}$	$\frac{2}{3}$
e_R^-	0	0	-1	-1	d_R	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$

ρ

In general ☛ $J_\mu^{\rm NC}$ (unlike $J_\mu^{\rm CC}$) is not pure $V-A$ current $(c_V \neq c_A)$

Neutral current interaction is described by a coupling $g/\cos\theta_w$



$$= \left(\frac{g}{\cos\theta_w} J_\mu^{\rm NC}\right)\left(\frac{1}{m_Z^2}\right)\left(\frac{g}{\cos\theta_w} J^{{\rm NC}\mu\dagger}\right) \quad ❖$$

$$= \frac{4G_F}{\sqrt{2}} 2\rho J_\mu^{\rm NC} J^{{\rm NC}\mu\dagger} \quad ☸$$

Identification of ⚡ and ✈ yields

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

Combining ❖ with ☸ gives

$$\rho\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_Z^2\cos^2\theta_w}$$

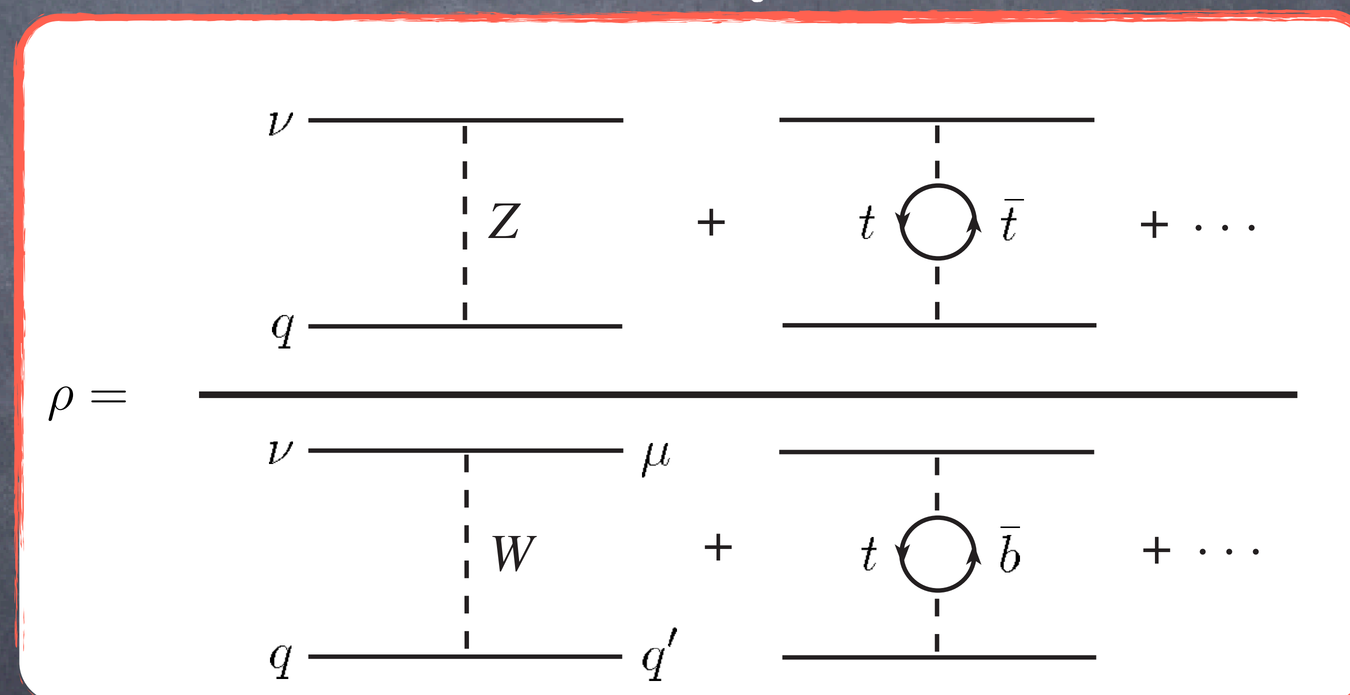
from last two equations and $m_W = \frac{g\,v}{2} = \frac{g}{2\sqrt{2}\,\lambda} m_H$ & $m_Z = \frac{m_W}{\cos\theta_w}$

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_w} = 1$$

IN OTHER WORDS...

- If model is successful ☛ all neutral current phenomena will be described by a common parameter
- For moment we will leave c_V^i, c_A^i and ρ as free parameters to be determined by experiment
- For further discussion it is useful to remember that:
  - neutral currents have a coupling ρG_F
  - ρ represents relative strength of neutral and charged weak currents

e.g. for neutrino-quark scattering:



- $\Delta\rho$ measures quantum corrections to ratio of neutral- and charged- current amplitudes at low energy

LO NC cross section

Calculation of inclusive cross sections $\nu N \to \nu X$ proceeds exactly as that for charged current processes

At LO in pQC we find

$$\frac{d^2\sigma_{\nu N}^{\rm NC}}{dx\,dy} = \frac{G_F^2\, M\, E_\nu}{2\pi}\left(\frac{m_Z^2}{Q^2+m_Z^2}\right)^2 \left[x q_\nu^{\rm NC}(x,Q^2) + (1-y)^2 x\overline{q}_\nu^{\rm NC}(x,Q^2)\right]$$

quark densities are given by

$$\begin{aligned} q_\nu^{\rm NC}(x,Q^2) &= \left[\frac{u_v(x,Q^2)+d_v(x,Q^2)}{2}\right]\left[(c_V^d+c_A^d)^2+(c_V^u+c_A^u)^2\right] \\ &+ 2\left[\frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}\right]\left[(c_V^d)^2+(c_A^d)^2+(c_V^u)^2+(c_A^u)^2\right] \\ &+ 2[s_s(x,Q^2)+b_s(x,Q^2)]\left[(c_V^d)^2+(c_A^d)^2\right] \\ &+ 2[c_s(x,Q^2)+t_s(x,Q^2)]\left[(c_V^u)^2+(c_A^u)^2\right] \end{aligned}$$

$$\begin{aligned} \overline{q}_\nu^{NC}(x,Q^2) &= \left[\frac{u_v(x,Q^2)+d_v(x,Q^2)}{2}\right]\left[(c_V^d-c_A^d)^2+(c_V^u-c_A^u)^2\right] \\ &+ 2\left[\frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}\right]\left[(c_V^d)^2+(c_A^d)^2+(c_V^u)^2+(c_A^u)^2\right] \\ &+ 2[s_s(x,Q^2)+b_s(x,Q^2)]\left[(c_V^d)^2+(c_A^d)^2\right] \\ &+ 2[c_s(x,Q^2)+t_s(x,Q^2)]\left[(c_V^u)^2+(c_A^u)^2\right] \end{aligned}$$

NC-to-CC ratio

A quantitative comparison of strength of NC to CC weak processes

obtained by NuTeV Collaboration scattering neutrinos off an iron target

Experimental values are

$$R_\nu^{\text{exp}} \equiv \frac{\sigma^{\text{NC}}_{\nu_\mu N \to \nu_\mu X}}{\sigma^{\text{CC}}_{\nu_\mu N \to \mu X}} = 0.3916 \pm 0.0007$$

$$R_{\bar\nu}^{\text{exp}} \equiv \frac{\sigma^{\text{NC}}_{\bar\nu_\mu N \to \bar\nu_\mu X}}{\sigma^{\text{CC}}_{\bar\nu_\mu N \to \mu X}} = 0.4050 \pm 0.0016$$

For $E_\nu > 10^7$ GeV theoretical prediction using CTEQ4 PDFs is

$$R_\nu = R_{\bar\nu} \simeq 0.4$$

Kaon decay

Leptons and quarks participate in weak interactions through $V-A$ CCs constructed from following pairs of (left- handed) fermion states

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} u \\ d \end{pmatrix}$$

All these charged currents couple with universal coupling G_F

It appears natural to try to extend this universality to embrace doublet

$$\begin{pmatrix} c \\ s \end{pmatrix}$$

formed from heavier quark states

However ☛ we already know that this cannot be quite correct

E.G. $K^+ \to \mu^+ \nu_\mu$ decay occurs ☛ K^+ is made of u and $\bar{s}$ quarks

implying there must be a weak current which couples a u to an $\bar{s}$ quark

This contradicts above scheme

which only allows weak transitions between $u \leftrightarrow d$ and $c \leftrightarrow s$

Quark Flavor Mixing

Instead of introducing new couplings to accommodate $K^+ \to \mu^+ \nu_\mu$

let's try to keep universality but modify quark doublets

We assume that charged current couples rotated quark states

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \quad \begin{pmatrix} c \\ s' \end{pmatrix}, \ldots$$

where

$$\begin{aligned} d' &= d\cos\theta_c + s\sin\theta_c \\ s' &= -d\sin\theta_c + s\cos\theta_c \end{aligned}$$

This introduces an arbitrary parameter θ_c quark mixing angle

-- known as Cabibbo angle --

Cabibbo Angle

In 1963 ☛ Cabibbo first introduced doublet u, d'
to account for weak decays of strange particles
Indeed mixing of d and s quark can be determined by comparing
$\Delta S = 1$ and $\Delta S = 0$ decays

E.G.

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} \sim \sin^2 \theta_c$$

$$\frac{\Gamma(K^+ \to \pi^0 e^+ \nu_e)}{\Gamma(\pi^+ \to \pi^0 e^+ \nu_e)} \sim \sin^2 \theta_c$$

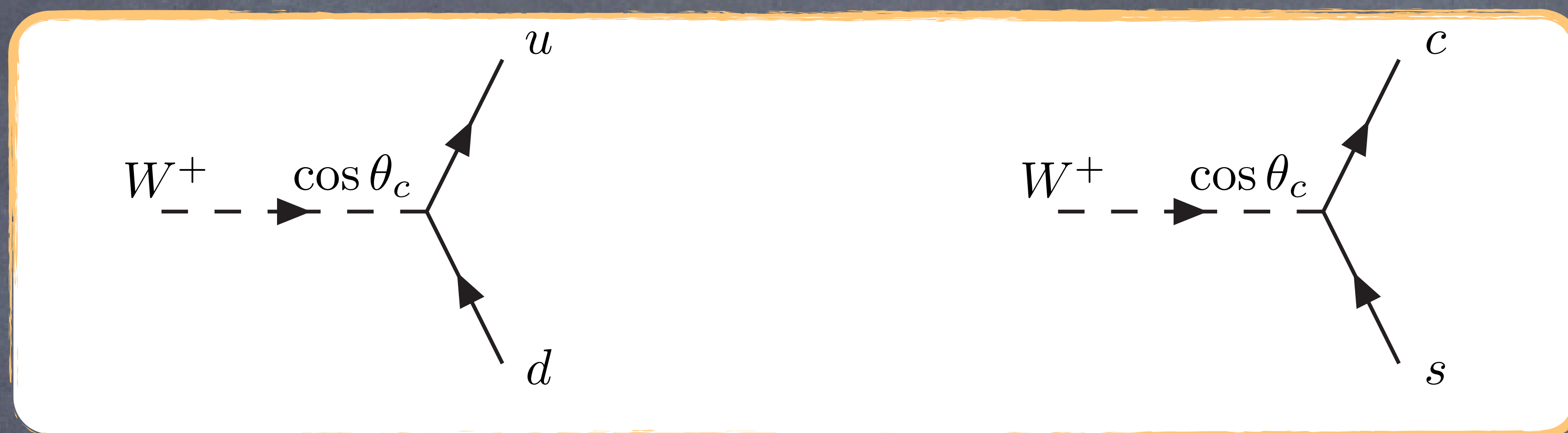
After allowing for kinematic factors arising from different particle masses
data show that $\Delta S = 1$ transitions are suppressed
by a factor of about 20 as compared to $\Delta S = 0$ transitions

This corresponds to $\sin \theta_c = 0.2255 \pm 0.0019$

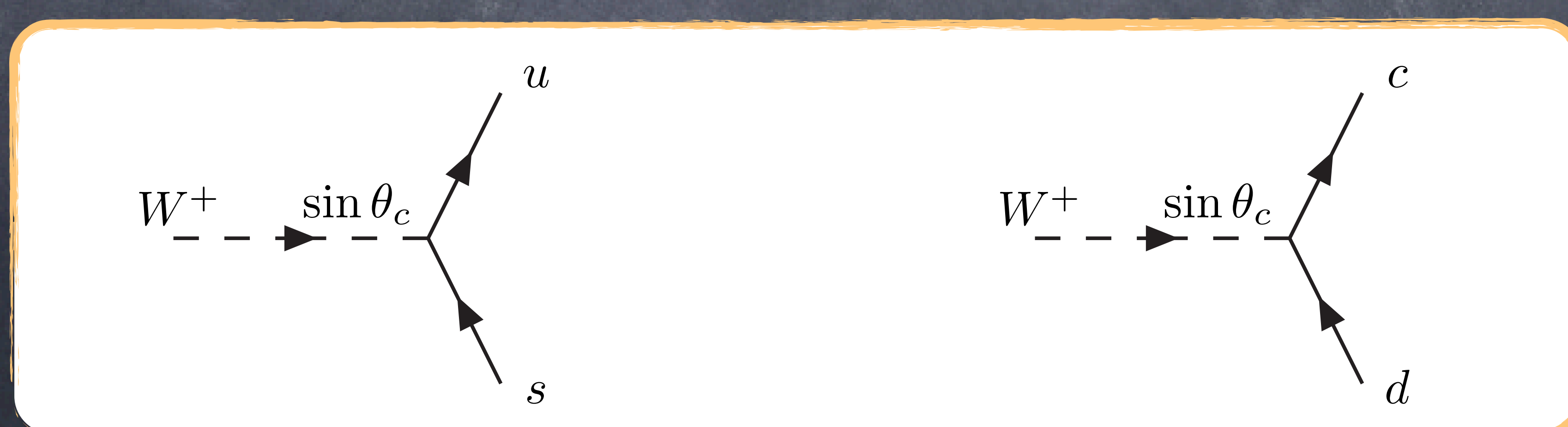
Cabibbo favored & suppressed transitions

What we have done is to change our mind about CC

We now have Cabibbo favored transitions (proportional to $\cos\theta_c$)



and Cabibbo suppressed transitions



We can summarize this...

by writing down explicit form of matrix element

describing the CC weak interactions of quarks

$$\mathfrak{M} = \frac{4G_F}{\sqrt{2}} J^\mu J^\dagger_\mu$$

with

$$J^\mu = (\bar{u} \quad \bar{c}) \frac{\gamma^\mu(\mathbb{I} - \gamma^5)}{2} U \begin{pmatrix} d \\ s \end{pmatrix}$$

Unitary matrix U performs rotation of d and s quarks states:

$$U = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

Amplitudes describing semileptonic decays
are constructed from product of a quark with a lepton current

J^μ (quark) $J^\dagger_\mu$ (lepton)

All this has implications for our previous calculations

We must replace G_F in * by $G_F = G_F \cos\theta_c$

BUT

purely leptonic μ-decay rate ※ (which involves no mixing) is unchanged

Detailed comparison of * and ※ rates supports Cabibbo's hypothesis

Cabibbo-Kobayashi-Maskawa matrix

Unitary matrix U in ♬ gives a zeroth-order approximation to weak interactions of u, d, s, c quarks

their coupling to third family (though non-zero) is very small

$$J^\mu = (\bar{u} \quad \bar{c} \quad \bar{t}) \frac{\gamma^\mu (\mathbb{I} - \gamma^5)}{2} \, U \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3×3 matrix U contains three real parameters and a phase factor $e^{i\delta}$

(Cabibbo-like mixing angles)

Original parametrization was due to Kobayashi and Maskawa

Easy-to-remember approximation

to observed magnitude of each element in 3-family matrix is

$$U = \begin{pmatrix} |U_{ud}| & |U_{us}| & |U_{ub}| \\ |U_{cd}| & |U_{cs}| & |U_{cb}| \\ |U_{td}| & |U_{ts}| & |U_{tb}| \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

with $\lambda = \sin\theta_c$

These are order of magnitude only

each element may be multiplied by a phase and a coefficient of $\mathcal{O}(1)$

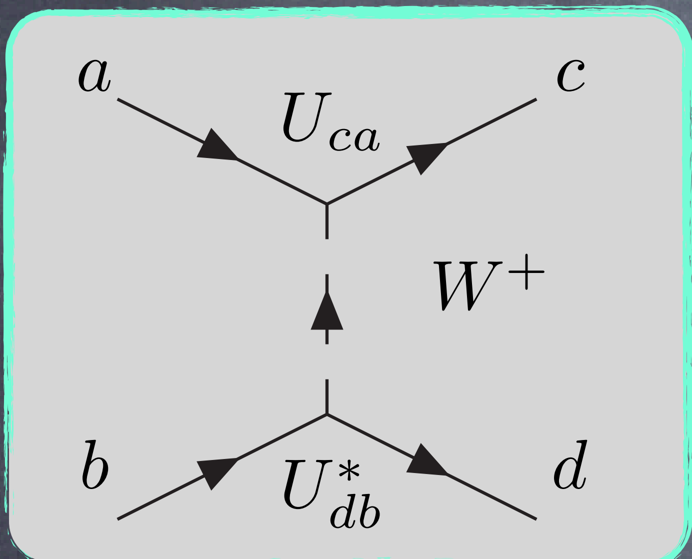
Properties of weak amplitude $\mathfrak{M}$

To investigate CP invariance we first compare:

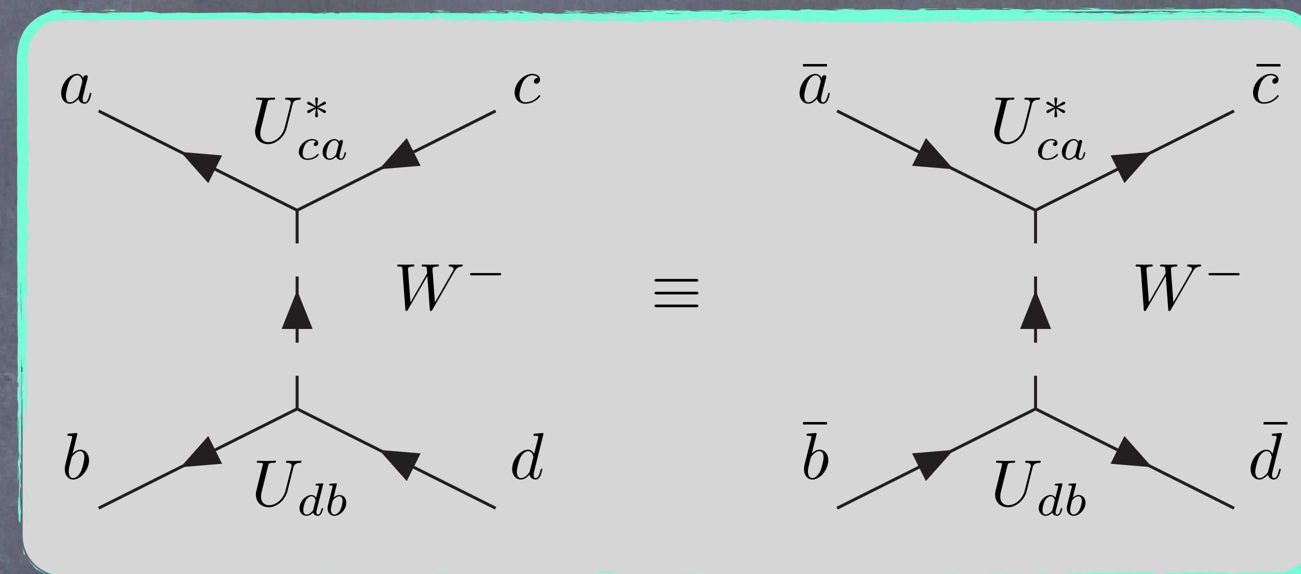
amplitude for weak process $ab \to cd$

with that for antiparticle reaction $\bar{a}\bar{b} \to \bar{c}\bar{d}$

We take $ab \to cd$ to be charged current interaction of processes described by



weak amplitude $\mathfrak{M}(ab \to cd)$



hermitian conjugate

Amplitude is

$$
\begin{aligned}
\mathfrak{M} &\sim J^{\mu}_{ca}\, J^{\dagger}_{\mu bd} \\
&\sim \left(\bar{u}_c \gamma^{\mu}(\mathbb{I}-\gamma^5) U_{ca} u_a\right)\left(\bar{u}_b \gamma_{\mu}(\mathbb{I}-\gamma^5) U_{bd} u_d\right)^{\dagger} \\
&\sim U_{ca} U^*_{db} \left(\bar{u}_c \gamma^{\mu}(\mathbb{I}-\gamma^5) u_a\right)\left(\bar{u}_d \gamma_{\mu}(\mathbb{I}-\gamma^5) u_b\right)
\end{aligned}
$$

because $U^{\dagger}_{bd} = U^*_{db}$

$\mathfrak{M}$ describes either $ab \to cd$ or $\bar{c}\bar{d} \to \bar{a}\bar{b}$

Properties of weak amplitude $\mathfrak{M}^\dagger$

Amplitude $\mathfrak{M}'$ for antiparticle process $\bar{a}\bar{b} \to \bar{c}\bar{d}$ (or $cd \to ab$) is

$$\begin{aligned} \mathfrak{M}' &\sim (J^\mu_{ca})^\dagger \, J_{\mu bd} \\ &\sim U^*_{ca} U_{db} \left(\bar{u}_a \gamma^\mu (\mathbb{I} - \gamma^5) u_c\right) \left(\bar{u}_b \gamma_\mu (\mathbb{I} - \gamma^5) u_d\right) \end{aligned}$$

that is $\mathfrak{M}' = \mathfrak{M}^\dagger$

This should not be surprising

It is demanded by hermiticity of Hamiltonian

By glancing back at
$$\begin{aligned} T_{fi} &= -i \int \phi^*_f(x)\, V(x)\, \phi_i(x)\; d^4x \\ &= -i \int \phi^*_f \, ie(A^\mu \partial_\mu + \partial_\mu A^\mu)\, \phi_i \; d^4x \end{aligned}$$
and

$$T_{fi} = -iN_A \, N_B \, N_C \, N_D \, (2\pi)^4 \, \delta^{(4)}(p_D + p_C - p_B - p_A)\, \mathfrak{M}$$

we see that $\mathfrak{M}$ is essentially interaction Hamiltonian V for process

Total interaction Hamiltonian must contain $\mathfrak{M} + \mathfrak{M}^\dagger$

$\mathfrak{M}$ describes $i \to f$ transition and $\mathfrak{M}^\dagger$ describes $f \to i$ transition

How to test CP invariance

- We have seen that weak interactions violate both P & C invariance
  BUT have indicated that
  invariance under combined CP operation may hold
- How do we verify that theory is CP invariant?
- We calculate from $\mathfrak{M}(ab \to cd)$
  amplitude $\mathfrak{M}_{CP}$ describing CP-transformed process
  and see whether or not Hamiltonian remains hermitian
- If it does ☛ that is if $\mathfrak{M}_{CP} = \mathfrak{M}^\dagger$ then theory is CP invariant

  If it does not ☛ then is CP violated
- $\mathfrak{M}_{CP}$ is obtained by substituting CP-transformed Dirac spinors

$$u_i \to P(u_i)^c, \qquad i = a, \ldots d$$

where u^c are charged conjugate spinors defined by

$$u^c = C\bar{u}^T$$

Hints for the calculation

To form $\mathfrak{M}_{CP}$ we need $\bar{u}^c$

and also to know how $\gamma^\mu(\mathbb{I}-\gamma^5)$ transforms under C

In standard representation of gamma matrices we have

$$\bar{u}^c = u^{c\dagger}\gamma^0 = (C\gamma^0 u^*)^\dagger\gamma^0 = u^T\gamma^0 C^\dagger\gamma^0 = -u^T C^\dagger\gamma^0\gamma^0 = -u^T C^{-1}$$

$$\gamma^\mu = -(C\gamma^0)\gamma^{\mu *}(C\gamma^0)^{-1} = -C\gamma^0\gamma^{\mu *}\gamma^0 C^{-1} = -C\gamma^{\mu T}C^{-1}$$

$$\begin{aligned}
C^{-1}\gamma^\mu\gamma^5 C &= -\gamma^{\mu T}C^{-1}i\gamma^0\gamma^1\gamma^2\gamma^3 C \\
&= -i\gamma^{\mu T}(C^{-1}\gamma^0 C)(C^{-1}\gamma^1 C)(C^{-1}\gamma^2 C)(C^{-1}\gamma^3 C) \\
&= -i\gamma^{\mu T}\gamma^{0T}\gamma^{1T}\gamma^{2T}\gamma^{3T} \\
&= -\gamma^{\mu T}(i\gamma^3\gamma^2\gamma^1\gamma^0)^T \\
&= -\gamma^{\mu T}(i\gamma^0\gamma^1\gamma^2\gamma^3)^T \\
&= -\gamma^{\mu T}\gamma^{5T} \\
&= -(\gamma^5\gamma^\mu)^T \\
&= (\gamma^\mu\gamma^5)^T
\end{aligned}$$

More hints for the calculation

With replacements ■ ☛ first charged current becomes

$$\begin{aligned}(J^{\mu}_{ca})^c &= U_{ca}(\bar{u}_c)^c\gamma^{\mu}(\mathbb{I}-\gamma^5)(u_a)^c \\ &= -U_{ca}u_c^T C^{-1}\gamma^{\mu}(\mathbb{I}-\gamma^5)C\bar{u}_a^T \\ &= U_{ca}u_c^T[\gamma^{\mu}(\mathbb{I}+\gamma^5)]^T\bar{u}_a^T \\ &= (-)U_{ca}\bar{u}_a\gamma^{\mu}(\mathbb{I}+\gamma^5)u_c\end{aligned}$$

Parity operation $P=\gamma^0$ ☛ and so $P^{-1}\gamma^{\mu}(\mathbb{I}+\gamma^5)P = \gamma^{\mu\dagger}(\mathbb{I}-\gamma^5)$

Thus ☛ $(J^{\mu}_{ca})_{CP} = (-)U_{ca}\bar{u}_a\gamma^{\mu\dagger}(\mathbb{I}-\gamma^5)u_c$

CP invariance?

- We can now compare

$$\mathfrak{M}_{CP} \sim U_{ca} U^*_{db} \left[\bar{u}_a \gamma^\mu (\mathbb{I} - \gamma^5) u_c\right] \left[\bar{u}_b \gamma_\mu (\mathbb{I} - \gamma^5) u_d\right]$$

with

$$\begin{aligned} \mathfrak{M}' &\sim (J^\mu_{ca})^\dagger J_{\mu bd} \\ &\sim U^*_{ca} U_{db} \left(\bar{u}_a \gamma^\mu (\mathbb{I} - \gamma^5) u_c\right) \left(\bar{u}_b \gamma_\mu (\mathbb{I} - \gamma^5) u_d\right) \end{aligned}$$

- Provided elements of matrix U are real

  we find $\mathfrak{M}_{CP} = \mathfrak{M}^\dagger$ and theory is CP invariant

  For (u, d, c, s) ☛ 2×2 matrix U is indeed real

- With addition of b and t matrix U becomes 3×3 CKM matrix

  U now contains a complex phase factor $e^{i\delta}$

- Therefore ☛ in general we have $\mathfrak{M}_{CP} \neq \mathfrak{M}^\dagger$

  and theory necessarily violates CP invariance

- CP violation was established many years before introduction of CKM matrix

  evidence was first revealed in mixing of neutral kaons

Electroweak Interference in e^+e^- Annihilation

Measurement of reaction $e^+e^- \to \mu^+\mu^-$ at PETRA energies

provides tests of validity of QED at small distances

Measurement also provides a unique test of asymmetry

(in angular distribution of muon pairs)

arising from interference of electromagnetic amplitude $\mathfrak{M}^{EM} \sim e^2/k^2$

with a small weak contribution

Size of this effect is found to be

$$\frac{|\mathfrak{M}^{\mathrm{EM}}\,\mathfrak{M}^{\mathrm{NC}}|}{|\mathfrak{M}^{\mathrm{EM}}|^2} \approx \frac{G_F}{e^2/k^2} \approx \frac{10^{-4}k^2}{m_N^2}$$

using $G_F \approx 10^{-5}/m_N^2$ and $e^2/4\pi = 1/137$

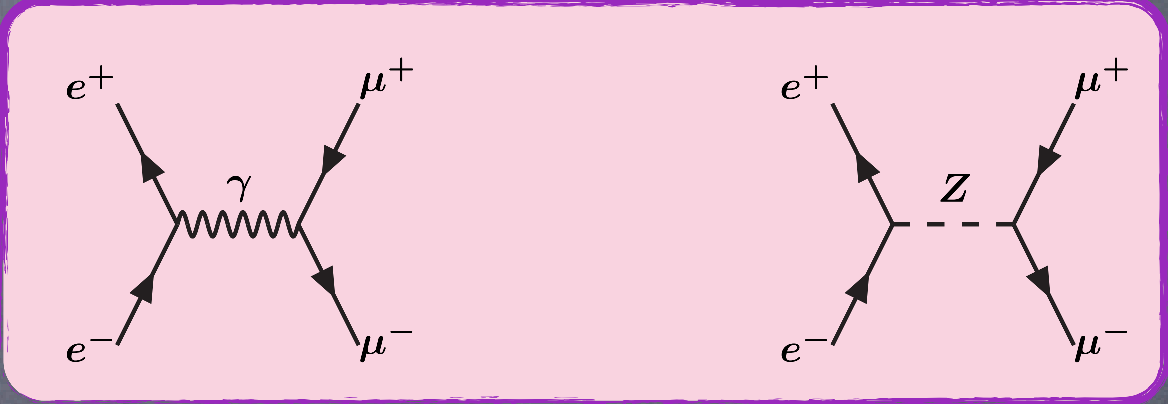
For PETRA e^+e^- beam energies $\sim 20\ \mathrm{GeV}$ ☛ $k^2 \approx s \approx (40\ \mathrm{GeV})^2$

and so predicts about a 15% effect ☛ which is readily observable

To make a detailed prediction...

Use Feynman rules to compute amplitudes $\mathfrak{M}_\gamma$ and $\mathfrak{M}_Z$

corresponding to diagrams of ☛



are

$$\mathfrak{M}_\gamma = -\frac{e^2}{k^2}(\overline{\mu}\gamma^\nu \mu)(\overline{e}\gamma_\nu e)$$

$$\mathfrak{M}_Z = -\frac{g^2}{4\cos^2\theta_w}\left[\overline{\mu}\gamma^\nu(c_V^\mu \mathbb{I} - c_A^\mu \gamma^5)\mu\right]\left(\frac{g_{\nu\sigma} - k_\nu k_\sigma/m_Z^2}{k^2 - m_Z^2}\right) \times \left[\overline{e}\gamma^\sigma(c_V^e \mathbb{I} - c_A^e \gamma^5)e\right]$$

where k is four-momentum of virtual γ (or Z) $s \simeq k^2$

With electron-muon universality ☛ superscripts on $c_{V,A}$ are superfluous

We ignore lepton masses ☛ Dirac equation for incident positron reads

$$(\tfrac{1}{2}k_\sigma)\overline{e}\gamma^\sigma = 0$$

and numerator of propagator simplifies to $g_{\mu\sigma}$

Chiral Couplings

Taking $\rho = 1$ ☛ $\mathfrak{M}_Z$ can be written as

$$\mathfrak{M}_Z = -\frac{\sqrt{2}G_F m_Z^2}{s - m_Z^2}\left[c_R^\mu(\bar{\mu}_R\gamma^\nu\mu_R) + c_L^\mu(\bar{\mu}_L\gamma^\nu\mu_L)\right]\left[c_R^e(\bar{e}_R\gamma_\nu e_R) + c_L^e(\bar{e}_L\gamma_\nu e_L)\right]$$

✧:

where

$$c_R \equiv c_V - c_A, \qquad c_L \equiv c_V + c_A$$

That is we have chosen to write

$$c_V\mathbb{I} - c_A\gamma^5 = (c_V - c_A)\tfrac{1}{2}(\mathbb{I} + \gamma^5) + (c_V + c_A)\tfrac{1}{2}(\mathbb{I} - \gamma^5)$$

$(\mathbb{I} \pm \gamma^5)$ are projection operators which enable $\mathfrak{M}_Z$ to be expressed explicitly in terms of right- and left-handed spinors

It is easier to calculate $|\mathfrak{M}_\gamma + \mathfrak{M}_Z|^2$ in this form

With definite electron and muon helicities we can apply results of QED calculation of $e^+e^- \to \mu^+\mu^-$

E.G.

$$\left.\frac{d\sigma}{d\Omega}\right|_{e_L^- e_R^+ \to \mu_L^- \mu_R^+} = \frac{\alpha^2}{4s}(1+\cos\theta)^2 \left[1 + r c_L^\mu c_L^e\right]^2$$ ⚑

$$\left.\frac{d\sigma}{d\Omega}\right|_{e_L^- e_R^+ \to \mu_R^- \mu_L^+} = \frac{\alpha^2}{4s}(1+\cos\theta)^2 \left[1 + r c_R^\mu c_L^e\right]^2$$ ⌘

r is ratio of coefficients in front of brackets in ✧: that is

$$r = \frac{\sqrt{2} G_F m_Z^2}{s - m_Z^2 + i m_Z \Gamma_Z}\left(\frac{s}{e^2}\right)$$

we have included finite resonance width Γ_Z

which is important for $s \sim m_Z^2$

Unpolarized Cross Section

Expressions similar to ⚑ and ⌘ hold
for other 2 non-vanishing helicity configurations

To calculate unpolarized $e^+e^- \to \mu^+\mu^-$ cross section
we average over four allowed L, R helicity combinations

We find

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\theta) + A_1\cos\theta\right]$$ ⏏

where (assuming electron-muon universality $c_i^\mu = c_i^e \equiv c_i$)

$$\begin{aligned} A_0 &\equiv 1 + \tfrac{1}{2}\Re\mathrm{e}(r)(c_L + c_R)^2 + \tfrac{1}{4}|r|^2(c_L^2 + c_R^2)^2 \\ &= 1 + 2\Re\mathrm{e}(r)c_V^2 + |r|^2(c_V^2 + c_A^2)^2 \end{aligned}$$

$$\begin{aligned} A_1 &\equiv \Re\mathrm{e}(r)(c_L - c_R)^2 + \tfrac{1}{2}|r|^2(c_L^2 - c_R^2)^2 \\ &= 4\Re\mathrm{e}(r)c_A^2 + 8|r|^2 c_V^2 c_A^2 \end{aligned}$$

Forward-Backward Asymmetry

Lowest-order QED result gives a symmetric regular distribution

$$(A_0 = 1,\ A_1 = 0)$$

Weak interaction introduces a forward-backward asymmetry

$$(A_1 \neq 0)$$

Let us calculate size of integrated asymmetry defined by

$$A_{FB} \equiv \frac{F - B}{F + B} \qquad \text{with } F = \int_0^1 \frac{d\sigma}{d\Omega} d\Omega\,, \qquad B = \int_{-1}^0 \frac{d\sigma}{d\Omega} d\Omega$$

Integrating ⏏ we obtain for $s \ll m_Z^2$ (i.e. ☛ $|r| \ll 1$)

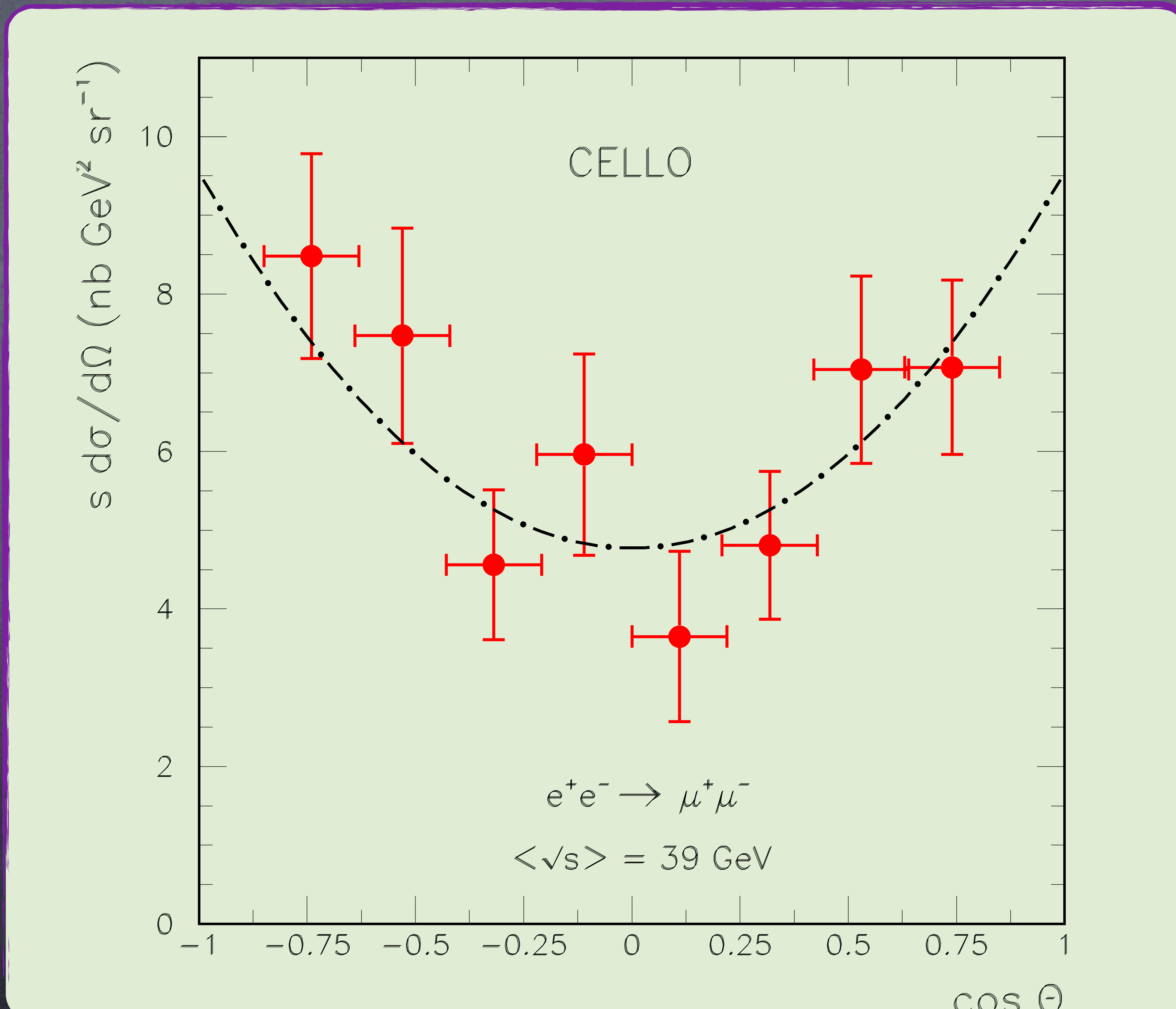
$$A_{FB} = \frac{A_1}{(8A_0/3)} \simeq \frac{2}{3} \Re\mathrm{e}(r) c_A^2 \simeq -\frac{3c_A^2}{\sqrt{2}} \left(\frac{G_F s}{e^2} \right)$$

This is in agreement with expectations of order of magnitude estimate

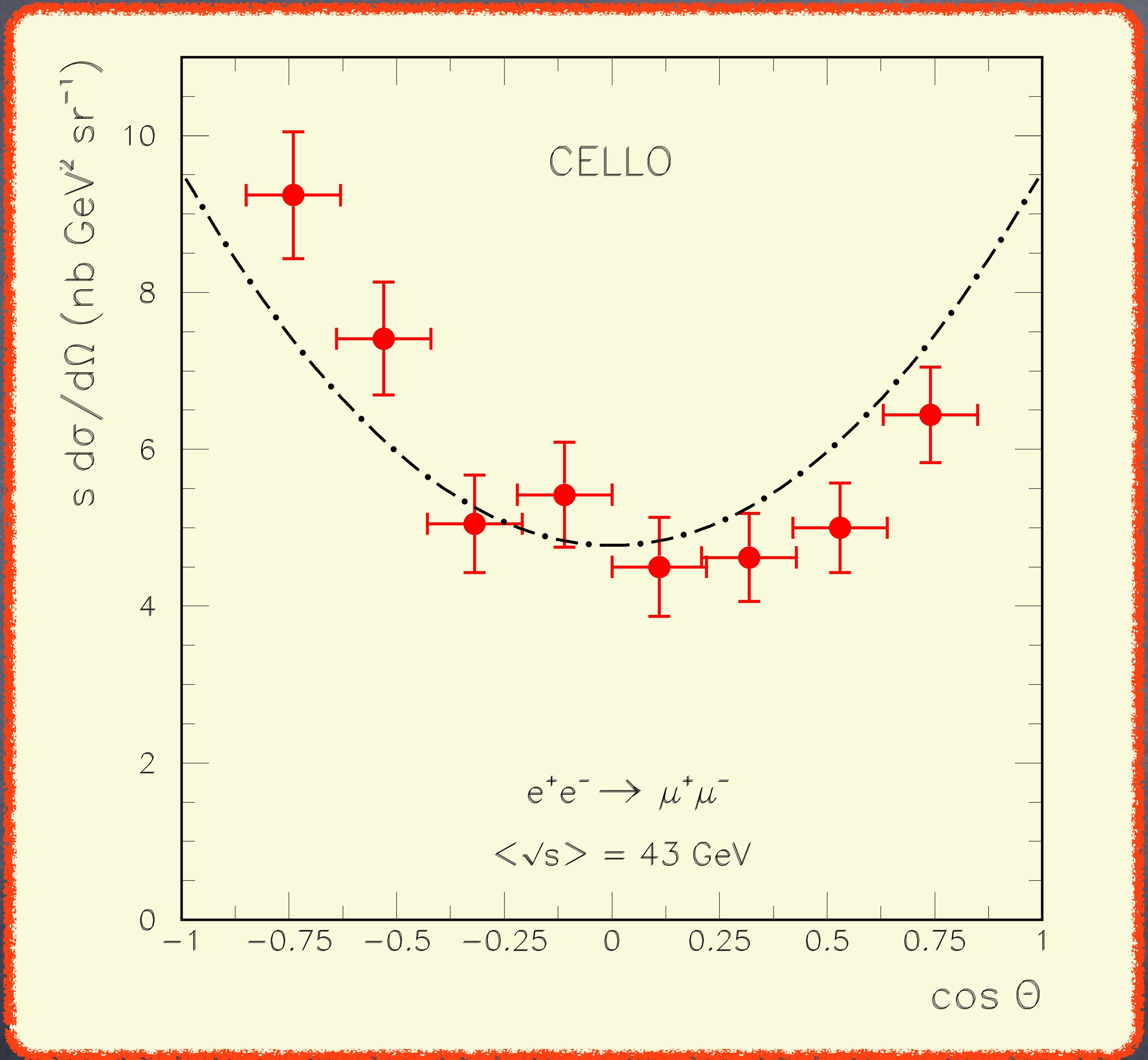
$$G_F s / e^2$$

PETRA-data

We may use standard model couplings $c_A = \frac{1}{2},\ c_V = -\frac{1}{2} + 2\sin^2\theta_w \simeq 0$
to compare with experimental measurements
of high-energy $e^+e^- \to \mu^+\mu^-$ angular distribution



PETRA-data (larger statistics)



$e^+e^- \rightarrow \mu^+\mu^-$ angular distribution for all CELLO data $\langle\sqrt{s}\rangle = 43$ GeV

$\cos\theta$ distribution does not follow $1 + \cos^2\theta$ QED prediction

CU NEXT WEEK