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Problems set # 9 **Physics 541-735** November 8, 2011

1. Verify that the inverse of the "momentum space operator" of (3.4.64) does not exist.

2. Show that the spin-averaged Compton amplitude is

$$\overline{|\mathfrak{M}|^2} = 2e^4 \left(-\frac{u}{s} - \frac{s}{u} \right)$$

3. Show that, in the high-energy limit, the spin-averaged amplitude of pair annihilation is

$$\overline{|\mathfrak{M}|^2} = 2e^4 \left(\frac{u}{t} + \frac{t}{u} \right)$$

4. We have seen that the charge is modified by the vacuum polarization loop in the photon propagator. We know that the loop will be repeated in higher orders and the geometric series can be summed to give

$$e^2 = e_0^2 \left[\frac{1}{1+\Pi(q^2)} \right] .$$

(i) Using a Feynman parameter to combine denominator factors, $Q = k + qx$, show that

$$\begin{aligned} i\Pi_{\mu\nu}(q) &= -\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} \left\{ (-ie)\gamma_\mu \frac{1}{(\not{k} - m)} (ie)\gamma_\nu \frac{1}{(\not{q} + \not{k} - m)} \right\} \\ &= -e^2 \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \mathrm{Tr} \left\{ \frac{\gamma_\mu (\not{Q} - \not{q}x + m) \gamma_\nu (\not{Q} + \not{q}(1-x) + m)}{[Q^2 - m^2 + q^2 x(1-x)]^2} \right\} \end{aligned}$$

(ii) Evaluate the traces and show that

$$\begin{aligned} i\Pi_{\mu\nu}(q) &= -e^2 \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} 4 \left\{ \frac{2Q_\mu Q_\nu}{[Q^2 - m^2 + q^2x(1-x)]^2} - \frac{2x(1-x)(q_\mu q_\nu - g_{\mu\nu} q^2)}{[Q^2 - m^2 + q^2 x(1-x)]^2} \right. \\ &\quad \left. - \frac{g_{\mu\nu}}{[Q^2 - m^2 + q^2 x(1-x)]} \right\} \end{aligned}$$

(iii) Using the tabulated integrals:

$$\int \frac{d^np\; p_\mu p_\nu}{(p^2 + 2pq - m^2)^\alpha} = \frac{i\pi^{n/2}}{\Gamma(\alpha)} \frac{1}{(-q^2 - m^2)^{\alpha - n/2}} \left[q_\mu q_\nu\, \Gamma(\alpha - n/2) + \frac{1}{2} g_{\mu\nu} (-q^2 - m^2) \Gamma(\alpha - 1 - n/2) \right]$$

and

$$\int \frac{d^np}{(p^2 + 2pq - m^2)^\alpha} = \frac{i\pi^{n/2}\Gamma(\alpha - n/2)}{\Gamma(\alpha)} \frac{1}{[-q^2 - m^2]^{\alpha - n/2}},$$

show that the first and third term in $\Pi_{\mu\nu}(q)$ cancel each other.
(iv) Obtain the expression for $\Pi(q^2)$ given in (C.0.8).