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Problems set # 7 **Physics 541-735** October 25, 2011

1. Determine the decay rate Γ of an unstable particle (assumed to be at rest) into a specified final state (of two or more particles).

2. Use (3.1.28) to show that for very high-energy "spinless" electron-muon scattering

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm cm} = \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2,$$

where θ is the scattering angle and $\alpha = e^2/4\pi$. Neglect the particle masses.

3.*(i)* Taking $e^-e^+ \to e^-e^+$ to be the s channel process, verify that

$$s = 4(k^2+m^2),$$

$$t = -2k^2(1-\cos\theta),$$

$$u = -2k^2(1+\cos\theta),$$

where θ is the center-of-mass scattering angle and $k = |\vec{k}_i| = |\vec{k}_f|$, where $\vec{k}_i$ and $\vec{k}_f$ are, respectively, the momenta of the incident and scattered electrons in the center-of-mass frame. Show that the process is physically allowed provided $s \geq 4m^2$, $t \leq 0$, and $u \leq 0$. Note that $t = 0$ ($u = 0$) corresponds to forward (backward) scattering.

(ii) For the cross reaction $A\bar{D} \to C\bar{B}\,(e^-e^- \to e^-e^-)$, show that u becomes the square of the total center-of-mass energy and that this process would become physical in a different kinematic region: $u \geq 4m^2$, $t \leq 0$ and $s \leq 0$ (Note that, for example, $-p_D = (E,\vec{p})$, where E and $\vec{p}$ refer to the incoming $\bar{D}$.)

4. Show that the invariant amplitude for "spinless" electron-positron scattering can be written as

$$\mathfrak{M} = e^2\left(\frac{s-u}{t} + \frac{t-u}{s}\right).$$

Comment on the symmetry of $\mathfrak{M}$ under $s \Leftrightarrow t$.

5. Deduce an expression for the energy of a γ-ray from the decay of the neutral pion, $\pi^0 \to \gamma\gamma$, in terms of the mass m, energy E, and velocity βc of the pion and the angle of emission θ^* in the pion rest frame. Show that if the pion has spin zero, so that the angular distribution is isotropic, the laboratory energy spectrum for the γ-rays will be flat extending from $E(1+\beta)/2$ to $E(1-\beta)/2$. For relativistic pions, find an expression for the disparity D (the ratio of energies) for the γ-rays and show that $D > 3$ is half the decays and $D > 7$ is one quarter of them.