Prof. Anchordoqui

**Problems set # 5** **Physics 541-735** October 11, 2011

1. The collision of two particles, each of mass $M$, is viewed in a Lorentz frame in which they hit head-on with momenta equal in magnitude but opposite in direction. We speak of this as the "center-of-mass" frame (though the name "center-of-momentum" would be more appropriate). The total energy of the system is $E_{\rm cm}$. Show that the Lorentz invariant

$$s \equiv (p_1 + p_2)_\mu (p_1 + p_2)^\mu \equiv (p_1 + p_2)^2 = E_{\rm cm}^2 .$$

If the collision is viewed in the "laboratory" frame where one of the particles is at rest, then show, by evaluating the invariant $s$, that the other has energy

$$E_{\rm lab} = \frac{E_{\rm cm}^2}{2M} - M\,.$$

2. One of the main physics goals of the experiments H1 and ZEUS, operating at the HERA (Hadron Electron Ring Anlage) collider at DESY (Hamburg) was the detailed investigation of the proton substructure. HERA followed the tradition of the Rutherford Scattering Experiments and used pointlike leptons to probe the substructure of the proton. Unlike earlier fixed target experiments, HERA collided beams of electrons at 30 GeV and protons up to 920 GeV. After successful operation for over a decade, the HERA collider was finally shut down on July 2007. *(i)* Compute the center-of-mass energy for a head on collision. *(ii)* Compute the boost, $\beta_{\rm cm}$, of the electron-proton center-of-mass frame relative to the laboratory frame. *(iii)* What should be the energy of an electron beam colliding with protons at rest if the center-of-mass energy were to be the same at HERA?

3. A high energy electron collides with an atomic electron which can be considered at rest. What is the threshold energy for producing an electron-positron pair?

4. An experiment which measured parity violation in weak interactions used a $\mu^+$ beam which was produced from the decay in flight of a $\pi^+$ beam ($\pi^+ \to \mu^+ \nu_\mu$). The kinetic energy of the $\pi^+$ beam was 85 MeV. The $\pi^+$ beam was produced by colliding a proton beam from an accelerator with a target and selecting the positively charged $\pi^+$ from the negatively charged $\pi^-$ using a magnetic field. Positive pions of a certain direction and momentum were selected by passing the $\pi^+$ beam through an appropriately positioned concrete block with a straight hole through it.

- Compute the decay length of the pions in the laboratory frame. The lifetime of the $\pi^+$ is 26 ns and the $\pi^+$ mass is about 140 MeV. The incident beam of the experiment was a mixture of 10% $\mu^+$ and 90% $\pi^+$. How far was the experiment from the point where the pions were produced?
- Compute the pion mean free path in carbon assuming that the pion carbon cross section at the relevant energy is 10 mb;
- Compute the muon and neutrino energies in the rest frame of the pion. Assume that the neutrino is massless and that the mass of the muon is $m_\mu = 106$ MeV;

- Compute the maximum and minimum energy of the muon in the lab frame;
- Given that pions and muons of this energy lose in carbon approximately 4.5 MeV $cm^{-1}$ due to ionization, would a carbon block 20 cm long stop the pions?
- Compute the maximum possible angle of the decay muons from pions in the Laboratory frame. Use the results from the previous item and explain how does the carbon block constrain this angle? Do the angles of the muons which are capable to penetrate the block extend up to this angle?

4. Show that the angular momentum is a conserved quantity.