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Problems set # 4 **Physics 541-735** October 4, 2011

1. *(i)* Show that

- the Weyl matrices

$$\vec{\alpha} = \begin{pmatrix} -\vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{1}$$

  satisfy the Dirac anticommutation relations. Hence, they form just another representation of the Dirac matrices;

- the Dirac matrices in the Weyl representation are

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} ; \tag{2}$$

- in the Weyl representation

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{3}$$

(ii) Solve the Dirac equation

$$(\vec{\alpha}\,.\,\vec{p} + \beta m)\psi = E\psi \tag{4}$$

in the particle rest frame using the Weyl representation. *(iii)* Determine the functional form of the chirality operators $(1 \pm \gamma_5)/2$ when they are acting on the Dirac solution in the Weyl representation.

2. Consider $\psi = \binom{u_A}{u_B}$ to be a solution of the Dirac equation where u_A, u_B are two-component spinors. Show that in the non-relativistic limit $u_B \sim v/c$.

3. Show that in the non-relativistic limit the motion of a spin half fermion of charge e in the presence of an electromagnetic field $A^\mu(A^0, \vec{A})$ is described by

$$\left[\frac{(\vec{p} - e\vec{A})^2}{2m} - \frac{e}{2m}\vec{\sigma}\,.\,vecB + eA^0\right]\chi = E\chi \tag{5}$$

where $\vec{B}$ is the magnetic field, σ^i are the Pauli matrices, and $E = p^0 - m$. Identify the g-factor of the fermion and show that the Dirac equation predicts the correct gyromagnetic ratio of the fermion. To write down the Dirac eqution in the presence of an electromagnetic field substitute $p^\mu \to p^\mu - eA^\mu$.

4 *(i)* Show that

- $\bar{\psi}\gamma_5\psi$ is a pseudoscalar;

- $\bar{\psi}\gamma_5\gamma^\mu\psi$ is an axial vector .

(ii) Comment on the Lorentz and parity properties of the quantities

- $\bar{\psi}\gamma_5\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi$;
- $\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$;
- $\bar{\psi}\psi\bar{\psi}\gamma_5\psi$;
- $\bar{\psi}\gamma_5\gamma^\mu\psi\bar{\psi}\gamma_5\gamma_\mu\psi$;
- $\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi$.

5. Obtain the form of the next correction to (1.6.143), i.e., the term third order in V.