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Problems set # 2 **Physics 541-735** September 20, 2011

1. The muons created by cosmic rays in the upper atmosphere rain down more-or-less uniformely on the earth's surface, although some of them decay on the way down, with a half life of about 2.2 μs (measured in their rest frame). A muon detector is carried in a balloon to an altitude of 2000 m, and in the course of an hour detects 650 muons traveling at $0.99c$ toward the earth. If an identical detector remains at sea level, how many muons should it register in one hour? Calculate the answer taking account of the relativistic time dilation and classically. (Remember that after n half-lives, 2^{-n} of the original particles survive.) Needles to say, the relativistic answer agrees with experiment.

2. A morphism is an abstraction derived from structure-preserving mappings between two mathematical structures. An endomorphism is a morphism from a mathematical object to itself. For example, an endomorphism of a vector space V is a linear map $f: V \to V$, and an endomorphism of a group G is a group homomorphism $f: G \to G$. An edomorphism $\Lambda: M \to M$ from a vector space M with non-degenerate inner product G is orthogonal if

$$\langle \Lambda(x), \Lambda(y) \rangle = \langle x, y \rangle.$$

It is easily seen that the endomorphism $\Lambda: M \to M$ is orthogonal if and only if

$$[\Lambda]_B^{-1} = [G]_B^{-1}\, [\Lambda]_B^t\, [G]_B,$$

where $[\Lambda]_B$ is the the matrix of the endomorphism Λ on the base B of M, and G is the metric tensor.
(i) Using this property show that Lorentz transformations satisfy

$$[G]_e = [\Lambda]_e^t\, [G]_e\, [\Lambda]_e,$$

where $[G]_e = (g_{ij})$ and $\{e\}$ is an orthonormal base.
(ii) Given the matrix $[\Lambda]_e{}^\mu{}_\nu$, with

$$\Lambda^0{}_0 = \gamma \qquad\qquad \Lambda^i{}_0 = \tfrac{\gamma v^i}{c}$$
$$\Lambda^i{}_j = \delta^i{}_j + (\gamma - 1)\frac{v^i\, v_j}{v^2} \qquad\qquad \Lambda^0{}_j = \tfrac{\gamma v_j}{c}\,,$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, show that $\Lambda^\mu{}_\nu$ is a matrix associated to a Lorentz transformation.

3. *(i)* Write down the definition of a parity transformation. *(ii)* Consider two Lorentz 4-vectors x^μ and y^μ which transform as polar and axial vectors, respectively; how do they transform under parity? *(iii)* Which of the follwoing Lorentz invariant quantities is invariant under parity and which one is not: a) $x^\mu x_\mu$; b) $y^\mu y_\mu$; c) $(x^\mu - y^\mu)(x_\mu - y_\mu)$.

4. In a certain reference frame a static, uniform, electric field E_0 is parallel to the x axis, and a static, uniform, magnetic induction $B_0 = 2E_0$ lies in the $x - y$ plane, making an angle θ with the x

axis. Determine the relative velocity of a reference frame in which the electric and magnetic fields are parallel. What are the fields in that frame for $\theta \ll 1$ and $\theta \to (\pi/2)$.

5. Introducing single particle creation and annihilation operators, verify (1.2.9) and determine the spectrum of the Klein-Gordon Hamiltonian.