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Problems set # 14 **Physics 541-735** December 13, 2011

1. *(i)* Write Gauss' law for a galaxy of mass m inside a homogeneous and isotrpic universe of mass density ρ and show that

$$\frac{d^2R/dt^2}{R} = -\frac{4\pi}{3}G\rho$$

where R is the distance of the galaxy to the center of the distribution. (ii) Obtain an expression for dR/dt from the fact that the total energy of the galaxy vanishes. P.S. You have now derived Friedman's equations for a flat universe in the Newtonian limit.

2. *(i)* Work out the redshift at the matter-radiation equality. *(ii)* Work out the redshift at the photon decoupling.

3. Consider the reactions $n\nu \rightleftharpoons pe^-$ and $ne^+ \rightleftharpoons p\bar{\nu}$. Assuming $T_\nu(t) = T$, derive the balance equation

$$\frac{dX_{n/N}}{dt} = -\lambda_{np}[1 + \exp(-\Delta m/T)](X_{n/N} - X^{\rm eq}_{n/N})\,, \qquad (1)$$

where $X_{n/N} = n_n/(n_p + n_n)$, $X^{\rm eq}_{n/N} = [1 + \exp(\Delta m/T)]^{-1}$, λ_{np} is the total reaction rate for the conversion of neutrons to protons, and $\Delta m = m_n - m_p = 1.293$ MeV. How do you have to modify the balance equation if neutron decay $n \to p\bar{\nu}e^-$ is not neglected? Using the explicit integral expression for the reactions, calculate the neutron freeze-out temperature $T^{\rm FO}_{n/N}$ for 3 relativistic neutrino species, i.e., $N^{\rm eff}_\nu = 3$. (You will have to integrate the reaction rates numerically.) How does $T^{\rm FO}_{n/N}$ change if $N^{\rm eff}_\nu = 2$, or $N^{\rm eff}_\nu = 4$?

4. Using the conservation of n_b/s and the current value $n_b/n = 5 \times 10^{-10}$, work out the asymmetry in the quark number

$$A_q \equiv \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \qquad (2)$$

before the QCD phase transition.