Prof. Anchordoqui

Problems set # 13 **Physics 541-735** December 6, 2011

1. Assume that a directional flux of cosmic ray neutrons originates at the Galactic center. These neutrons decay *en route* to Earth to pure $\overline{\nu}_e$, giving an initial ratio $\omega_e : \omega_\mu : \omega_\tau = 1 : 0 : 0$. Determine the mass-eigenstate ratios and the Earthly flavor ratios.

2. One of the most acute problems connected with ultraviolet divergences concerns radiative corrections to the mass appearing in the Higgs potential, $V = \mu^2 \Phi\Phi^\dagger + \lambda(\Phi^\dagger\Phi)^2$. The fermion, W, and Higgs self-coupling radiative corrections to the Higgs mass (shown in the figure below) are respectively given by:

$$i\frac{Y_f^2}{2}\int^\Lambda \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\Big(\frac{i}{\not k - m_f}\frac{i}{\not k + \not p - m_f}\Big) \quad\sim\quad -\Lambda^2\ \mathrm{tr}(\mathbb{1})\ \frac{Y_f^2}{32\pi^2}\,,$$

$$i\frac{g^2}{4}\int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_W^2} \quad\sim\quad \Lambda^2\,\frac{g^2}{64\pi^2}\,,$$

$$i6\lambda\int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_H^2} \quad\sim\quad \Lambda^2\,\frac{3\lambda}{8\pi^2}\,,$$

where $m_W^2 = \frac{1}{4}g^2v^2$, $v = 246$ GeV, $m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$, $m_f^2 = \frac{1}{2}Y_f^2v^2$, Y_f is the Yukawa coupling, $m_H^2 = 2\lambda v^2$, g and g' are the $SU(2)_L \times U(1)_Y$ gauge couplings, λ is the quartic Higgs coupling, and Λ is a cutoff. Using the different contributions for the one loop one-particle-irreducible diagrams verify (5.7.167). Show that to avoid the destabilization of the electroweak scale, the Higgs mass must be fine-tunned to an accuracy $\mathcal{O}(10^{32})$. [Hint: the contribution from the Z boson is obtained with the substitutions $m_W \to m_Z$ and $g^2 \to g^2 + g'^2$.]

3. Consider the forward scattering process in which two particles of 4-momenta p_a and p_b and masses $M_R \gg 1$ GeV and $m_N \simeq 1$ GeV scatter two particles of momenta p_c and p_d and masses M_R and M_X, respectively. Show that the invariant quantity $(E_c - E_a)/E_a$ that describes the inelasticity of the process can be written as $K_{\rm inel} \approx (M_R/\mathrm{GeV})^{-1}$.

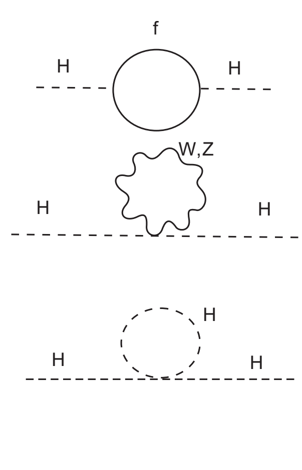

Table 1: Observation times and inferred neutrino energies in the Kamioka experiment

Event No.	Time t_{obs} [s]	Energy E [MeV]
1	0 (def)	21.3 ± 2.9
2	0.107	14.8 ± 3.2
3	0.303	8.9 ± 2.0
4	0.324	10.6 ± 2.7
5	0.507	14.4 ± 2.9

4. On 23 February 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by two neutrino-detection experiments. The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. *(i)* Suppose light takes a time t_0 to reach us from the location of the supernova. How long would it take a neutrino of energy E and mass $m \ll E$ to reach the Earth (work to lowest non-trivial order in m)? For future reference, the light travel time is approximately $t_0 = 5.3 \times 10^{12}$ s. *(ii)* The observation times and neutrino energies for the first 5 neutrinos observed by the Kamioka detector in Japan are given in Table 1. The first neutrino is defined to have arrived at time $t_{\text{obs}} = 0$. For any given neutrino, t_{obs} is the sum of its emission time (compared to neutrino #1) and its travel time (again, subtracting neutrino #1's travel time). Assume that all the neutrinos have the same mass and plot their emission times vs. the common value of m^2c^4 (in eV^2). From this plot, argue that, if all these neutrinos were emitted within 4 s of each other (a conservative upper limit), the maximum neutrino mass is no more than 13 eV/c^2. [Hint: For simplicity you can dropped error bars.]