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Problems set # 11 **Physics 541-735** November 22, 2011

1. Show that momentum conservation at the QCD vertex requires (for $z < 1$)

$$P_{qq}(z) = P_{gq}(1-z),$$

$$P_{qg}(z) = P_{qg}(1-z),$$

$$P_{gg}(z) = P_{gg}(1-z).$$

Check that the explicit formulas for the splitting functions satisfy these relations.

2. Fragmentation functions are often parametrized by the form

$$D_q^h(z) = N\frac{(1-z)^n}{z}$$

where N and n are constants. Show that

$$N = (n+1)\langle z\rangle\,,$$

where $\langle z\rangle$ is the average fraction of the quark energy carried by hadrons of type h after fragmentation. Compare these predictions with those from

$$\frac{dn_h}{dz} \approx \frac{15}{16} z^{-3/2}(1-z)^2\,.$$

3. Determine the color factor for the following processes: *(i)* $\gamma g \to q\bar{q}$, *(ii)* $q\bar{q} \to \gamma g$, *(iii)* $qg \to q\gamma$, *(iv)* $q\bar{q} \to \ell^+\ell^-$, *(v)* $\ell^+\ell^- \to q\bar{q}$, *(vi)* $gg \to gg$, *(vii)* $gg \to q\bar{q}$, *(viii)* $q\bar{q} \to gg$, and *(ix)* $gq \to gq$.

4. Ignoring the Z mass (i.e., keeping only transverse Z's) show that the ratio of the $Z+$ jet and $\gamma+$ jet invariant mass spectra is roughly 0.92 for processes involving u (or $\bar{u}$) quarks and 4.7 for processes involving d (or $\bar{d}$) quarks.