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Problems set # 1 **Physics 541-735** September 13, 2011

1. In a unit system where $\hbar = c = 1$ show that

- $1\ kg = 5.6 \times 10^{26}$ GeV;
- $1\ \text{GeV}^{-2} = 0.389$ mb;
- $1\ \text{m} = 5.068 \times 10^{15}\ \text{GeV}^{-1}$;
- $1\ \text{s} = 1.5 \times 10^{24}\ \text{GeV}^{-1}$;
- the Compton wavelength for an electron is $\lambda_c = m_e^{-1}$ (calculate the numerical value) ;
- the Bohr radius of a hydrogen atom is $\lambda_c = (\alpha m_e)^{-1}$ (calculate the numerical value);
- the velocity of an electron in the lowest Bohr orbit is α (calculate the numerical value);
- due to the fact that the electromagnetic interaction is relatively weak we can use the non-relativistic Schrödinger equation to describe the hydrogen atom.
- the energy scale where quantum gravity effects become important is $M_{\rm Pl} \sim 10^{19}$ GeV. Estimate the length scale at which this happens.

[Hint: use $\hbar c = 197.3$ MeV fm, $\alpha = 1/137$, $m_e = 0.511$ MeV, $G = 6.67 \times 10^{-11}\ \text{J m/kg}^2$.]

2. The non-relativistic electromagnetic differential cross section for scattering beam of charged particles with charge $q = e$ off a heavy nucleus of charge $Q = Z|e|$ is found to be

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z\alpha}{4E}\right)^2 \sin^{-4}(\theta/2),$$

where α is the fine structure constant, θ is the scattering angle, and E is the beam energy. This calculation assumes that $\hbar = c = 1$. Compute the differential cross section $d\sigma/d\Omega$ in μb for $E = 1$ GeV, $\theta = 45°$, and $Z = 12$.

3. The LHC beam is not continuous but it has a bunch structure. Bunches of particles will collide every 25 ns at 14 TeV center-of-mass energy with a luminosity (number of particles per second per square centimeter) of $10^{34}\ \text{cm}^{-2}\, s^{-1}$. The total inelastic proton-proton cross section at 14 TeV is 70 mb. *(i)* Compute the number of interactions occuring per second as well as every time two bunches collide. These are called minimum bias event and are of almost no interest (background). The Higgs particle is expected to be produced at the LHC mainly through gluon fusion ($gg \to H$) which has a cross section of about 40 pb (at low Higss mass). One of the Higgs discovery channels involves searching for the Higgs decaying to two photons which has a probability (branching ratio) of $\sim 0.2 \times 10^{-2}$. *(ii)* How many bias events are expected to be produced for every Higgs event

observed via the two photon channel if one ignores detector effects?

4. Coyote notices that the Road Runner is about to take a train and he decides to look for a good location for blowing up the train. He notices a bridge that crosses a great ravine that the train will pass over. It appears to the Coyote, while he is standing on the side of the bridge and as the train is coming towards him, that the train takes up the entire length of the bridge in one moment. He knows the train will then pass by him to go to the station to pick up the Road Runner among other passengers, and will make a return trip, thus once again passing over the same bridge. The Coyote decides to place three Acme bombs strategically on the bridge, one on each end and one directly in the middle. He plans on blowing them up all at the same time while the train is occupying the entire bridge. He lines up the Acme wiring to the bombs in such a way that they are synchronized with *infinite* precision in his reference frame. The wiring is connected to an Acme switch placed on his side of the bridge. He now returns to the train station to ensure his plans are in place and that the Road Runner has taken his seat on the train. The Road Runner spots the Coyote just as the train is leaving and leaves his seat to run onto the track in front of the train. The Coyote realizes that to catch the Road Runner he must take the train to follow him and climbs to the top of the train and is soon following right behind the Road Runner. As the train approaches the bridge the Coyote sees the Road Runner is right in front of the switch and ready to push it with his beak. Since the train is moving at relativistic speed v, the Coyote sees the bridge contraction and he is sure that most of the train cars (and especially his car in the front of the train) will make it across the bridge safely. The Road Runner flips the switch and the three bombs blow up syncronized in his (the Road Runner's) reference frame when the train occupies the entire bridge. Describe precisely how the artist will draw the cartoon's events from the view of a passenger riding the train. "Beep! Beep!"

5. *(i)* Verify that Maxwell's equations remain invariant under arbitrary Lorentz boosts. Consider that the fields $\vec{E}$ and $\vec{B}$ are the components of a second rank tensor,

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix},$$

and that $j^\mu = (\rho, \vec{\jmath})$ is a fourth-vector. Maxwell's equations of classical electrodynamics are, in vacuo,

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0\,,$$

$$\vec{\nabla} \, . \, \vec{E} = \rho\,,$$

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{\jmath},$$

$$\vec{\nabla} \, . \, \vec{B} = 0\,,$$

where we are using Heaviside-Lorentz rationalized units.

(ii) Show that Maxwell's equations are equivalent to the following covariant equation for A^μ

$$\Box^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = j^\mu \,,$$

where $A^\mu = (\phi, \vec{A})$, the four-vector potential, is related to the electric and magnetic fields by

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi, \qquad \vec{B} = \vec{\nabla} \times \vec{A}\,.$$

Further, show that in terms of the antisymmetric field strength tensor

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

Maxwell's equations take the compact form

$$\partial_\mu F^{\mu\nu} = j^\mu$$

and the current conservation

$$\partial_\nu j^\nu = \frac{\partial \rho}{\partial t} + \vec{\nabla}\,.\,\vec{\jmath} = 0\,,$$

follows as a natural compatibility. [Note that $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla}(\nabla\,.\,\vec{A})$.]