Astronomy, Astrophysics, and Cosmology

Luis A. Anchordoqui

Department of Physics and Astronomy
Lehman College, City University of New York

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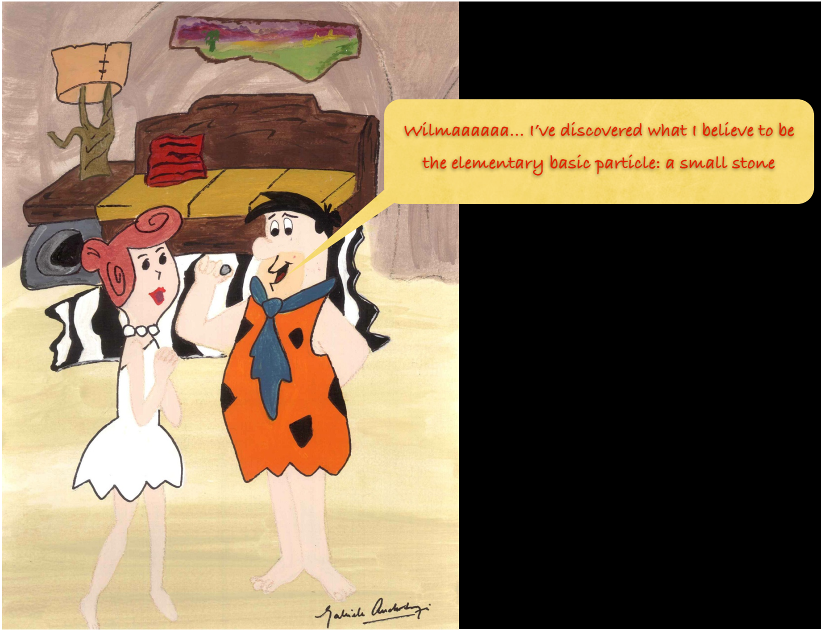
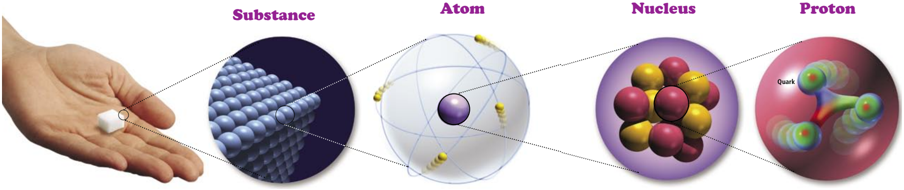
Table of Contents

The Standard Model is our most modern attempt to answer two simple questions that have been perplexing (wo)mankind throughout the epochs:

What is the Universe made of? Why is our world the way it is?


Wilmaaaaaa... I've discovered what I believe to be
the elementary basic particle: a small stone

- If we look deep inside Fred's rock ☞ we can see that

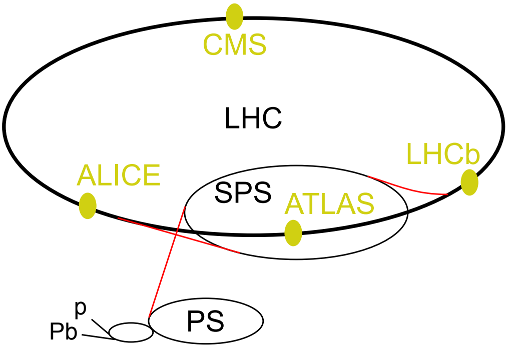


it is made up of only a few types of elementary "point-like" particles

- Elementary-particle model accepted today
  views quarks and leptons as basic constituents of ordinary matter

- By "pointlike"
  we understand that quarks and leptons show no evidence
  of internal structure at the current limit of our resolution

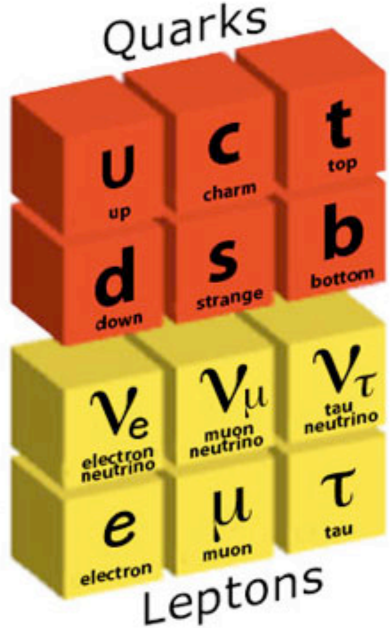
- World's largest microscope ☞ Large Hadron Collider



- Remarkably ☞ 70% of energy carried into collision by protons emerges perpendicular to incident beams
- @ transverse energy $E_\perp$ ☞ rough estimate of resolution length

$$\ell \approx \hbar c / E_\perp \approx 2 \times 10^{-19}\ \mathrm{TeV\,m}/E_\perp$$

- For collisions @ $\sqrt{s} = 13$ TeV ☞ $\ell_{\rm LHC} \approx 2 \times 10^{-20}$ m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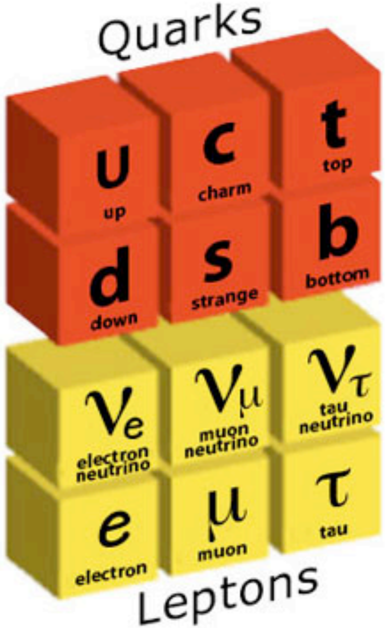
Leptons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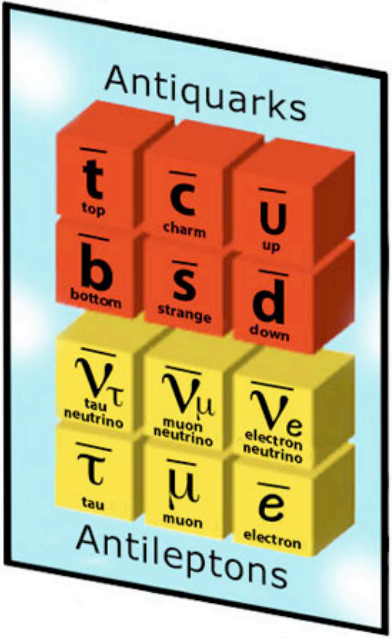
Leptons


Antiquarks
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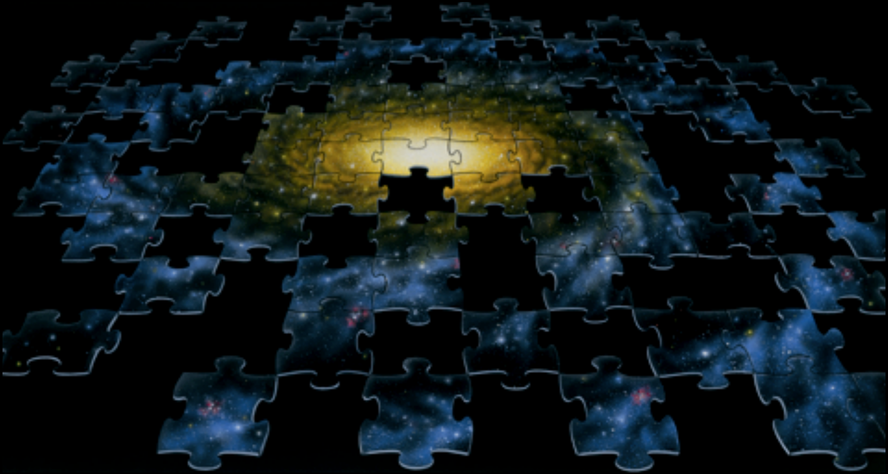
$\bar{\nu}_e$
electron neutrino
$\bar{\tau}$
tau
$\bar{\mu}$
muon
$\bar{e}$
electron
Antileptons

Three generations of quarks and leptons

	Fermion	Short-hand	Generation	Charge	Mass	Spin
Quarks	up	u	I	$+\frac{2}{3}$	$2.3^{+0.7}_{-0.5}$ MeV	$\frac{1}{2}$
	charm	c	II		1.275 ± 0.025 GeV	
	top	t	III		173.21 ± 0.51 GeV	
	down	d	I	$-\frac{1}{3}$	$4.8^{+0.5}_{-0.3}$ MeV	$\frac{1}{2}$
	strange	s	II		$95^{\pm}5$ MeV	
	bottom	b	III		4.18 ± 0.03 GeV	
Leptons	electron neutrino	ν_e	I	0	< 2 eV 95% CL	$\frac{1}{2}$
	muon neutrino	ν_μ	II		< 0.19 MeV 90% CL	
	tau neutrino	ν_τ	III		< 18.2 MeV 95%CL	
	electron	e	I	-1	0.511 MeV	$\frac{1}{2}$
	muon	μ	II		105.7 MeV	
	tau	τ	III		1.777 GeV	

Now ☞ an understanding of how world is put together



needs theory of how quarks and leptons interact with one another

4 fundamental forces of Nature

Forces can be characterized on basis of 4 criteria

1. types of particles that experience force
2. relative strength of force
3. range over which force is effective
4. nature of particles that mediate force

- electromagnetic force is carried by the photon
- strong force is mediated by gluons
- W and Z bosons transmit weak force
- quantum of gravitational force is called graviton

Force carriers

Force	Boson	Short-hand	Charge	Mass	Spin
Electromagnetic	photon	γ	0	0	1
Weak	W	$W^\pm$	± 1	80.385 ± 0.015 GeV	1
Weak	Z	Z^0	0	91.1876 ± 0.0021 GeV	1
Strong	gluon	g	0	0	1
Gravitation	graviton	G	0	0	2

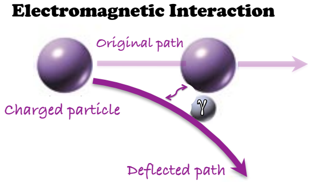
- Gravitation and electromagnetism have unlimited range
  largely for this reason they are familiar to everyone
- Weak force and strong force cannot be perceived directly
  because their influence extends only over a short range
  no larger than radius of atomic nucleus

Relative force strength for protons inside a nucleus

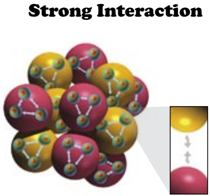
Force	Relative Strength
Strong	1
Electromagnetic	10^{-2}
Weak	10^{-6}
Gravitational	10^{-38}

- Though gravity is most obvious force in daily life
  on nuclear scale it is weakest of four forces
  and its effect at particle level can nearly always be ignored

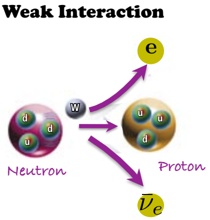
- electromagnetic and gravity force
  can be felt directly as agencies that pull or push



- Strong force binds together quarks inside hadrons
- Indirectly ☞ also binds protons and neutrons
  into atomic nuclei

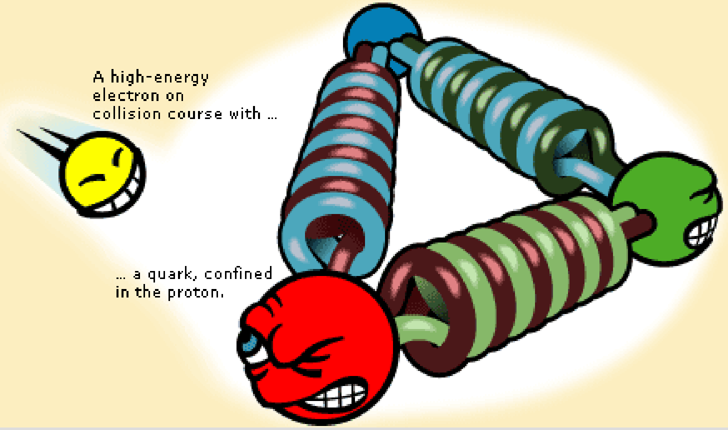


- Weak force
  is mainly responsible for decay of certain particles
- Its best-known effect
  is to transmute a down quark into an up quark
  which causes neutron to become proton
  plus electron and antineutrino



Hadrons

$q\bar{q}$	(quark + antiquark)	mesons	integral spin $\rightarrow$ Bose-Eisntein statistics
qqq	(three quarks)	baryons	half-integral spin $\rightarrow$ Fermi-Dirac statistics



✧ Keystone of any theory of strong interactions

explain peculiar rules for building hadrons out of quarks

Baryons and Mesons

- Structure of meson is not so hard to account for:
  since meson is made out of quark and antiquark
  assume quarks carry some property analogous to electric charge
- Binding of quark and antiquark
  explained on principle that opposite charges attract
  just as they do in electromagnetism
- Structure of baryons is far profound enigma
- To describe how three quarks can produce bound state
  we must assume that three like charges attract

Color

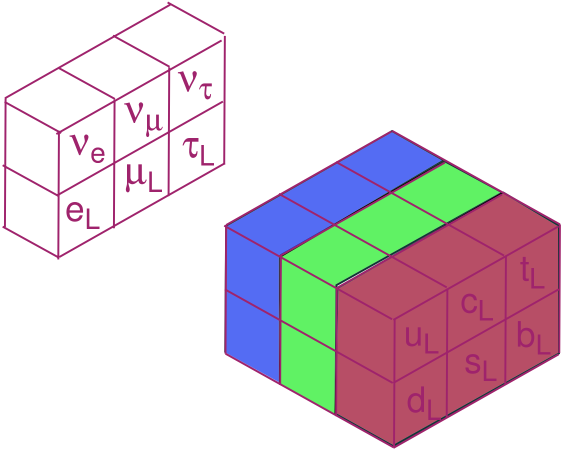
- Analogue of electric charge is property called color
- Rules for forming hadrons
  require combinations of quarks to be "white" or colorless
- Quarks are assigned the primary colors ☞ red, green, and blue
- Antiquarks have complementary "anticolors"
  cyan, magenta and yellow
- Each of the quark flavors comes in all three colors
  introduction of color charge triples number of distinct quarks

Gluons

- Quanta of color fields are called gluons
  (because they glue the quarks together)
- There are 8 of them: they are all massless
  they have a spin angular momentum 1
  they are massless vector bosons like the photon
- Also like photons ☞ gluons are electrically neutral
  but they are not color-neutral
- Each gluon carries one color and one anticolor
- There are nine possible combinations of a color and an anticolor but one of them is equivalent to white and is excluded
  leaving eight distinct gluon fields

Weak Interactions

- Charge-changing weak interactions mediated by $W^{\pm}$ are chiral

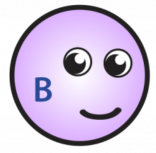


- Neutral weak interactions involving Z^0 act on both left-handed and right-handed particles

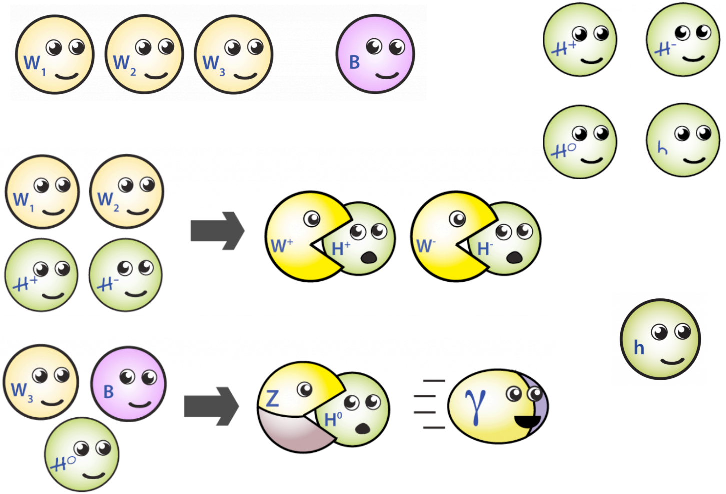
The electroweak theory then implies two sets of gauge bosons

a weak isovector ☛

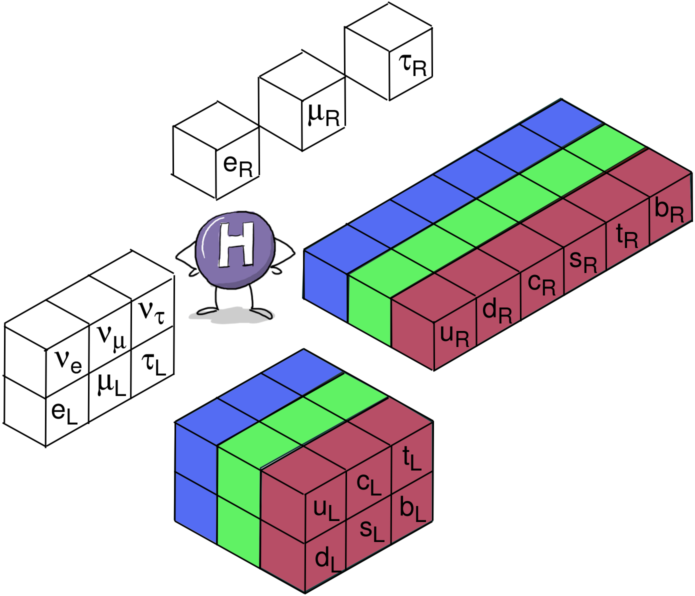
a weak isoscalar ☛



Electroweak symmetry breaking



Yukawa coupling



- Because early universe was in thermal equilibrium particle reactions can be modeled using tools of thermodynamics

- Number density, energy density, and pressure of weakly-interacting gas of particles with g degrees of freedom written in terms of its phase space distribution function $f(\vec{p}\,)$

$$
\begin{aligned}
n &= \frac{g}{(2\pi)^3} \int f(\vec{p}\,) d^3p \\
\rho &= \frac{g}{(2\pi)^3} \int E(\vec{p}\,)\, f(\vec{p}\,) d^3p \\
P &= \frac{g}{(2\pi)^3} \int \frac{|\vec{p}\,|^2}{3E} f(\vec{p}\,)\, d^3p
\end{aligned}
\tag{1}
$$

- $c = 1$ and $\hbar = 1$ ☞ $E = \sqrt{m^2 + p^2}$

- For a particle species in kinetic equilibrium ☞ occupancy f given by Fermi-Dirac or Bose-Einstein distrubutions

$$f(\vec{p}\,) = \frac{1}{e^{(E-\mu)/T} \pm 1} \tag{2}$$

  - T ☞ temperature
  - μ ☞ chemical potential (if present)
  - $\pm$ corresponds to either Fermi or Bose statistics

- Take $|\mu| \ll T$ and neglect all chemical potentials when computing total thermodynamic quantities

- All evidence indicates ☞ this is good approximation to describe particle interactions in super-hot primeval plasma

For particle species of mass m

$$\begin{aligned} \rho &= \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}}{e^{E/T} \pm 1} E^2 \, dE \\ n &= \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}}{e^{E/T} \pm 1} E \, dE \\ P &= \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{3/2}}{e^{E/T} \pm 1} \, dE \end{aligned} \tag{3}$$

Useful formulae

$$\int_0^\infty \frac{z^{n-1}}{e^z - 1} \, dz = \Gamma(n) \, \zeta(n) \tag{4}$$

$$\int_0^\infty \frac{z^{n-1}}{e^z + 1} \, dz = \frac{1}{2^n} \, (2^n - 2) \, \Gamma(n) \, \zeta(n) \tag{5}$$

- For nondegenerate ($T \gg \mu$) relativistic species ($T \gg m$)

$$\begin{aligned} n &= \begin{cases} \frac{1}{\pi^2}\,\zeta(3)\,gT^3 & \text{for bosons} \\ \frac{3}{4}\,\frac{1}{\pi^2}\,\zeta(3)\,gT^3 & \text{for fermions} \end{cases} \\ \rho &= \begin{cases} \frac{\pi^2}{30}\,g\,T^4 & \text{for bosons} \\ \frac{7}{8}\,\frac{\pi^2}{30}\,g\,T^4 & \text{for fermions} \end{cases} \\ P &= \rho/3 \end{aligned} \tag{6}$$

$$\zeta(3) = 1.20206...$$

- For nonrelativistic particle species ($T \ll m$)
  statistical quantities follow Maxwell-Boltzmann distribution
  ☞ there is no difference between fermions and bosons

$$\begin{aligned} n &= g\left(\frac{mT}{2\pi}\right)^{3/2}\, e^{-m/T} \\ \rho &= m\,n \\ P &= nT \ll \rho \end{aligned} \tag{7}$$

- Internal energy U can be considered to be function of 2 thermodynamic variables among P, V , and T
- These variables are related by equation of state
- Choose V and T to be fundamental variables
- Internal energy ☞ $U(V,T)$
- Differentiate U

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT \tag{8}$$

- Combine (8) with first law

$$dU = TdS - PdV \tag{9}$$

to obtain

$$TdS = \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV + \left(\frac{\partial U}{\partial T}\right)_V dT \tag{10}$$

- Since internal energy is function of T and V
  we may also choose to view S as a function of T and V
- This gives rise to differential relation

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \tag{11}$$

- Substituting (11) into (10) and equating dV and dT parts

$$\frac{\partial U}{\partial T} = T \frac{\partial S}{\partial T} \tag{12}$$

and

$$S = \frac{U + PV}{T} \tag{13}$$

- Define entropy density $s = S/V$

$$s = \rho + P \tag{14}$$

Average energy per particle

- For nondegenerate relativistic species

$$\langle E \rangle = \rho/n = \begin{cases} \frac{\pi^4}{30\zeta(3)}\, T & \simeq 2.701\, T \text{ for bosons} \\ \frac{7\pi^4}{180\zeta(3)}\, T & \simeq 3.151\, T \text{ for fermions} \end{cases} \quad (15)$$

- For non-relativistic species

$$\langle E \rangle = m + \frac{3}{2} T \quad (16)$$

For photons ☞ thermodynamic quantities computed *rather easily*

$$\rho_\gamma = \frac{\pi^2}{15} T_\gamma^4; \quad p_\gamma = \frac{1}{3}\rho_\gamma; \quad s_\gamma = \frac{4\rho_\gamma}{3T_\gamma}; \quad n_\gamma = \frac{2\zeta(3)}{\pi^2} T_\gamma^3 \quad (17)$$

For $T \gg m_i$ ☞ total energy density is

$$\rho_{\rm rad} = \left(\sum_B g_B + \frac{7}{8}\sum_F g_F\right)\frac{\pi^2}{30}T^4 \equiv \frac{\pi^2}{30}\,g_\rho(T)\,T^4 \tag{18}$$

- $g_{B(F)}$ ☞ total number of boson (fermion) degrees of freedom
- sum runs over all boson (fermion) states with $m_i c^2 \ll kT$
- $7/8$ due to difference between Fermi and Bose integrals
- (18) ☞ defines effective number of degrees of freedom $g_\rho(T)$ by taking into account new particle degrees of freedom as temperature is raised
- Change in $g_\rho(T)$ (ignoring mass effects) is ☞

Effective numbers of degrees of freedom in the standard model

Temperature	New particles	$4g_\rho(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c^*$	π's	69
$T_c < T < m_{\rm charm}$	- π's + $u, \bar{u}, d, \bar{d}, s, \bar{s}$ + gluons	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\rm bottom}$	$\tau^\pm$	303
$m_b < T < m_{W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\rm Higgs}$	$W^\pm, Z$	381
$m_H < T < m_{\rm top}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

*T_c ☞ confinement–deconfinement transition between quarks and hadrons