Astronomy, Astrophysics, and Cosmology

Luis A. Anchordoqui

Department of Physics and Astronomy
Lehman College, City University of New York

Lesson VII
March 29, 2016

arXiv:0706.1988

Table of Contents

- Rate of change for proper distance between us and distant galaxy

$$\dot{d}_{\rm p} = \dot{a} r = \frac{\dot{a}}{a} d_{\rm p} \tag{1}$$

- @ present time $(t = t_0)$ ☞ there is linear relation between proper distance to galaxy and its recession speed

$$v(t_0) = H_0 \, d_{\rm p}(t_0) \tag{2}$$

where

$$v(t_0) \equiv \dot{d}_{\rm p}(t_0) \tag{3}$$

and

$$H_0 = \left(\frac{\dot{a}}{a}\right)_{t=t_0} \tag{4}$$

in agreement with Hubble's Law

- In expanding universe wavelength of radiation is proportional to a

$$\lambda_0/\lambda_{\rm em} = a_0/a(t_{\rm em}) \tag{5}$$

- Redshift of a galaxy

$$z = \frac{\lambda_0 - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{a_0}{a(t_{\rm em})} - 1 \tag{6}$$

  expresses how much scale factor changed since light was emitted

- Light detected today was emitted at some time $t_{\rm em}$
- According to (6) ☞ 1-to-1 correspondence between z and $t_{\rm em}$
- z can be used instead of t to parametrize history of universe
- A given z corresponds to time
  when our universe was $1 + z$ times smaller than now

- Expressions for $a(t)$ are rather complicated
- One cannot directly invert (6) to express $t \equiv t_{\rm em}$ in terms of z
- It is useful to derive general integral expression for $t(z)$
- Differentiating (6)

$$dz = -\frac{a_0}{a^2(t)}\dot{a}(t)dt = -(1+z)H(t)dt \tag{7}$$

from which follows that

$$t = \int_z^\infty \frac{dz}{H(z)(1+z)} \tag{8}$$

- Integration constant has been chosen so that $z \to \infty$ corresponds to initial moment of $t = 0$
- Since photons travel on null geodesics of zero proper time we see directly from FRW metric

$$r = -\int \frac{cdt}{a(t)} = -\int c\frac{dt}{dz}(1+z) = c\int \frac{dz}{H(z)} \tag{9}$$

- For the moment ☞ set $c = 1$ and take $\rho = \rho_m c^2$
- To obtain expression for $H(z, H_0, \Omega_{m,0})$ rewrite Friedmann equation

$$H^2(z) + \frac{k}{a_0^2 R_0}(1+z)^2 = \Omega_{m,0} H_0^2 \frac{\rho_m(z)}{\rho_{m,0}} \tag{10}$$

- At $z = 0$ ☞ this reduces to

$$\frac{k}{a_0 R_0} = (\Omega_{m,0} - 1) H_0^2 \tag{11}$$

allowing to express current value $a_0 R_0$
in a spatially curved universe ($k \neq 0$)
in terms of H_0 and $\Omega_{m,0}$

- Taking this into account

$$H(z) = H_0 \left((1 - \Omega_{m,0})(1+z)^2 + \Omega_{m,0} \frac{\rho_m(z)}{\rho_{m,0}} \right)^{1/2} \tag{12}$$

Hubble's Law ☞ approximation for small redshift

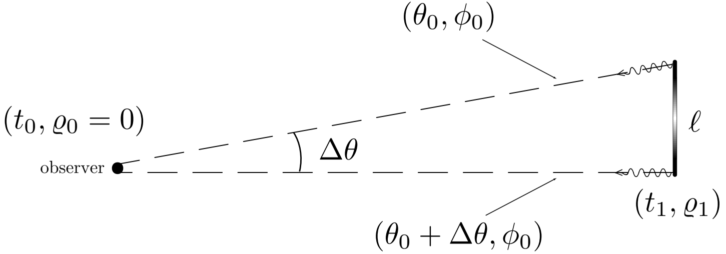
- Taylor expansion

$$a(t) = a(t_0) + (t - t_0)\dot{a}(t_0) + \frac{1}{2}(t - t_0)^2 \ddot{a}(t_0) + \cdots$$
$$= a(t_0)\left[1 + (t - t_0)H_0 - \frac{1}{2}(t - t_0)^2 q_0 H_0^2 + \cdots\right] \quad (13)$$

- $q_0 \equiv -\ddot{a}(t_0)a(t_0)/\dot{a}^2(t_0)$ ☞ deceleration parameter
- If expansion is slowing down ☞ $\ddot{a} < 0$ and $q_0 > 0$
- For not too large time-differences
  we can use Taylor expansion of $a(t)$ and write

$$1 - z \approx \frac{1}{1+z} = \frac{a(t)}{a(t_0)} \approx 1 + (t - t_0)H_0 \quad (14)$$

- Hubble's law ☞ $z = (t_0 - t)H_0 = d/cH_0$
  is valid as long as $z \ll H_0(t_0 - t) \ll 1$
- Deviations from its linear form arises for $z \gtrsim 1$
  and can be used to determine q_0



- Light source of size ℓ at $\varrho = \varrho_1$ and $t = t_1$
  subtending angle $\Delta\theta$ at origin ($\varrho = 0$, $t = t_0$)
- Proper distance ℓ between two ends of object is related to $\Delta\theta$ by

$$\Delta\theta = \frac{\ell}{a(t_1)\varrho_1} \tag{15}$$

- Angular diameter distance

$$d_A = \frac{\ell}{\Delta\theta} \tag{16}$$

so that ☞

$$d_A = a(t_1)\varrho_1 = \frac{\varrho_1}{1+z} \tag{17}$$

Angular diamter distance as function of redshift

- Recall light travel on null geodesics ☞ following similar derivation

$$d_A(z) = \frac{c\, f_k\left(H_0 \sqrt{|\Omega_{k,0}|} \int_0^z dz/H(z)\right)}{H_0 \sqrt{|\Omega_{k,0}|}\,(1+z)} \tag{18}$$

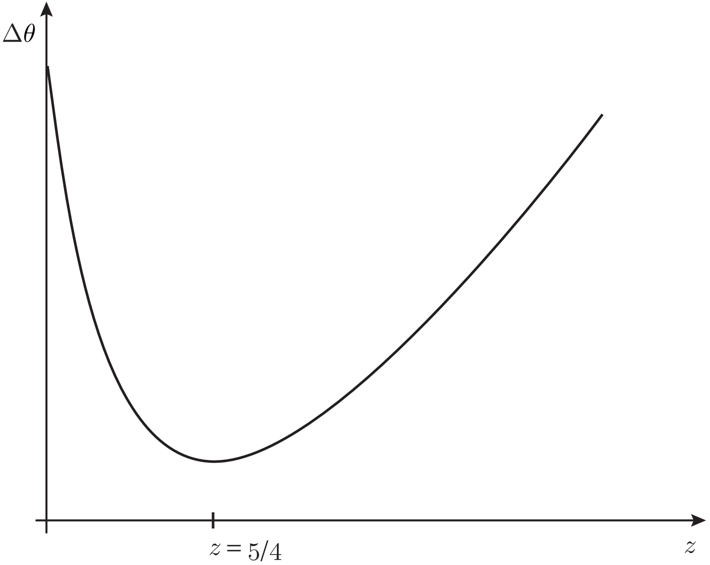
with

$$f_k(x) = \begin{cases} \sin x & \text{for } k = +1 \\ x & \text{for } k = 0 \\ \sinh x & \text{for } k = -1 \end{cases} \tag{19}$$

- For flat universe filled with dust ☞ angular diameter is

$$\Delta\theta(z) = \frac{\ell H_0}{2c} \frac{(1+z)^{3/2}}{(1+z)^{1/2} - 1} \tag{20}$$

At low redshifts ($z \ll 1$) ☞ $\Delta\theta$ decreases in inverse proportion to z reaches a minimum at $z = 5/4$ and then scales as z for $z \gg 1$



Relation between monochromatic flux density and luminosity

- Assume isotropic emission ☞ photons emitted by source pass with uniform flux density any sphere surrounding source
- Shift origin and consider FRW metric as being centred on source
- Because of homogeneity
  same comoving distance ϱ_1 between source and observer
- Photons from source pass through sphere of proper surface area
  $$4\pi a_0^2 \varrho_1^2$$
- Redshift still affects flux density in four further ways
  - Photon energies are redshifted
    reducing flux density by factor $1+z$
  - Photon arrival rates are time dilated
    reducing flux density by factor $1+z$
  - Bandwidth $d\nu$ is reduced by a factor $1+z$
    increasing energy flux per unit bandwidth by one power of $1+z$
  - Observed photons at frequency ν_0
    were emitted at frequency $(1+z)\nu_0$

- Overall ☞ flux density is luminosity at frequency $(1+z)\nu_0$ divided by total area and divided by $(1+z)$

$$\mathcal{F}_\nu(\nu_0) = \frac{L_\nu([1+z]\nu_0)}{4\pi a_0^2 \varrho_1^2(r)(1+z)} = \frac{L_\nu(\nu_0)}{4\pi a_0^2 \varrho_1^2 (1+z)^{1+\alpha}} \tag{21}$$

(second expression assumes power-law spectrum $L \propto \nu^{-\alpha}$)

- Integrate over ν_0 to obtain bolometric formulae

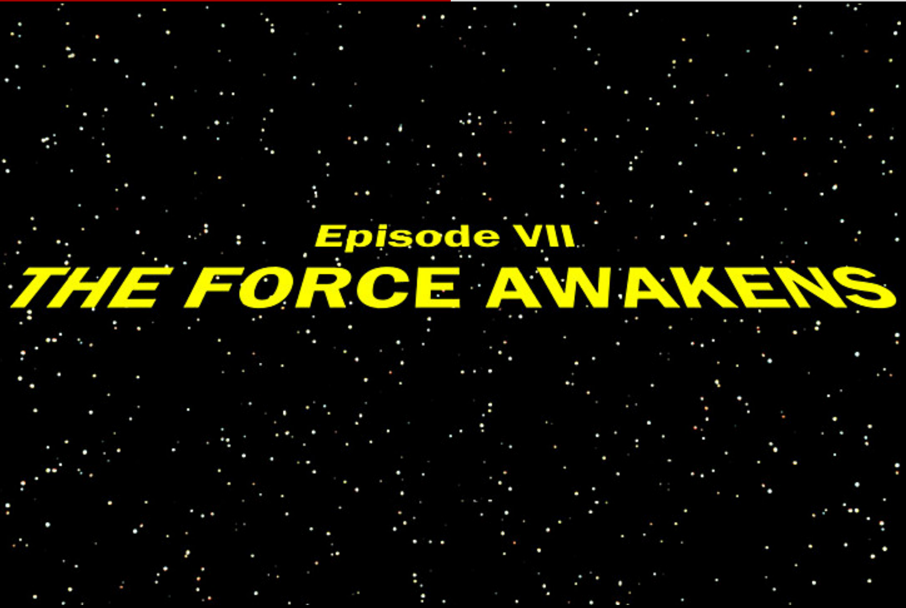
$$\mathcal{F} = \frac{L}{4\pi a_0^2 \varrho_1^2 (1+z)^2} \tag{22}$$

- Luminosity distance d_L is defined to satisfy relation

$$\mathcal{F} = \frac{L}{4\pi d_L^2} \tag{23}$$

- If we normalize scale factor today $a_0 = 1$

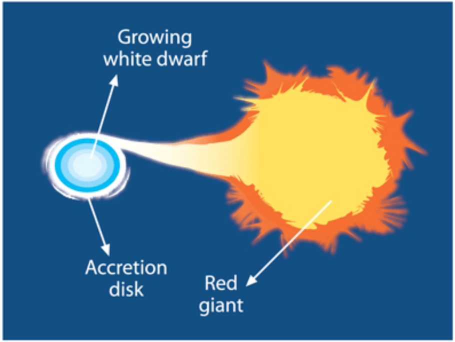
$$d_L = (1+z)\varrho_1 = (1+z)^2 d_A \tag{24}$$


Episode VII
THE FORCE AWAKENS

- Independent cosmological observations have unmasked presence of some unknown form of energy density related to otherwise empty space ☞ which appears to dominate recent gravitational dynamics of universe
  and yields a stage of cosmic acceleration
- We still have no solid clues as to nature of such dark energy
  (or perhaps more accurately dark pressure)
- Cosmological constant is simplest possible form of dark energy because it is constant in both space and time
  and provides good fit to experimental data as of today

- Expansion history determined using as standard candle any class of objects of known intrinsic brightness that can be identified over a wide distance range
- As light from such beacons travels to Earth expansion stretches not only distances between galaxy clusters but also wavelengths of photons en route
- Recorded redshift and brightness of each these candles provide a measurement of total integrated exansion since time light was emitted
- Collection of measurements over sufficient range of distances would yield an entire historical record of universe's expansion

- SNe Ia are best cosmological yard sticks in market
- They are precise distance indicators
  because they have a uniform intrinsic brightness
  due to similarity of triggering white dwarf mass ☞ $M_{\rm Ch} = M_\odot$
  and consequently amount of nuclear fuel available to burn
- This makes SNe Ia best (or at least most practical)
  example of standardizable candles in distant universe



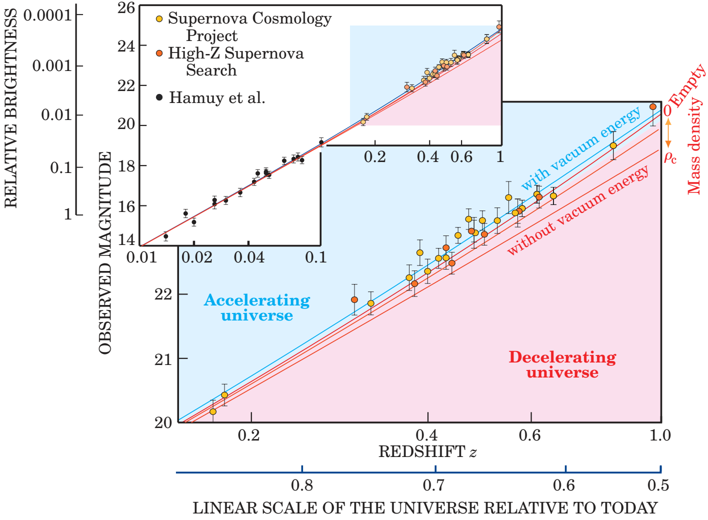
- Apparent magnitude m of celestial object
  measure of its apparent brightness as seen by observer on Earth
- The smaller the magnitude ☞ the brighter a star appears
- Magnitude scale originates in Hellenistic practice
  of dividing stars visible to the naked eye into six magnitudes
- Brightest stars in night sky were said to be $m = 1$
  faintest were $m = 6$ (limit of human visual perception)
- Pogson formalized system ☞ apparent magnitude in band x

$$m_x - m_{x,0} = -2.5 \log_{10}(\mathcal{F}_x / \mathcal{F}_{x,0}) \tag{25}$$

- Difference in magnitudes☞ $\Delta m = m_1 - m_2$
  can be converted to relative brightness

$$\frac{I_2}{I_1} = 2.5^{\Delta m}$$

Observed magnitude (and relative brightness) versus redshift

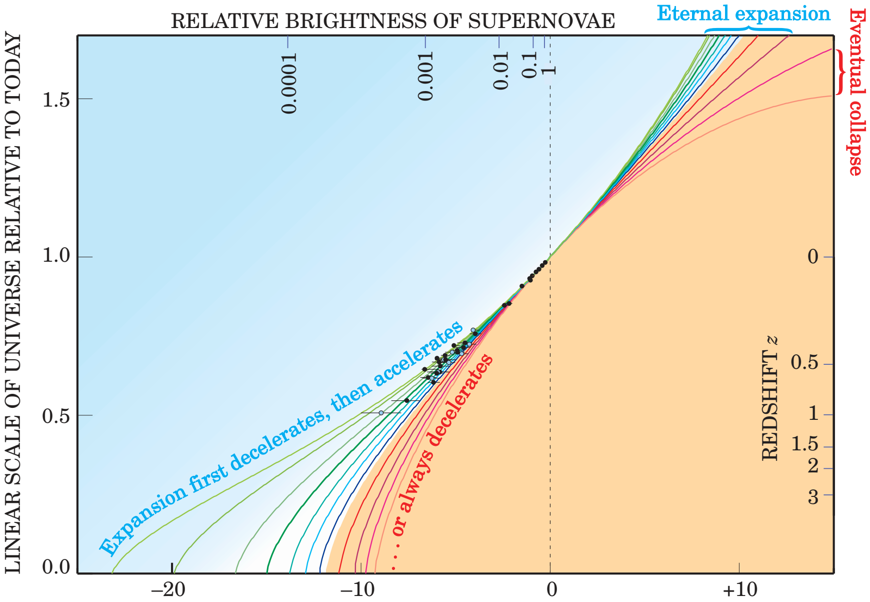


for well-measured distant and (in the inset) nearby SNe Ia

- Faintness (or distance) of high-redshift supernovae comes as a dramatic surprise
- In (simplest) standard cosmological models expansion history ☞ determined entirely by its mass density
- The greater density ☞ the more expansion is slowed by gravity
- In past ☞ high-mass-density universe would have been expanding much faster than it does today
- We shouldn't have to look far back in time to distant (faint) SNe Ia to find given integrated expansion (redshift)
- Conversely ☞ in low-mass-density universe we would have to look farther back
- But ☞ there is a limit to how low mean mass density could be
- After all ☞ we are here and stars and galaxies are here
- All that mass surely puts a lower limit on how far-that is to what level of faintness we must look to find a given redshift
- However ☞ high-redshift SNe Ia are fainter than would be expected even for empty cosmos

- If these data are correct
  obvious implication is that three simplest models of cosmology
  must be too simple
- Next-2-simplest model include expansionary term in eq. of motion
  driven by the cosmological constant Λ
  which competes against gravitational collapse
- Best fit to 1998 supernova data implies
  in present epoch vacuum energy density ρ_Λ
  is larger than energy density attributable to mass ρ_m
- Cosmic expansion is now accelerating

History of cosmic expansion as measured by high-redshift SNe Ia



- To accommodate SNe Ia data ☞ new term in Friedmann eq.

$$H^2 = \frac{8\pi}{3} G \frac{\rho}{c^2} - \frac{kc^2}{a^2 R_0^2} + \frac{\Lambda c^2}{3} \quad (26)$$

- Λ term also modifies aceleration equation

$$\frac{\ddot{a}}{a} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3c^2}(\rho + 3P) \quad (27)$$

- $H(z)$ is now given by

$$H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda} \quad (28)$$

- $\Omega_{m,0} + \Omega_\Lambda + \Omega_k = 1$
- Ω_k ☞ dimensionless density that measures curvature of space