Astronomy, Astrophysics, and Cosmology

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COSMOLOGY

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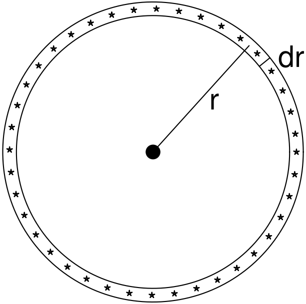
Copernican revolution ☞ Olbers paradox

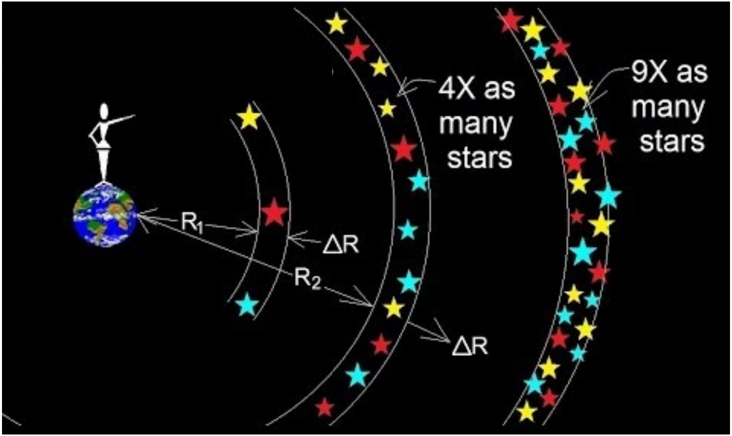
- Simplest and most ancient of all astronomical observations
  sky grows dark when Sun goes down
- When idea of unending unchanging space filled with stars like Sun was widespread ☞ question of dark night sky became a problem
- if absorption is neglected ☞ ☞ $b = L/4\pi r^2$
- If number density of stars is constant n
  number of stars between r and $r + dr$ ☞ $dN = 4\pi n r^2 dr$



- Total radiant energy density due to all stars

$$\rho_s = \int b\, dN = \int_0^\infty \left(\frac{L}{4\pi r^2}\right) 4\pi\, n\, r^2 dr = Ln \int_0^\infty dr \qquad (1)$$

- Integral diverges ☞ leading to infinite energy density of starlight!



To avoid this paradox . . .

- Olbers postulated existence of interstellar medium that absorbs light from very distant stars
- However ☞ this resolution of the paradox is unsatisfactory
  - In eternal universe ☞ interstellar medium T would have to rise until medium was in thermal equilibrium with starlight
  - In such case ☞ it would be emitting as much energy as it absorbs and hence could not reduce average radiant energy density
- Stars themselves are of course opaque and totally block out the light from sufficiently distant sources
- However ☞ if this were solution to paradox every line of segment must terminate at surface of star so whole sky should have T equal to surface of typical star

In the late 1920's . . .

- Hubble discovered that
  spectral lines of galaxies were shifted towards red
  by an amount proportional to their distances

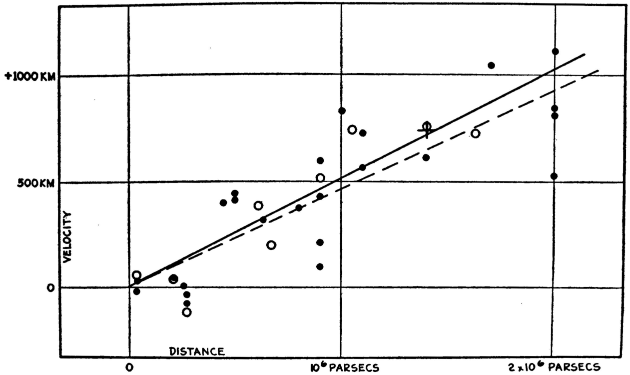
- If redshift is due to Doppler effect
  this means galaxies move away from each other
  with velocities proportional to their separations

- This what we expect according to simplest possible picture
  of flow of matter in expanding universe

- Measuring galaxy's redshift ☞ relatively easy
  $$z \equiv (\lambda' - \lambda)/\lambda$$
  and can be done with high precision

- Measuring galaxy's distance ☞ difficult

- Hubble knew z for nearly 50 galaxies
- However ☞ had estimated distances for only 20 of them
- From plot of redshift versus distance he found ☞ $z = H_0 r/c$



- Hubble's original plot ☞ redshift (vertical) and distance (horizontal)
- Note that ☞ vertical axis he actually inplots cz rather than z and that units are accidentally written as km rather than km/s

- Since in Hubble's study all redshift were small ☞ $z < 0.04$
  he was able to use non-relativistic realtion

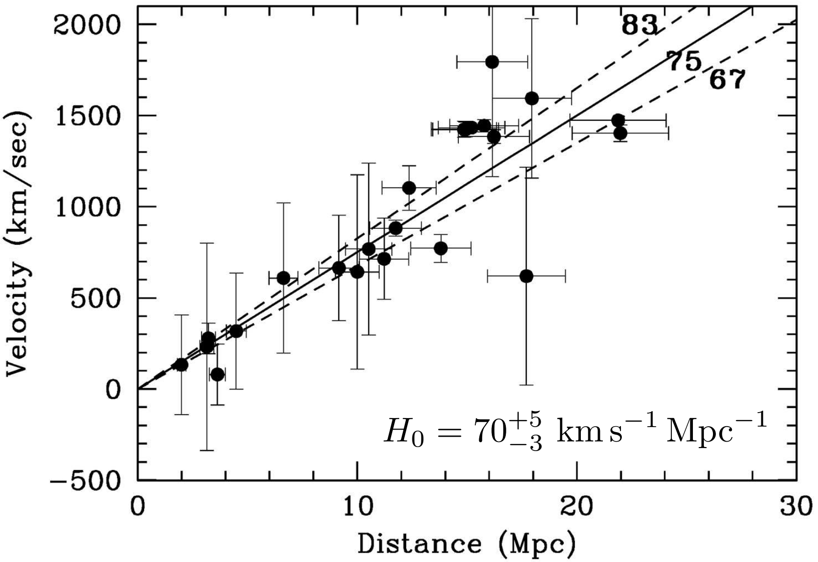
- For $v \ll c$ ☞ Doppler redshift $z \approx v/c$ and Hubble's law takes form

$$v = H_0 \, r \tag{2}$$

- From Hubble's diagram it follows that $H_0 = 500 \ \mathrm{km \, s^{-1} \, Mpc^{-1}}$

- However ☞ Hubble severely underestimated distances to galaxies

HST determination of the Hubble constant

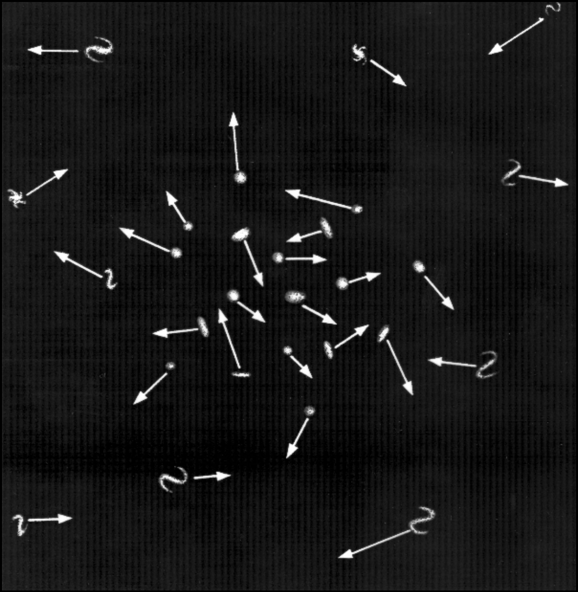


Peculiar velocities

- Galaxies do not follow Hubble's law exactly
- In addition to expansion of universe
  galaxy motions affected by gravity of specific nearby structures
  (such as pull of Milky Way and Andromeda on each other)
- Each galaxy therefore has a peculiar velocity
  peculiar ☞ used in the sense of "individual" or "specific to itself"
- Recession velocity of a galaxy

$$v = H_0 d + v_{\rm pec} \tag{3}$$

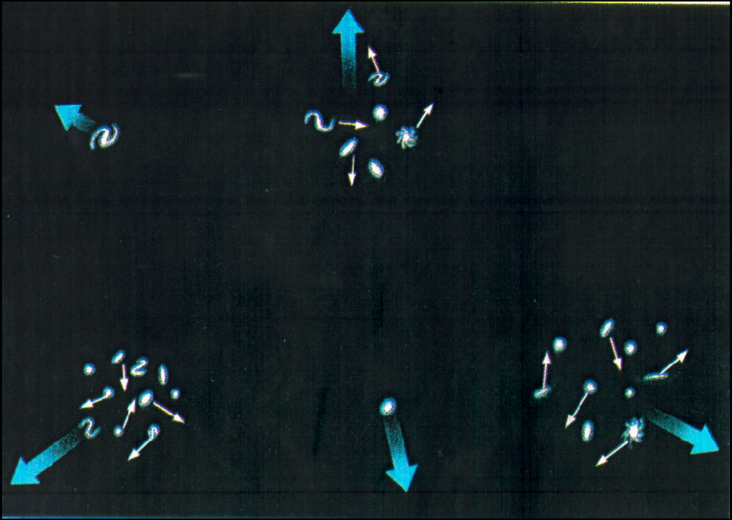
Local motions (peculiar velocities) ☞ Doppler shifts



More on peculiar velocities

- If peculiar velocities could have any value this would make Hubble's law useless
- However ☞ peculiar velocities are typically only about 300 km/s and they very rarely exceed 1000 km/s
- Hubble's law becomes accurate for galaxies that are far away when $H_0 d$ is much larger than 1000 km/s
- We can often estimate what a galaxy's peculiar velocity will be by looking at the nearby structures that will be pulling on it

Expansion velocities (Hubble flow) ☞ cosmological redshift

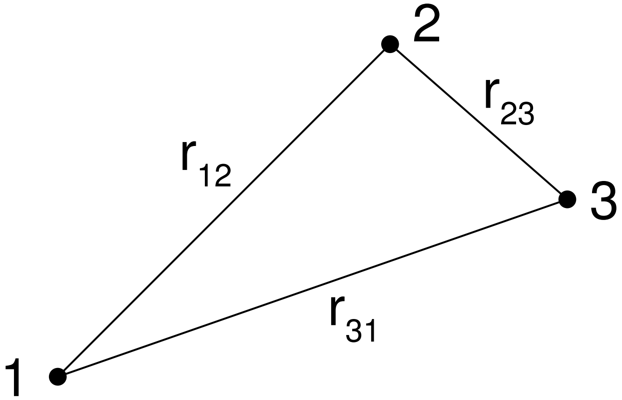


- We would expect intuitively that at any given time universe must be same to observers in all typical galaxies and in whatever direction they look
- Hereafter we will use label *typical* to indicate galaxies that don't have any large peculiar motion of their own but are simply carried along with general cosmic flow of galaxies
- This hypothesis is so natural (at least since Copernicus) that it has been called *Cosmological Principle* by Milne
- As applied to galaxies themselves ☞ Cosmological Principle requires that observer in typical galaxy should see all other galaxies moving with the same pattern of velocities whatever typical galaxy observer happens to be riding in
- It is a direct mathematical consequence of this principle that relative speed of any two galaxies must be proportional to distance between them

  just as found by Hubble

- Consider three typical galaxies at positions $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$
- They define triangle with sides of length

$$\begin{aligned} r_{12} &\equiv |\vec{r}_1 - \vec{r}_2| \\ r_{23} &\equiv |\vec{r}_2 - \vec{r}_3| \\ r_{31} &\equiv |\vec{r}_3 - \vec{r}_1| \end{aligned} \quad (4)$$



- Homogeneous and uniform expansion means that triangle shape is kept as galaxies move away from each other
- Maintaining correct relative lengths for sides of triangle requires expansion law of the form

$$\begin{aligned} r_{12}(t) &= a(t) r_{12}(t_0) \\ r_{23}(t) &= a(t) r_{23}(t_0) \\ r_{31}(t) &= a(t) r_{31}(t_0) \, . \end{aligned} \tag{5}$$

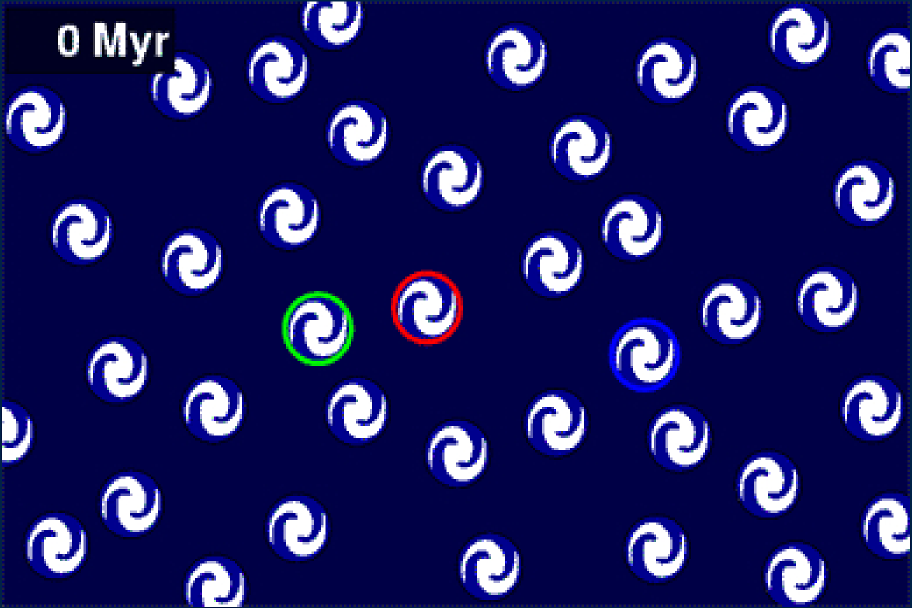
- $a(t)$ ☞ scale factor totally independent of location or direction
- At present moment $(t = t_0)$ ☞ $a(t_0) = 1$
- Scale factor $a(t)$ tells us how expansion (or possibly contraction) of universe depends on time

- At any time t ☞ an observer in galaxy 1
  will see other galaxies receding with speed

$$\begin{aligned} v_{12}(t) &= \frac{dr_{12}}{dt} = \dot{a}\, r_{12}(t_0) = \frac{\dot{a}}{a} r_{12}(t) \\ v_{31}(t) &= \frac{dr_{31}}{dt} = \dot{a} r_{31}(t_0) = \frac{\dot{a}}{a} r_{31}(t) \end{aligned} \tag{6}$$

- Easily seen that observers in galaxy 2 or galaxy 3
  find same relation between recession speed and distance
  with $\dot{a}/a$ playing role of Hubble constant

- Since argument can be applied to any trio of galaxies☞ implies:
  in any universe where distribution of galaxies
  is undergoing homogeneous and isotropic expansion
  velocity-distance relation ☞ $v = Hr$ with $H = \dot{a}/a$


0 Myr

- If galaxies are currently moving away from each other
  this implies they were closer together in past

- Consider pair of galaxies currently separated by r
  with $v = H_0 r$ relative to each other

- If no forces accelerate or decelerate their relative motion
  then their velocity is constant

- Independent of current separation r
  time that has elapsed since they were in contact

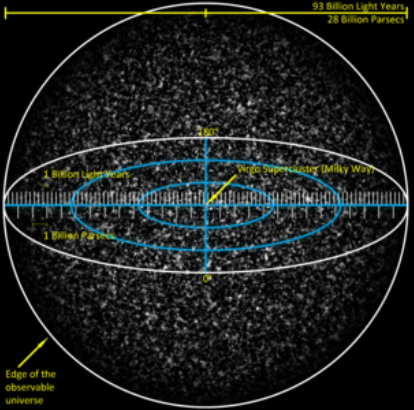
$$t_{\rm H} = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1} \tag{7}$$

- Time H_0^{-1} ☞ referred to as the Hubble time

- For $H \approx 70\ {\rm Mpc}^{-1}$ ☞ Hubble time is $H_0^{-1} \approx 14.0\ {\rm Gyr}$

- If relative velocities of galaxies have been constant in past
  a $t_{\rm H}$ time ago all galaxies were packed into small volume
- Observation of galaxy redshifts ☞ big bang model
  for evolution of universe
- Big bang model ☞ model in which universe expands
  from initially highly dense state to current low-density state
- $t_{\rm H} \sim 14\ {\rm Gyr}$ ☞ comparable to ages of oldest stars in universe.
- Age of universe ☞ time elapsed since original highly dense state
  is not necessarily $t_{\rm H}$
- If energy density of universe is dominated by matter
  attractive force of gravity slows down expansion
  universe was expanding more rapidly in past than now
  and universe is younger than H_0^{-1}
- If energy density of the universe is dominated by Λ
  dominant gravitational force is repulsive
  and universe is older than H_0^{-1}

- Horizon distance:
  greatest distance photon can travel during age of universe
- Hubble's $\mathcal{R}_\mathrm{H} = c/H_0 \approx 4,300\ \mathrm{Mpc}$ ☞ provides natural scale
- In most big bang models ☞ exact horizon distance
  depends on expansion history of universe
- Hubble's volume ☞ $(c/H_0)^3 \sim 10^{31}$ cubic light years



Caveats

- Cosmological Principle is obviously not true on small scales
  - Our Galaxy belongs to small local group of other galaxies
  - Of 33 galaxies in Messier's catalogue
    almost half are in one small part of sky ☞ constellation of Virgo
  - Cosmological Principle ☞ if at all valid
    comes into play when we view universe
    on scale at least as large as distance between clusters of galaxies
    or about 100 million light years
- Local universe approximation in deriving Hubble's Law
  - None of galaxies Hubble studied
    had speed anywhere near speed of light
  - When one thinks about really large distances
    needs theoretical framework
    capable of dealing with velocities approaching speed of ligh

Hubble's Law ties in with Olbers' Paradox

- If universe is of finite age $t_{\rm H} \sim H_0^{-1}$
  night sky can be dark ☞ even if universe is infinitely large
  because light from distant galaxies hasn't yet had time to reach us
- Galaxy surveys ☞ luminosity density of galaxies in local universe

$$nL \approx 2 \times 10^8 L_\odot \ {\rm Mpc}^{-3} \quad (8)$$

- By terrestrial standards ☞ universe is not a well-lit place
  luminosity density $\equiv$ 40 watt light bulb within sphere 1 AU in radius

- Total flux of light received from all stars within horizon

$$F_{\rm gal} \approx nL\int_0^{\mathcal{R}} dr \sim nL\frac{c}{H_0} \sim 9\times 10^{11} L_\odot \ {\rm Mpc}^{-2} \sim 2\times 10^{-11} L_\odot \ {\rm AU}^{-2} \qquad (9)$$

- By the cosmological principle ☞ this is total flux of starlight you'd expect at any randomly located spot in universe
- Flux of light we receive from the Sun

$$F_\odot = \frac{L_\odot}{4\pi \ {\rm AU}^2} \approx 0.08 L_\odot \ {\rm AU}^{-2} \qquad (10)$$

- Comparing ☞ $F_{\rm gal}/F_\odot \sim 3\times 10^{-10}$
- For entire universe to be as well-lit as Earth it would have to be over a billion times older than it is and you'd have to keep stars shining during all that time