Astronomy, Astrophysics, and Cosmology

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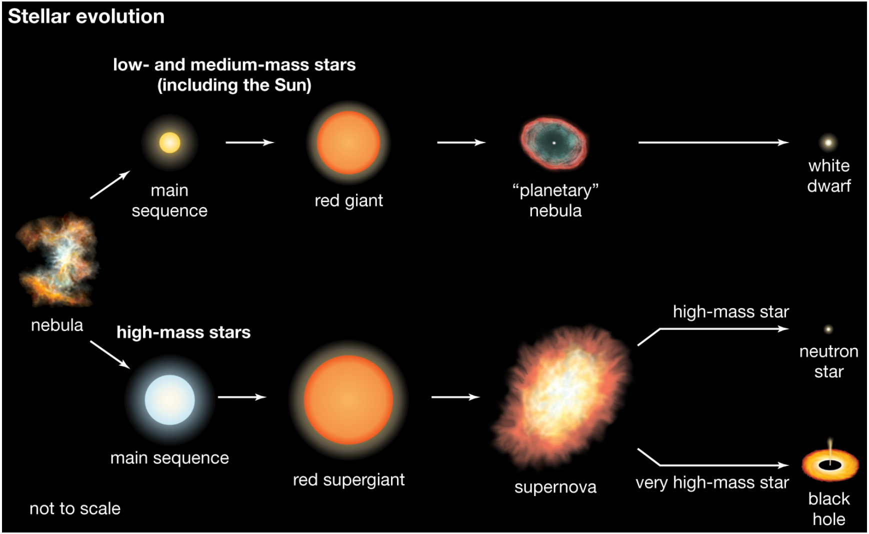
Table of Contents

In previous class we have seen that...

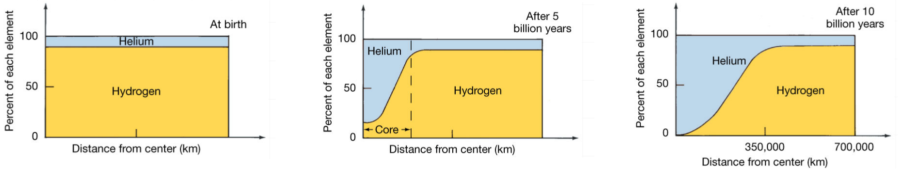


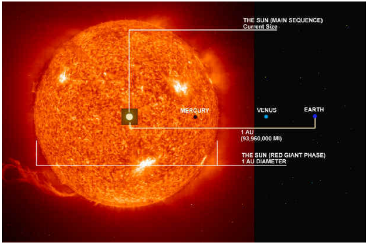
- ✧ Star form when gaseous (mostly ^{1}H) clouds contract due to pull of gravity
- ✧ Energy releasy in ^{1}H fusion reactions
  produces outward pressure to halt inward gravitational contraction

- As hydrogen fuses to form helium @ star's core
  helium formed is denser and tends to accumulate in central core
- As core of helium grows
  hydrogen continues to fuse in a shell around it
- When much of hydrogen within core has been consumed
  production of energy decreases at center and ...
  cannot prevent gravitational force to contract and heat up core
- Hydrogen in shell around core fuses more fiercely
  as T rises causing outer envelope to expand and cool
- Surface T reduces ☞ spectrum peaks at longer wavelength
  (reddish)
- By this time the star has left the main sequence:
  - It has become redder
  - It has grown in size
  - It has become more luminous
  - It enters red giant stage
- Model explains origin of red giants as step in stellar evolution

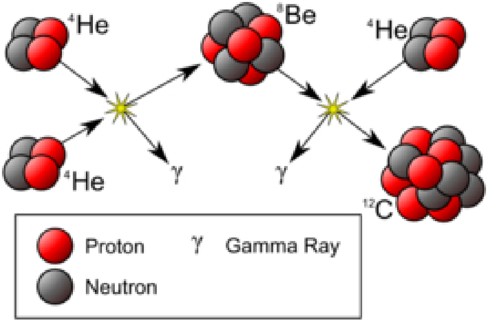
Example

- Sun has been on main sequence for $\sim$ four and a half billion years
- It will probably remain there another 4 or 5 billion years
- As becomes red giant expected to grow out to Mercury's orbit



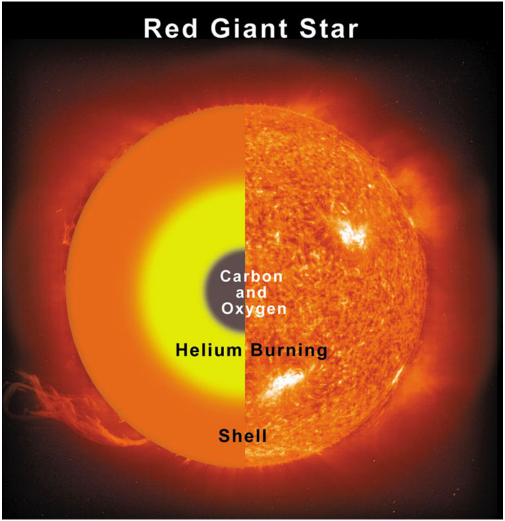


- If star is like our Sun or larger ☞ further fusion can occur
- As star's outer envelope expands ☞ core shrinks and heats up
- When the temperature reaches about 10^8 K
  helium nuclei reach each other and undergo fusion
- Reactions are



- Two reactions must occur in quick succession
  because ${}^{8}_{4}$Be is very unstable
- Net energy release of the triple-α process is 7.273 MeV

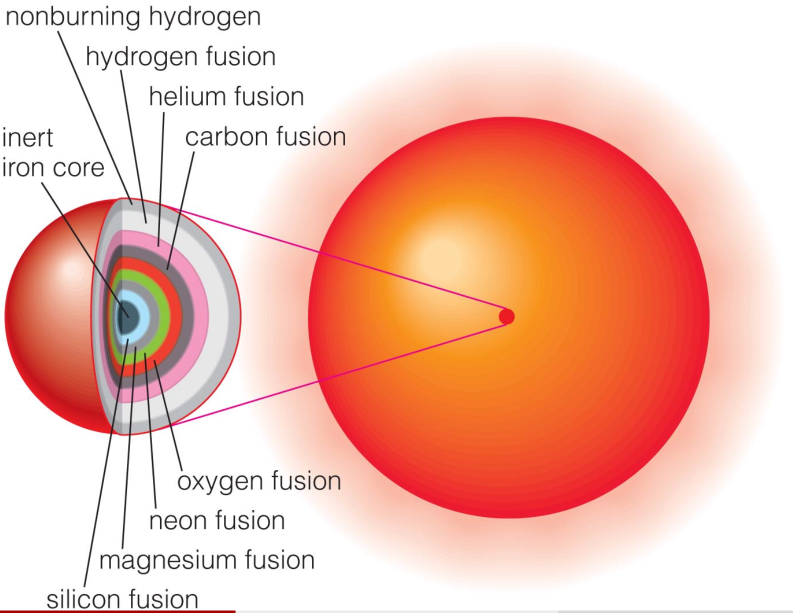
- Further fusion reactions are possible ☞ ${}^{4}_{2}$He fusing with ${}^{12}_{6}$C to form ${}^{16}_{8}$O.
- In very massive stars ☞ higher Z elements (e.g. ${}^{20}_{10}$Ne or ${}^{24}_{12}$Mg) can be made



Red supegiants

- As massive red supergiants age
  stars produce "onion layers" of heavier elements in their interiors
- @ $T = 5 \times 10^9$ K nuclei as heavy as $^{56}_{26}$Fe and $^{56}_{28}$Ni can be made
- Average binding energy per nucleon
  begins to decrease beyond iron group of isotopes
- Formation of heavy nuclei by fusion ends at iron group
- As a consequence
  core of iron builds up in centers of massive supergiants
- Process of creating heavier nuclei from lighter ones
  or by absorption of neutrons at higher Z
  is called nucleosynthesis

Red Supergiant



Sirius

- Sirius @ 2.6 pc ☞ fifth closest stellar system to Sun
- Analyzing motions of Sirius Bessel concluded it had an unseen companion with an orbital period $T \sim 50$ yr
- In 1862 ☞ Clark discovered this companion ☞ Sirius B
- Following-up observations showed that for Sirius B $M \approx M_\odot$
- Sirius B's peculiar properties were not established until 1915
- Adams noted high temperature of Sirius B ☞ $T \simeq 25,000$ K which together with its small luminosity ☞ $L = 3.84 \times 10^{26}$ W requires extremely small radius and thus large density of this star

- From Stefan-Boltzmann law we have

$$\frac{R}{R_\odot} = \left(\frac{L}{L_\odot}\right)^{1/2} \left(\frac{T}{T_\odot}\right)^2 \approx 10^{-2} \tag{1}$$

- Mean density of Sirius B ☞ $\rho = 3 \times 10^6 \ \mathrm{g/cm^3}$
- Lower limit for central pressure of a star in hydrostatic equilibrium

$$P_c > \frac{M^2}{8\pi R^4} = 4 \times 10^{16} \ \mathrm{bar} \tag{2}$$

- What would be central temperature T_c needed
  if pressure is dominated by ideal gas?
- From ideal gas law

$$T_c = \frac{P_c}{nk} \sim 10^2 T_{c,\odot} \approx 10^9 \ \mathrm{K} \tag{3}$$

- For such high central temperature
  dT/dr in Sirius B would be a factor 10^4 larger than in Sun
- This would in turn require larger luminosity $L(R)$
  and larger energy production rate than in main sequence star

- Stars like Sirius B are called white dwarfs
- They have very long cooling times because of their small surface luminosity
- White dwarfs are numerous ☞ mass density in solar neighborhood
  - main-sequence stars ☞ $0.04 M_\odot/\mathrm{pc}^3$
  - white dwarfs ☞ $0.015 M_\odot/\mathrm{pc}^3$
- Typical mass in range $0.4 - 1 M_\odot$ ☞ peaking @ $0.6 M_\odot$
- For white dwarfs ☞ no further fusion energy can be obtained
- White dwarf continues to lose internal energy by radiation decreasing in T and becoming dimmer until its light goes out
- Star has then become cold dark chunk of ash

- For a classical gas ☞ $P = nkT$
  ☞ in limit of zero T also P inside star goes to zero
- How can star be stabilized after fusion processes
  and thus energy production stopped?
- Solution to this puzzle:
  ☞ main source of P in such compact stars has different origin
- Pauli principle forbids fermions to occupy same quantum state
- In statistical mechanics
  - Heisenbergs uncertainty principle ☞ $\Delta x \Delta p \geq \hbar$
  - together with Pauli's principle

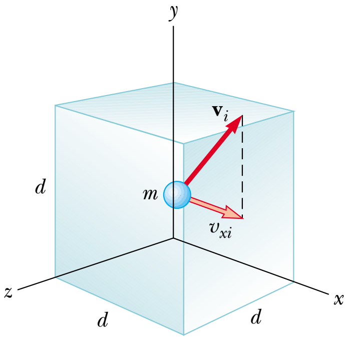
  imply that each phase-space volume $\hbar^{-1}\, dx\, dp$
  can be occupied by only one fermionic state

- A (relativistic or non-relativistic) particle in box of volume d^3 collides per time interval $\Delta t = d/v_x$ once with yz-side of box
- Thereby it exerts $F_x = \Delta p_x/\Delta t = p_x v_x/d$
- Pressure produced by N particles

$$P = F/A = N p_x v_x/(dA) = n p_x v_x$$

- For isotropic distribution with $\langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle = 3\langle v_x^2\rangle$

$$P = \frac{1}{3} n v p \qquad (4)$$



- If we use $\Delta x = n^{-1/3}$ and $\Delta p \approx \hbar/\Delta x \approx \hbar n^{1/3}$ together with $v = p/m$ (valid for non-relativistic particles)

$$P \approx nvp \approx \frac{\hbar^2 n^{5/3}}{m} \Rightarrow P \propto \rho^{5/3} \quad (5)$$

- For relativistic particles ☞ $v = c$

$$P \approx ncp \approx c\hbar n^{4/3} \Rightarrow P \propto \rho^{4/3} \quad (6)$$

- Note the following important points:
  - Both pressure laws are polytropic equations of state ☞ $P = K\rho^\gamma$
  - Non-relativistic degenerate Fermi gas
    has the same adiabatic index as an ideal gas ☞ $\gamma = 5/3$
  - Relativistic degenerate Fermi gas
    has the same adiabatic index as radiation ☞ $\gamma = 4/3$
  - In non-relativistic limit ☞ $P \propto 1/m$
    degeneracy will become important first to electrons

Pressure of degenerate non-relativistic electron gas

- To good approximation ☞ e^- in Sirius B are non-relativistic

$$n_e = \frac{\rho}{\mu_e \, m_p} \tag{7}$$

$\mu_e \equiv A/Z$ ☞ average number of nucleon per free electron

- For metal-poor stars ☞ $\mu_e = 2$

$$\begin{aligned} P &\approx \frac{h^2 n_e^{5/3}}{m_e} \approx \frac{(1.05 \times 10^{27}\ \mathrm{erg\,s})^2}{9.11 \times 10^{-28}\ \mathrm{g}} \left(\frac{10^6\ \mathrm{g/cm^3}}{2 \times 1.67 \times 10^{-24}\ \mathrm{g}}\right)^{5/3} \\ &\approx 10^{23}\ \mathrm{dyn/cm^2} = 10^{17}\ \mathrm{bar} \end{aligned} \tag{8}$$

- Consistent with lower limit for central pressure of Sirius B

✧ Relate mass of star to its radius by combining

$$P_c \sim \frac{GM^2}{R^4} \tag{9}$$

and

$$P = K\rho^{5/3} \sim K\left(\frac{M}{R^3}\right)^{5/3} = \frac{KM^{5/3}}{R^5} \tag{10}$$

we have

$$\frac{GM^2}{R^4} = \frac{KM^{5/3}}{R^5} \tag{11}$$

or

$$R = \frac{M^{(10-12)/6}}{K} = \frac{1}{KM^{1/3}} \tag{12}$$

✧ If small differences in chemical composition ☞ neglected

there is unique relation between mass and radius

✧ Radius of white dwarf stars decreases for increasing masses

suggesting that there exists maximal mass

- Assume pressure described by non-relativistic degenerate Fermi gas

- Total kinetic energy $U_{\rm kin} = Np^2/(2m_e)$ ☞ $n \sim N/R^3$ and $p \sim \hbar n^{1/3}$

$$U_{\rm kin} \sim N \frac{\hbar^2 n^{2/3}}{2m_e} \sim \frac{\hbar^2 N^{(3+2)/3}}{2m_e R^2} = \frac{\hbar^2 N^{5/3}}{2m_e R^2} \tag{13}$$

- Potentail gravitational energy approximated by $U_{\rm pot} \sim GM^2/R$

- Balance equation

$$U(R) = U_{\rm kin} + U_{\rm pot} \sim \frac{\hbar^2 N^{5/3}}{2mR^2} - \frac{GM^2}{R} \tag{14}$$

- For small R ☞ positive term dominates
  and so stable minimum $R_{\rm min}$ exists for each M

- If Fermi gas in star becomes relativistic ☞ $U_{\rm kin} = Ncp$

$$U_{\rm kin} \sim Nc\hbar n^{1/3} \sim \frac{c\hbar N^{4/3}}{R} \tag{15}$$

and

$$U(R) = U_{\rm kin} + U_{\rm pot} \sim \frac{c\hbar N^{4/3}}{R} - \frac{GM^2}{R} \tag{16}$$

- Both terms scale like $1/R$

- For fixed chemical composition ☞ ratio N/M remains constant

- If M is increased ☞ negative term increases faster than first one

- If U becomes negative
  and can be made arbitrary small by decreasing R
  star collapses

- Critical mass for $U = 0$ ☞ Chandrasekhar mass $M_{\rm Ch}$
- Using $M = N_N m_N$ ☞ $c\hbar N_{\rm max}^{4/3} = G N_{\rm max}^2 m_N^2$

$$N_{\rm max} \sim \left(\frac{c\hbar}{G m_p^2}\right)^{3/2} \sim \left(\frac{M_{\rm Pl}}{m_p}\right)^3 \sim 2 \times 10^{57} \,. \tag{17}$$

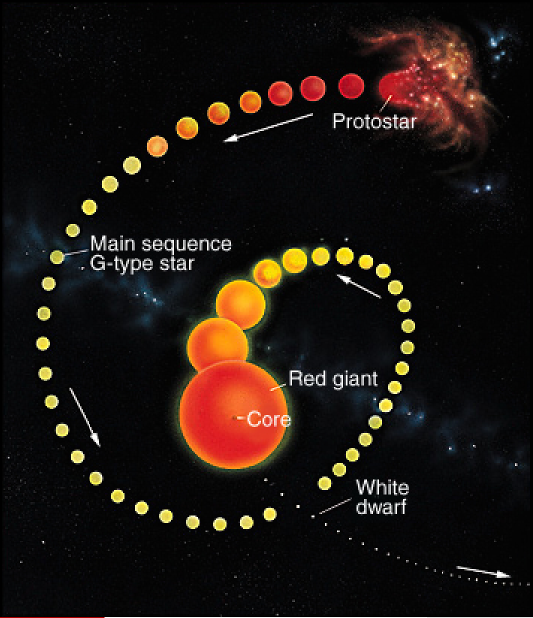
- This leads to ☞ $M_{\rm Ch} = N_{\rm max} m_p \sim 1.5 M_\odot$
- Critical size determined by two conditions:
  gas becomes relativistic $U_{\rm kin} \lesssim N m_e c^2$ and $N = N_{\rm max}$

$$N_{\rm max} m_e c^2 \gtrsim \frac{c\hbar N_{\rm max}^{4/3}}{R} \Rightarrow m_e c^2 \gtrsim \frac{c\hbar}{R}\left(\frac{c\hbar}{G m_N^2}\right)^{1/2} \tag{18}$$

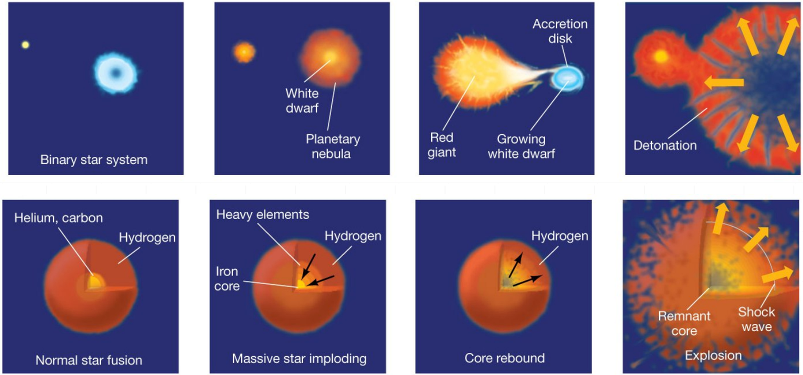
- This corresponds to radii found for white dwarf stars

$$R \gtrsim \frac{\hbar}{m_e c}\left(\frac{c\hbar}{G m_N^2}\right)^{1/2} \sim 5 \times 10^8 \text{ cm} \tag{19}$$

Life cycle of the Sun



- Supernovae are massive explosions that take place at end of star's life cycle
- They can be triggered by one of two basic mechanisms:
  - I by sudden re-ignition of nuclear fusion in degenerate star
  - II by the sudden gravitational collapse of massive star's core



Supernova explosion

- Sudden release of a large amount E into fluid of density ρ_1 creates strong explosion characterized by strong shock wave emanating from the point where the energy was released

- Sedov andTaylor solved the problem of point explosion in context of atomic bomb explosions

Order of magnitude estimates

- Mass of swept up material ☞ order $M(t) \sim \rho_1 R^3(t)$
- Fluid velocity behind shock ☞ order mean radial velocity

$$u_{\rm sh}(t) \sim R(t)/t \tag{20}$$

- Kinetic energy

$$E_{\rm kin} \sim M u_{\rm sh}^2 \sim \rho_1 R^3 \frac{R^2}{t^2} = \rho_1 \frac{R^5}{t^2} \tag{21}$$

- What about thermal energy in bubble created by explosion?
- If P is postshock pressure ☞ thermal energy should be of order

$$E_{\rm them} \sim \frac{3}{2} PV \tag{22}$$

Jumping conditions across a shock

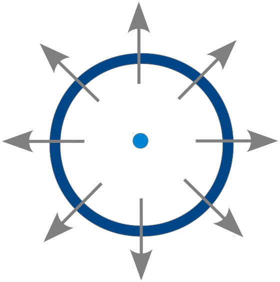
- If shock moves to right with velocity $u_{\rm sh}(t)$ ☞ in shock rest-frame background gas streams with velocity $u_1 = u_{\rm sh}(t)$ to the left and comes out of the shock with
  - higher density ρ_2
  - higher pressure P_2
  - lower velocity u_2
- For strong explosion ☞ sound-speed of medium $c_1 \ll 1$
- Rankine-Hugonoit relations for shock tell us

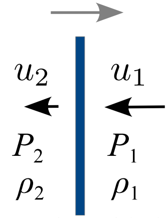
$$\frac{P_2}{P_1} = \frac{2\gamma(u_1/c_1)^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \quad (23)$$

- As background pressure ☞ $P_1 = \rho_1 c_1^2/\gamma$ ➽ in strong shock limit

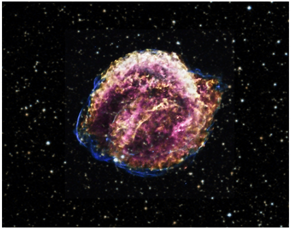
$$P_2 \simeq \frac{2\rho_1 u_1^2}{\gamma+1} \quad (24)$$

Blast wave ☞ spherically symmetric



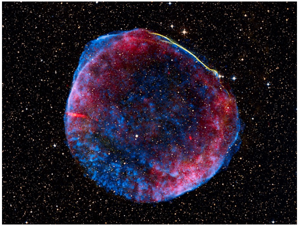


SN1604 (Kepler)



SNR-0509-67.5 (LMC)

SN1006



More about order of magnitude estimates

- With postshock pressure we estimate thermal energy in shocked bubble

$$E_{\rm therm} \sim P_2 R^3 \sim \rho_1 u_1^2 R^3 \sim \rho_1 \frac{R^5}{t^2} \tag{25}$$

- Total energy

$$E = E_{\rm kin} + E_{\rm therm} \sim \rho_1 \frac{R^5}{t^2} \tag{26}$$

- Shock front radius

$$R(t) \sim \left(\frac{Et^2}{\rho_1}\right)^{1/5} \tag{27}$$

- Expanding shock wave slows down as it expands

$$u_{\rm sh} = \frac{2}{5}\left(\frac{E}{\rho_1 t^3}\right)^{1/5} = \frac{2}{5}\left(\frac{E}{\rho_1}\right)^{1/2} R^{-3/2} \tag{28}$$