Astronomy, Astrophysics, and Cosmology

Luis A. Anchordoqui

Department of Physics and Astronomy
Lehman College, City University of New York

Lesson II
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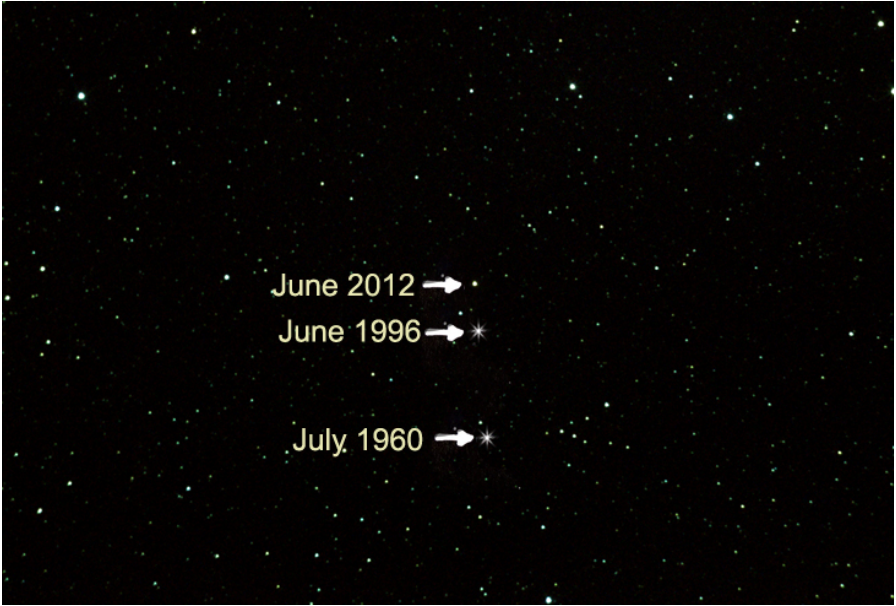
- Observational evidence that stars move
  at speeds ranging up to a few hundred kilometers per second
- E.g. ☞ relatively fast moving Barnard's star @ $D \sim 56 \times 10^{12}$ km
  moves across line of sight @ $v \sim 89$ km/s
- Consequence ☞ proper motion shifts $\sim 0.0029^\circ$/yr
- HST has measured proper motions as low as about 1 marcs/yr
- In radio ☞ VLBA relative motions
  can be measured to an accuracy of ~ 0.2 marcs/yr
- Apparent position in sky of more distant stars changes so slowly that proper motion can't be detected
  with even most patient observation
- However ☞ rate of approach or recession of star in line of sight can be measured much more accurately
  than its $\perp$ motion to line of sight
- Technique uses familiar property of any sort of wave motion
  Doppler effect

Barnard's star



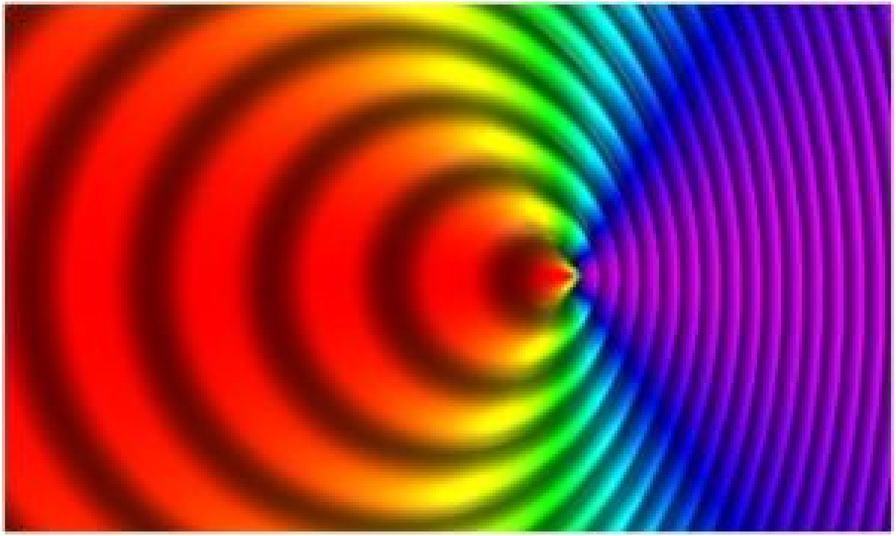
- When we observe sound or light wave from source at rest
time between arrival wave crests at our instruments
is same as time between crests as they leave source

- However ☞ if source is moving away from us
time between arrivals of successive wave crests
increases over time between departures from source
because each crest has little farther to go on its journey to us
than crest before

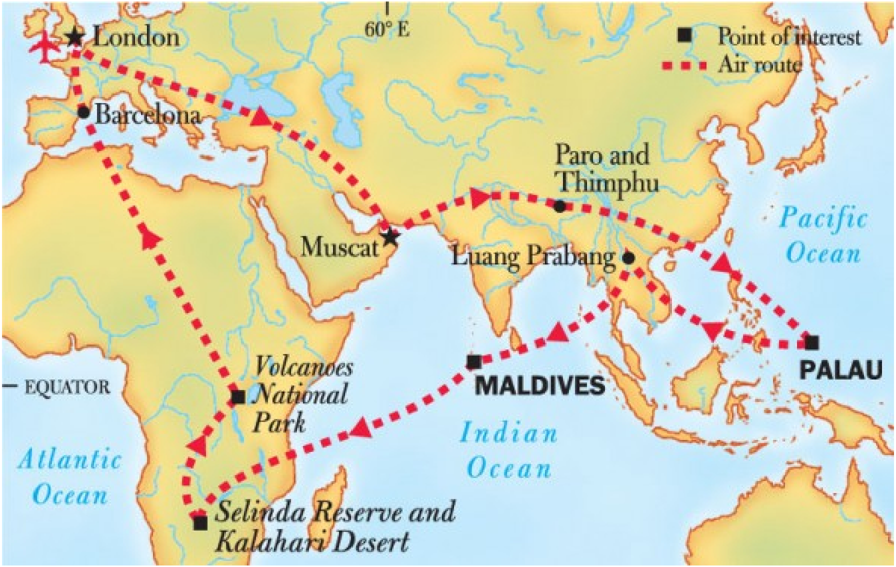
- Time between crests ☞ wavelength divided by wave speed
so a wave sent out by a source moving away from us
will appear to have longer wavelength than source @ rest

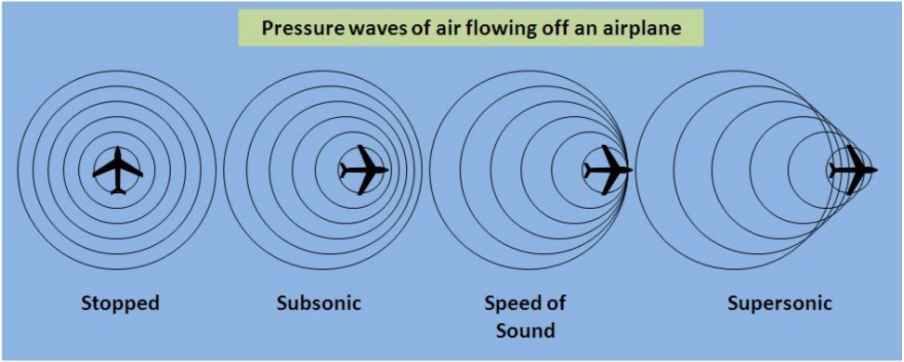
- Likewise ☞ if source is moving toward us
time between arrivals of wave crests is decreased
because each successive crest has shorter distance to go
and waves appear to have shorter wavelength

redshift and blueshift

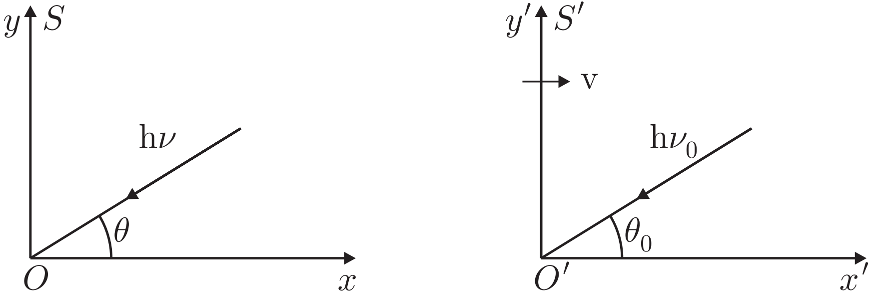


Weinberg's analogy




Pressure waves of air flowing off an airplane
Stopped
Subsonic
Speed of Sound
Supersonic

✧ Consider two inertial frames S and S' moving with relative velocity v



✧ Assume star @ rest in S' emits light @ (ν_0, θ_0) with respect to O'

✧ Momentum 4-vector for photon as seen in S

$$p^\mu = \left(\frac{h\nu}{c}, -\frac{h\nu}{c}\cos\theta, -\frac{h\nu}{c}\sin\theta, 0\right) \tag{1}$$

✧ Momentum 4-vector for photon as seen in S'

$$p_0^\mu = \left(\frac{h\nu_0}{c}, -\frac{h\nu_0}{c}\cos\theta_0, -\frac{h\nu_0}{c}\sin\theta_0, 0\right) \tag{2}$$

✧ Apply inverse LT to get 4-momentum relation from $S' \to S$

$$\begin{aligned} \frac{h\nu}{c} &= \gamma \left[\frac{h\nu_0}{c} + \beta \left(-\frac{h\nu_0}{c} \cos\theta_0 \right) \right] \\ -\frac{h\nu}{c}\cos\theta &= \gamma \left(-\frac{h\nu_0}{c}\cos\theta_0 + \beta \frac{h\nu_0}{c} \right) \\ \frac{h\nu}{c}\sin\theta &= \frac{h\nu_0}{c}\sin\theta_0 \end{aligned} \tag{3}$$

✧ First expression gives relativistic Doppler formula

$$\nu = \nu_0 \gamma (1 - \beta \cos\theta_0) \tag{4}$$

✧ For observational astronomy (4) is not useful
because (ν_0, θ_0) refer to the star's frame not that of observer

✧ Apply instead direct LT $S \to S'$ to photon energy

$$\nu_0 = \gamma \nu (1 + \beta \cos\theta) \tag{5}$$

which gives ν_0 in terms of quantities measured by observer

Special cases

- $\theta_0 = 0$ ☞ source moving away from the observer

$$\nu = \nu_0 \sqrt{(1-\beta)/(1+\beta)} \tag{6}$$

Non-relativistic limit ☞ $\nu = \nu_0(1-\beta)$

- $\theta_0 = \pi$ ☞ source moving towards observer

$$\nu = \nu_0 \sqrt{(1+\beta)/(1-\beta)} \tag{7}$$

- $\theta_0 = \pi/2$ ☞ transverse Doppler effect

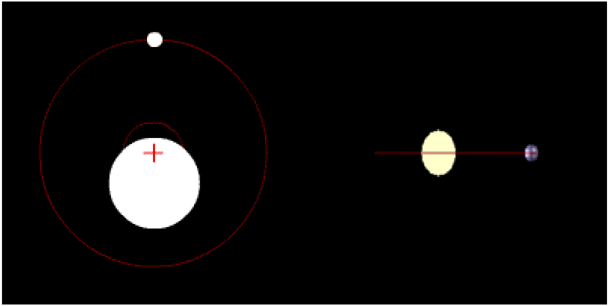
$$\nu = \nu_0 \gamma \tag{8}$$

2nd order relativistic effect
arising from dilation of time in moving frame

Search for exoplanets

✧ As planet orbits star ☞ star has its own orbit around CM system

✧ Radial velocity method: Variations in star's radial velocity detected from displacements in star's spectral lines due to Doppler effect



✧ Photometric method: If planet crosses in front of its parent star's disk observed brightness drops by small amount

- Stars appear unchanging
- Night after night heavens reveal no significant variations
- On human time scales ☞ majority of stars change very little
- We cannot follow any but tiniest part of star life cycle

Star formation

- Stars are born when gaseous clouds (mostly hydrogen)
  contract due to pull of gravity
- Huge gas cloud fragments into numerous contracting masses
- Each mass is centered in area
  where density is only slightly greater than @ nearby points
- Once such "globules" formed
  gravity would cause each to contract in towards its CM
- As particles of such protostar accelerate inward
  their kinetic energy increases
- When kinetic energy is sufficiently high
  Coulomb repulsion ☞ not strong enough to keep ^{1}H nuclei appart
  and nuclear fussion can take place
- In star like our Sun
  "burning" of ^{1}H occurs when $4p$ fuse to form ^{2}He nucleus
  with release of: γ, e^+, ν_e

M16 a.k.a. Eagle Nebula located $\approx 7,000$ ly away

HST's pillars of creation

Sun's energy output

- pp cycle due to following sequence of fusion reactions:

$$^1_1\mathrm{H} + ^1_1\mathrm{H} \to ^2_1\mathrm{H} + 2\,e^+ + 2\,\nu_e \qquad (0.42\ \mathrm{MeV}) \tag{9}$$

$$^1_1\mathrm{H} + ^2_1\mathrm{H} \to ^3_2\mathrm{He} + \gamma \qquad (5.49\ \mathrm{MeV}) \tag{10}$$

$$^3_2\mathrm{He} + ^3_2\mathrm{He} \to ^4_2\mathrm{He} + ^1_1\mathrm{H} + ^1_1\mathrm{H} \qquad (12.86\ \mathrm{MeV}) \tag{11}$$

- Released energy ➢ mass difference between initial & final states
  ➢ carried off by outgoing particles
- Net effect $$4\,^1_1\mathrm{H} \to ^4_2\mathrm{He} + 2e^+ + 2\nu_e + 2\gamma \tag{12}$$
- Takes 2 of each of first 2 reactions to produce two $^3_2$He
- Total energy released for net reaction ☞ 24.7 MeV
- e^+ quickly annihilates with e^- to produce $2m_ec^2 = 1.02$ MeV
  so total energy released ☞ 26.7 MeV
- Deuterium formation has very low probability
  infrequency of reaction limits rate at which Sun produces energy

In more massive stars...

- Energy output comes from the carbon (or CNO) cycle
- CNO cycle comprises following sequence of reactions:

$$ {}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma \tag{13} $$

$$ {}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \nu \tag{14} $$

$$ {}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma \tag{15} $$

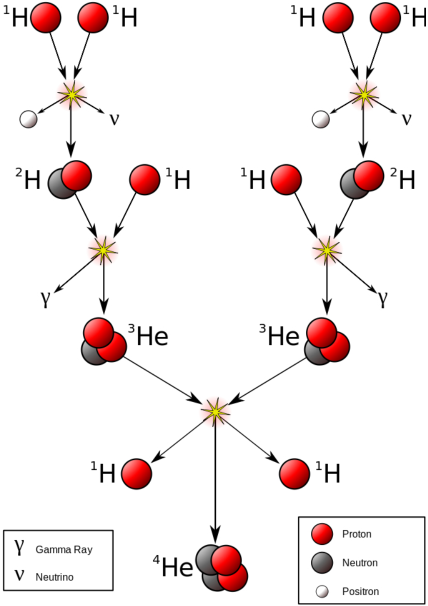
$$ {}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + \gamma \tag{16} $$

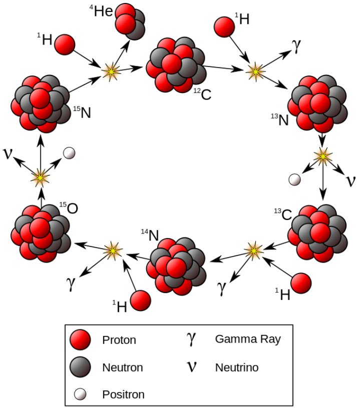
$$ {}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + e^{+} + \nu \tag{17} $$

$$ {}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \tag{18} $$

- No carbon is consumed in this cycle (see first and last equations)
- Net effect is the same as the pp cycle
- Theory of the pp and CNO cyles first worked out by Bethe in 1939

Nuclear Burning





- Fusion reactions take place in star core ☞ $T \sim 10^7$ K
- Surface temperature is much lower ☞ $\mathcal{O}(\text{few thousand K})$
- Tremendous release of energy in these fusion reactions produces outward pressure to halt inward gravitational contraction protostar (now really a young star) stabilizes in main sequence
- Stellar structure on main sequence ☞ described by spherically symmetric system in hydrostatic equilibrium
- This requires that rotation, convection, magnetic fields, and other effects that break rotational symmetry have only a minor influence on the star

✧ $M(r)$ ☞ mass enclosed inside sphere with radius r and density $\rho(r)$

$$M(r) = 4\pi \int_0^r dr' \, r'^2 \, \rho(r') \Rightarrow \frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \tag{19}$$

✧ Gravitational acceleration produced by $M(r)$ is

$$g(r) = -\frac{GM(r)}{r^2} \tag{20}$$

✧ If star is in equilibrium ☞ acceleration balanced by pressure gradient from center of star to its surface

✧ Since $P = F/A$ ☞ pressure change along distance dr yields

$$\begin{aligned} dF &= dAP - (P + dP)dA \\ &= -\underbrace{dAdP}_{\text{force}} = -\underbrace{\rho(r) dAdr}_{\text{mass}} \quad \underbrace{a(r)}_{\text{acceleration}} \end{aligned} \tag{21}$$

✧ For increasing r ☞ gradient $dP < 0$ and resulting dF is positive and directed outward

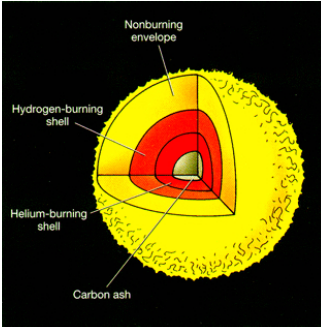
✧ Hydrostatic equilibrium ☞ $g(r) = -a(r)$

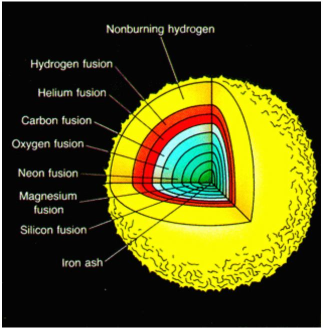
$$\frac{dP}{dr} = \rho(r) g(r) = -\frac{GM(r)\,\rho(r)}{r^2} \tag{22}$$

✧ If the pressure gradient and gravity do not balance each other layer at position r is accelerated

$$a(r) = \frac{GM(r)}{r^2} + \frac{1}{\rho(r)}\frac{dP}{dr} \tag{23}$$

✧ In general ☞ equation of state $P = P(\rho, T, Y_i)$





✧ Estimate of central pressure ☞ $P_c = P(0)$
integrate (22) using (19) and obtain with $P(R) \approx 0$

$$P_c = \int_0^R \frac{dP}{dr} dr = G \int_0^M dM \frac{M}{4\pi r^4}\,, \tag{24}$$

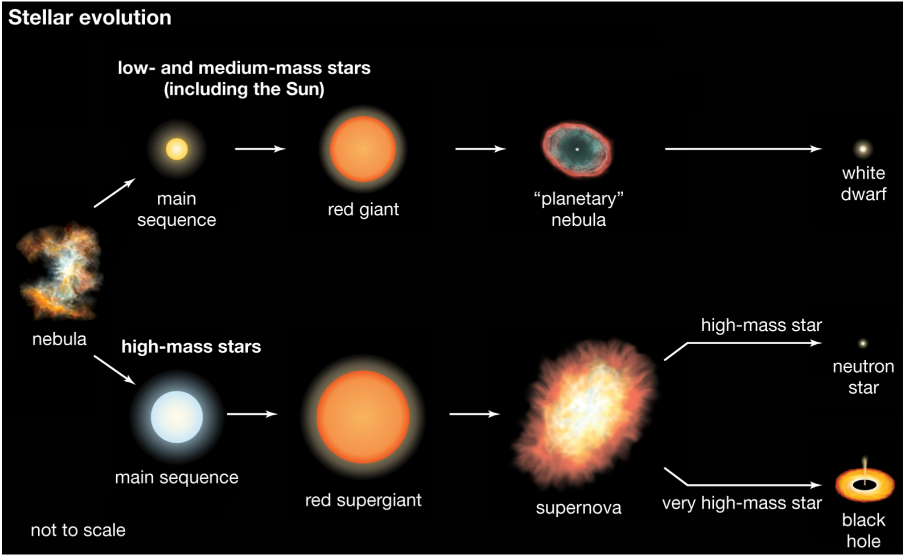
✧ P_c lower limit ☞ replace r by stellar radius $R \geq r$

$$\begin{aligned} P_c &= G \int_0^M dM \frac{M}{4\pi r^4} \\ &> G \int_0^M dM \frac{M}{4\pi R^4} = \frac{M^2}{8\pi R^4} \end{aligned} \tag{25}$$

✧ Inserting values for Sun

$$P_c > \frac{M^2}{8\pi R^4} = 4 \times 10^8 \text{ bar} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right)^4 \tag{26}$$

✧ Integrating hydrostatic equation using “solar standard model”
$P_c = 2.48 \times 10^{11}$ bar ☞ factor 500 larger



to be continued . . .