Astronomy, Astrophysics, and Cosmology

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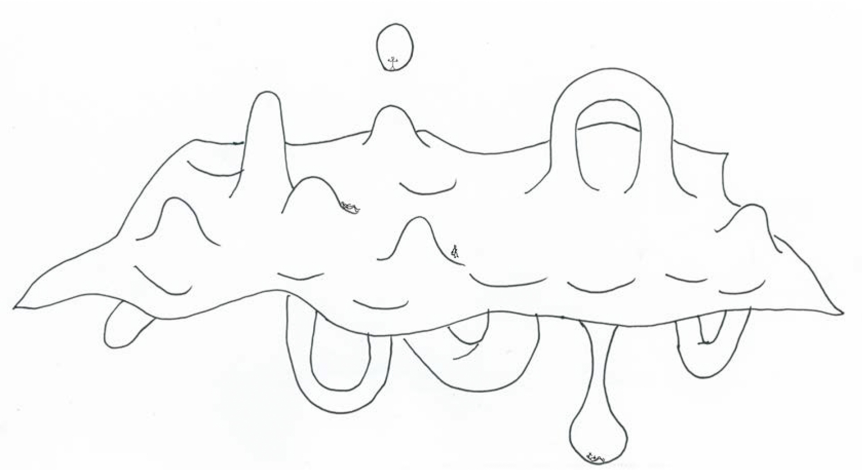
Spacetime-foam



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1. The Early Universe
   - Neutrino decoupling and BBN
   - Quantum black holes

- After first tenth of a second ☞ $T \sim 3 \times 10^{10}$ K
  universe filled with ☞ p, n, γ, e^-, e^+, ν, and $\overline{\nu}$
- Baryons are of course nonrelativistic
  while all other particles are relativistic
- Particles kept in thermal equilibrium by EM and weak processes
  $\bar{\nu}\nu \rightleftharpoons e^+e^-$, $\nu e^- \rightleftharpoons \nu e^-$, $n\nu_e \rightleftharpoons pe^-$, $\gamma\gamma \rightleftharpoons e^+e^-$, $\gamma p \rightleftharpoons \gamma p$, etc.
- Complying with precision demanded by phenomenological study sufficient to consider cross section of reactions involving left-handed neutrinos, right-handed antineutrinos, and electrons

$$\sigma_{\rm weak} \sim G_F^2 E^2$$

- $G_F = 1.16 \times 10^{-5}\ {\rm GeV}^{-2}$ ☞ Fermi constant

- If we approximate E of all particle species by T and v by c their density by $n \sim T^3$ ☞ neutrino interaction rate

$$\Gamma_{\mathrm{int},\nu}(T) \approx \langle v\sigma \rangle \, n_\nu \approx G_F^2 T^5 \tag{1}$$

- Comparing (1) with

$$H = \left(\frac{8\pi G \rho_{\mathrm{rad}}}{3}\right)^{1/2} = \left(\frac{8\pi^3}{90} g_\rho(T)\right)^{1/2} T^2/M_{\mathrm{Pl}}$$
$$\sim 1.66\sqrt{g_\rho(T)}\, T^2/M_{\mathrm{Pl}} \tag{2}$$

calculated for $g(T) = 10.75$

we see that when T drops below characteristic temperature $T_{\nu_L}^{\mathrm{dec}}$ neutrinos *decouple* ☞ they lose thermal contact with electrons

- Condition

$$\Gamma_{\mathrm{int},\nu}(T_{\nu_L}^{\mathrm{dec}}) = H(T_{\nu_L}^{\mathrm{dec}}) \tag{3}$$

sets decoupling temperature for ν_L ☞ $T_{\nu_L}^{\mathrm{dec}} \sim 1\ \mathrm{MeV}$

- Much stronger electromagnetic interaction continues to keep protons, neutrons, electrons, positrons, and photons in equilibrium
- Reaction rate per nucleon ☞ $\Gamma_{\text{int},N} \sim T^3 \alpha^2 / m_N^2$ larger than expansion rate as long as

$$T > \frac{m_N^2}{\alpha^2 M_{\rm Pl}} \sim \text{a very low temperature} \qquad (4)$$

- $\sigma_{\rm EM} \sim \alpha^2 / m_N^2$ ☞ non-relativistic form of EM cross section
- Nucleons ☞ mantianed in kinetic equilibrium
- Average kinetic energy per nucleon ☞ $\frac{3}{2} T$
- One must be careful to distinguish between: kinetic equilibrium and chemical equilibrium
- Reactions like $\gamma\gamma \to p\bar{p}$ have long been suppressed as there are essentially no anti-nucleons around

- For $T > m_e \sim 0.5$ MeV $\sim 5 \times 10^9$ K ☞ $n_{e^-} \sim n_{e^+} \sim n_\gamma$.
- Exact ratios ☞ easily supplied by inserting appropriate "g-factors"
- Because universe is electrically neutral ☞ $n_{e^-} - n_{e^+} = n_p$
  and so there is a slight excess of electrons over positrons
- When T drops below m_e ☞ $\gamma\gamma \to e^+e^-$ suppressed by $e^{-m_e/T}$
  as only γ's in tail-end of Bose distribution can participate
- e^+ and e^- annihilate via $e^+e^- \to \gamma\gamma$ and are not replenished
  leaving a small number of electrons $n_{e^-} \sim n_p \sim 5 \times 10^{-10} n_\gamma$
- As long as thermal equilibrium was preserved
  total entropy remained fixed
- We have seen that $sa^3 \propto g(T)T^3a^3 =$ constant
- For $T \gtrsim m_e$ ☞ particles in thermal equilibrium include:
  photon ($g_\gamma = 2$) and $e^\pm$ pairs ($g_{e^\pm} = 4$)
- Effective number of particle species B4 annihilation ☞ $g_{\rm before} = \frac{11}{2}$
- After e^+e^- annihilation ☞ γ only remaining particle in equilibrium
- Effective number of particle species ☞ $g_{\rm after} = 2$

- From entropy conservation

$$\frac{11}{2}\left(T_\gamma a\right)^3\Big|_{\rm before} = 2\left(T_\gamma a\right)^3\Big|_{\rm after} \qquad (5)$$

- Heat produced by e^+e^- annihilation increases quantity $T_\gamma a$

$$\frac{(T_\gamma a)|_{\rm after}}{(T_\gamma a)|_{\rm before}} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.4 \qquad (6)$$

- Before e^+e^- annihilation ☞ $T_\nu = T_\gamma$
- But from then on ☞ T_ν dropped as a^{-1} so

$$(T_\nu a)|_{\rm after} = (T_\nu a)|_{\rm before} = (T_\gamma a)|_{\rm before} \qquad (7)$$

- Conclusion ☞ after annihilation process is over
  photon temperature is higher than neutrino temperature by

$$\left(\frac{T_\gamma}{T_\nu}\right)\Big|_{\rm after} = \frac{(T_\gamma a)|_{\rm after}}{(T_\nu a)|_{\rm after}} \simeq 1.4 \qquad (8)$$

- Therefore ☞ though out of thermal equilibrium
  ν_L and $\bar\nu_R$ make important contribution to energy density

- Energy density stored in relativistic species given by "effective number of neutrino species" ☞ $N_{\rm eff}$

$$\rho_{\rm rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\rho_\gamma \tag{9}$$

and so

$$N_{\rm eff} \equiv \left(\frac{\rho_{\rm rad} - \rho_\gamma}{\rho_\nu}\right) \simeq \frac{8}{7}\sum_B{}' \frac{g_B}{2}\left(\frac{T_B}{T_\nu}\right)^4 + \sum_F{}' \frac{g_F}{2}\left(\frac{T_F}{T_\nu}\right)^4 \tag{10}$$

  - ρ_ν ☞ energy density of single species of massless neutrinos
  - $T_{B(F)}$ ☞ effective temperature of boson (fermion) species
  - primes ☞ electrons and photons are excluded from sums
- Normalization of $N_{\rm eff}$ is such that it gives $N_{\rm eff} = 3$ for three families of massless left-handed SM neutrinos
- For practical purposes ☞ ν's freeze-out completely @ $1\ {\rm MeV}$
- Non-instantaneous neutrino decoupling gives a correction to normalization $N_{\rm eff} = 3.046$

- Near 1 MeV ☞ CC weak interactions

$$n\nu_e \rightleftharpoons pe^- , \quad ne^+ \rightleftharpoons p\bar{\nu}_e, \quad n \rightleftharpoons pe^- \bar{\nu}_e \tag{11}$$

guarantee neutron-proton chemical equilibrium

- Defining λ_{np} ☞ summed rate of reactions which convert neutrons to protons

$$\lambda_{np} = \lambda(n\nu_e \to pe^-) + \lambda(ne^+ \to p\bar{\nu}_e) + \lambda(n \to pe^-\bar{\nu}_e) \ , \tag{12}$$

- Rate λ_{pn} for reverse reactions which convert protons to neutrons given by detailed balance:

$$\lambda_{pn} = \lambda_{np} \ e^{-\Delta m/T(t)} \tag{13}$$

$\Delta m \equiv m_n - m_p = 1.293 \ \mathrm{MeV}$

- Evolution of fractional neutron abundance $X_{n/N} \equiv n_n/n_N$ described by balance equation

$$\frac{dX_{n/N}(t)}{dt} = \lambda_{pn}(t)[1 - X_{n/N}(t)] - \lambda_{np}(t) X_{n/N}(t) \qquad (14)$$

- $n_N = n_n + n_p$ ☞ total nucleon density at this time
- Equilibrium solution ☞ obtained by setting $dX_{n/N}(t)/dt = 0$

$$X^{\rm eq}_{n/N}(t) = \frac{\lambda_{pn}(t)}{\lambda_{pn}(t) + \lambda_{np}(t)} = \left[1 + e^{\Delta m/T(t)}\right]^{-1} \qquad (15)$$

- Neutron abundance tracks its value in equilibrium until inelastic neutron-proton scattering rate decreases sufficiently so as to become comparable to Hubble expansion rate
- At this point neutrons freeze-out ☞ go out of *chemical* equilibrium
- Neutron abundance @ freeze-out temperature $T^{\rm FO}_{n/N} = 0.75~{\rm MeV}$ can be approximated by its equilibrium value (15)

$$X_{n/N}(T^{\rm FO}_{n/N}) \simeq X^{\rm eq}_{n/N}(T^{\rm FO}_{n/N}) = \left[1 + e^{\Delta m/T^{\rm FO}_{n/N}}\right]^{-1} \qquad (16)$$

- Since ratio $\Delta m / T^{\rm FO}_{n/N}$ is of $\mathcal{O}(1)$
  substantial fraction of neutrons survive
  when chemical equilibrium between n and p is broken
- At this time ☞ T_γ below deuterium binding energy $\Delta_{\rm D} \simeq 2.2~{\rm MeV}$ expect sizable amounts of D to be formed via $n\,p \to {\rm D}\,\gamma$ process.
- Large photon-nucleon density ratio η^{-1} delays D synthesis until the photo–dissociation process become ineffective
- Defining onset of nucleosynthesis by criterion

$$e^{\Delta_D/T_{\rm BBN}}\eta \sim 1 \qquad (17)$$

  we obtain $T_{\rm BBN} \approx 89~{\rm keV}$
- (17) ensures that below $T_{\rm BBN}$ high energy tail in γ distribution with $E_\gamma > \Delta_D$ has been sufficiently diluted by expansion
- At this epoch ☞$N(T) = 3.36$ hence time-temperature relation dictates that BBN begins at

$$t_{\rm BBN} \simeq 167~{\rm s} \approx 180~{\rm s} \qquad (18)$$

- Once D starts forming ☞ a whole nuclear process network sets in
- When T dropped below ~ 80 keV ☞ universe cooled sufficiently cosmic nuclear reactor can begin in earnest ☞ building

$$\begin{array}{lll} p\,n \to \gamma\,\mathrm{D}, & & \\ p\,\mathrm{D} \to {}^3\mathrm{He}\,\gamma, & \mathrm{D}\,\mathrm{D} \to {}^3\mathrm{He}\,n, & \mathrm{D}\,\mathrm{D} \to p\,\mathrm{T}, \\ \mathrm{T}\mathrm{D} \to {}^4\mathrm{He}\,n, & {}^4\mathrm{He}\,\mathrm{T} \to {}^7\mathrm{Li}\,\gamma, & \\ {}^3\mathrm{He}\,n \to p\,\mathrm{T}, & {}^3\mathrm{He}\,\mathrm{D} \to {}^4\mathrm{He}\,p, & {}^3\mathrm{He}\,{}^4\mathrm{He} \to {}^7\mathrm{Be}\,\gamma, \\ {}^7\mathrm{Li}\,p \to {}^4\mathrm{He}\,{}^4\mathrm{He}, & {}^7\mathrm{Be}\,n \to {}^7\mathrm{Li}\,p, & \\ \vdots & & \end{array} \tag{19}$$

- By this time ☞ neutron abundance surviving at freeze-out has been depleted by β-decay to

$$X_{n/N}(T_{\rm BBN}) \simeq X_{n/N}(T^{\rm FO}_{n/N})\, e^{-t_{\rm BBN}/\tau_n} \tag{20}$$

$\tau_n \simeq 887$ s ☞ neutron lifetime

- Nearly *all* of these surviving neutrons are captured in ^{4}He
  (because of its large binding energy $\Delta_{^4\mathrm{He}} = 28.3~\mathrm{MeV}$)
  via reactions listed in (19)
- Heavier nuclei do not form in any significant quantity
  - because of absence of stable nuclei with A=5 or 8
    impeding nucleosynthesis via n ^{4}He, p ^{4}He or ^{4}He ^{4}He reactions
  - because of large Coulomb barrier for reactions such as
    ^{4}He T $\to$ ^{7}Li γ and ^{3}He ^{4}He $\to$ ^{7}Be γ
- By time T has dropped below ~ 30 keV ☞ time $\sim$ neutron lifetime
  average thermal energy of the nuclides and nucleons
  is too small to overcome the Coulomb barriers
  any remaining free neutrons decay and BBN ceases
- Resulting *mass* fraction of helium ☞ conventionally called Y_p is

$$Y_\mathrm{p} \simeq 2 X_{n/N}(t_\mathrm{BBN}) = 0.251 \tag{21}$$

subscript p ☞ primordial

- After a bit of algebra, (21) can be rwerittten as

$$Y_{\rm p} \simeq 0.251 + 0.014 \Delta N_\nu^{\rm eff} + 0.0002 \Delta \tau_n + 0.009 \ln \left(\frac{\eta}{5 \times 10^{-10}} \right) \quad (22)$$

- BBN has a single adjustable parameter ☞ baryon density
- Observationally-inferred primordial fractions of ^{4}He baryonic mass $Y_{\rm p} = 0.2472 \pm 0.0012$, $Y_{\rm p} = 0.2516 \pm 0.0011$, $Y_{\rm p} = 0.2477 \pm 0.0029$ have been constantly favoring ☞ $N_\nu^{\rm eff} \lesssim 3$
- Unexpectedly ☞ two recent independent studies yield $Y_{\rm p} = 0.2565 \pm 0.001({\rm stat}) \pm 0.005({\rm syst})$ and $Y_{\rm p} = 0.2561 \pm 0.011$
- For $\tau_n = 885.4 \pm 0.9$ s and $\tau_n = 878.5 \pm 0.8$ s
  $N_{\rm eff} = 3.68^{+0.80}_{-0.70}$ (2σ) and $N_{\rm eff} = 3.80^{+0.80}_{-0.70}$ (2σ), respectively
- Most recent estimate of $Y_{\rm p}$ yields
  $N_{\rm eff} = 3.58 \pm 0.25 (68\%{\rm CL}), \pm 0.40 (95.4\%{\rm CL}), \pm (99\%{\rm CL})$
- Conclusion ☞ non-standard value of $N_{\rm eff}$ is preferred at 99% CL implying $\exists$ of additional types of relativistic species

Macroscopic black holes

- We have seen ☞ BH are evolutionary endpoints of massive stars that undergo a supernova explosion
  leaving behind fairly massive burned out stellar remnant
- With no outward forces to oppose gravitational forces
  remnant will collapse in on itself
- Density to which the matter must be squeezed
  scales as inverse square of mass
- E.g. ☞ Sun would have to be compressed to a radius of 3 km
  to become a black hole
- Body lighter than Sun resists collapse
  because it becomes stabilized by repulsive quantum forces
  between subatomic particles

Quantum black holes

- Stellar collapse is not only way to form black holes
- Known laws of physics allow matter densities up to Planck value
- 10^{97} kg/m^3 ☞ density at which force of gravity becomes so strong that quantum mechanical fluctuations can break down spacetime creating BH with radius $\sim 10^{-35}$ m and mass of 10^{-8} kg
- This is the lightest black hole that can be produced according to conventional description of gravity
- It is more massive but much smaller in size than a proton

- High densities of early universe
  were prerequisite for BH formation ☞ but did not guarantee it

- For region to stop expanding and collapse
  it must have been denser than average
  so density fluctuations were also necessary

- Hawking considered quantum effects and showed that
  black holes not only swallow particles but also spit them out

- Strong gravitational fields around black hole
  induce spontaneous creation of pairs near event horizon

- While particle with positive energy can escape to infinity
  particle with negative energy must tunnel through horizon into BH
  where there are particle states with negative energy w.r.t. infinity

- As the black holes radiate ☞ they lose mass
  and so will eventually evaporate completely and disappear

Hawking evaporation

- Evaporation is generally regarded as being thermal in character
  - temperature inversely proportional to its mass $M_{\rm BH}$

$$T_{\rm BH} = \frac{1}{8\pi G M_{\rm BH}} = \frac{1}{4\ \pi\ r_s} \tag{23}$$

  - entropy $S = 2\,\pi\,M_{\rm BH}\,r_s$ ☞ $r_s \equiv$ Schwarzschild radius

- For solar mass black hole ☞ $T_{\rm BH} \sim 10^{-6}$ K
  which is completely negligible in today's universe

- For black holes of 10^{12} kg ☞ $T_{\rm BH} \sim 10^{12}$ K
  hot enough to emit both massless particles ☞ such as γ-rays
  and massive ones ☞ such as e^+ and e^-

Greybody factor

- BH produces effective potential barrier near horizon that backscatters part of the outgoing radiation modifing blackbody spectrum
- Adopt geometric optics approximation where BH acts as perfect absorber of slightly larger radius with emitting area given by

$$A = 27\pi r_s^2 \qquad (24)$$

- We conveniently write greybody factor as dimensionless constant normalized to the black hole surface area seen by the SM fields

$$\Gamma_s = \sigma_s / A_4$$

- We have ☞ $\Gamma_{s=0} = 1$, $\Gamma_{s=1/2} \approx 2/3$, and $\Gamma_{s=1} \approx 1/4$

- BH emits particles with energy between $(Q, Q+dQ)$ at rate

$$\frac{d\dot{N}_i}{dQ} = \frac{\sigma_s}{8\,\pi^2}\, Q^2 \left[\exp\left(\frac{Q}{T_{\rm BH}}\right) - (-1)^{2s}\right]^{-1} \quad (25)$$

per degree of particle freedom i

- Change of variables $u = Q/T$ brings (25) into familar form

$$\dot{N}_i = \frac{27\,\Gamma_s\,T_{\rm BH}}{128\,\pi^3} \int \frac{u^2}{e^u - (-1)^{2s}}\, du \quad (26)$$

- This expression can be easily integrated using

$$\int_0^\infty \frac{z^{n-1}}{e^z - 1}\, dz = \Gamma(n)\,\zeta(n) \quad (27)$$

and

$$\int_0^\infty \frac{z^{n-1}}{e^z + 1}\, dz = \frac{1}{2^n}\,(2^n - 2)\,\Gamma(n)\,\zeta(n) \quad (28)$$

- Integration leads to

$$\dot{N}_i = \mathcal{A}_\pm \frac{27\,\Gamma_s}{128\,\pi^3}\,\Gamma(3)\,\zeta(3)\,T_{\rm BH}\,. \tag{29}$$

- Black hole emission rate is found to be

$$\dot{N}_i \approx 7.8 \times 10^{20} \left(\frac{T_{\rm BH}}{\rm GeV}\right) \ {\rm s}^{-1} \tag{30}$$

$$\dot{N}_i \approx 3.8 \times 10^{20} \left(\frac{T_{\rm BH}}{\rm GeV}\right) \ {\rm s}^{-1} \tag{31}$$

$$\dot{N}_i \approx 1.9 \times 10^{20} \left(\frac{T_{\rm BH}}{\rm GeV}\right) \ {\rm s}^{-1} \tag{32}$$

for particles with $s = 0$, $1/2$, 1, respectively

- At any given time
  rate of decrease in BH mass is total power radiated

$$\frac{d\dot{M}_{\rm BH}}{dQ} = -\sum_i g_i \, \frac{\sigma_s}{8\pi^2} \, \frac{Q^3}{e^{Q/T_{\rm BH}} - (-1)^{2s}} \tag{33}$$

g_i ☞ number of internal d.o.f of particle species i

- A straightforward calculation gives

$$\dot{M}_{\rm BH} = -\sum_i g_i \; \mathcal{B}_\pm \; \frac{27\,\Gamma_s}{128\,\pi^3}\,\Gamma(4)\,\zeta(4)\;T_{\rm BH}^2 \tag{34}$$

- Assuming effective high energy theory
  contains approximately same number of modes as SM
- $g_{s=1/2} = 90$ and $g_{s=1} = 27$

$$\frac{dM_{\rm BH}}{dt} = 8.3 \times 10^{73}\ {\rm GeV}^4 \frac{1}{M_{\rm BH}^2} \tag{35}$$

- Ignoring thresholds ☞ assuming $M_{\rm BH}$ evolves according to (35) during the entire process of evaporation

$$\tau_{\rm BH} = 1.2 \times 10^{-74}\ {\rm GeV}^{-4} \int M_{\rm BH}^2\ dM_{\rm BH} \tag{36}$$

- Using $\hbar = 6.58 \times 10^{-25}\ {\rm GeV\,s}$ ☞ (36) can then be re-written as

$$\begin{aligned} \tau_{\rm BH} &\simeq 2.6 \times 10^{-99}\ (M_{\rm BH}/{\rm GeV})^3\ {\rm s} \\ &\simeq 1.6 \times 10^{-26}\ (M_{\rm BH}/{\rm kg})^3\ {\rm yr} \end{aligned} \tag{37}$$

- For solar mass black hole ☞ lifetime is unobservably long 10^{64} yr
- For 10^{12} kg BH ☞ lifetime is $\sim 1.5 \times 10^{10}$ yr
  about the present age of the universe
- Any primordial black hole of this mass
  would be completing its evaporation and exploding right now

- Questions raised by primordial BH motivate empirical search for them
- Most of BH mass would go into gamma rays
  with energy spectrum that peaks around 100 MeV
- q's and g's would hadronize mostly into π
  which in turn would decay to γ-rays and ν's
- In 1976 ☞ Hawking and Page realized that γ-ray background observations
  place strong upper limits on number of such black holes
- By looking at observed γ-ray spectrum
  for masses near 5×10^{11} kg
  they set upper limit ding43 $10^4/\mathrm{pc}^3$