Astronomy, Astrophysics, and Cosmology

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Lesson X
April 19, 2016

arXiv:0706.1988

At the beginning... there was the hand of God

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- History of universe from 10^{-10} seconds to today
  is based on observational facts
- Fundamental laws of high energy physics are well-established
  up to energies reached by LHC
- Before 10^{-10} seconds ☞ energy of universe exceeds 13 TeV
  and we lose comfort of direct experimental guidance
- Physics of that era is as speculative as it is fascinating
- Today ☞ we will go back to the earliest of times
  as close as possible to the *big bang*
  and follow evolution of Universe

- As $a \to 0$ ☞ temperature increases without limit $T \to \infty$ but there comes point @ which extrapolation of classical physics breaks down
- Realm of quantum black holes ☞ thermal energy of particles is such that their de Broglie wavelength is smaller than their Schwarzschild radius
- Equating h/mc to $2Gm/c^2$ yields system of Planck units

$$\begin{aligned} M_{\rm Pl} &\equiv \sqrt{\frac{\hbar c}{G}} \simeq 10^{19}~{\rm GeV} \\ \ell_{\rm Pl} &\equiv \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35}~{\rm m} \\ t_{\rm Pl} &\equiv \sqrt{\frac{\hbar G}{c^5}} \simeq 10^{-43}~{\rm s} \end{aligned} \tag{1}$$

- $t_{\rm Pl}$ ☞ sets origin of time for *classical big bang* era
- It is inaccurate to extend solution of Friedmann equation to $a = 0$ and conclude universe began in singularity of infinite density

- @ $t \sim 10^{-43}$ s ☞ *phase transition* is thought to have occured during which gravitational force *condensed out* as separate force
- Symmetry of four forces was broken
  but the strong, weak, and electromagnetic forces were still unified and there were no distinctions between quarks and leptons
- This is unimaginably short time
  and predictions can be only speculative
- Temperature would have been about 10^{32} K ☞ corresponding to *particles* moving about every which way with average energy

$$kT \approx \frac{1.4 \times 10^{-23}\ \mathrm{J/K}\ \ 10^{32}\ \mathrm{K}}{1.6 \times 10^{-10}\ \mathrm{J/GeV}} \approx 10^{19}\ \mathrm{GeV} \quad (2)$$

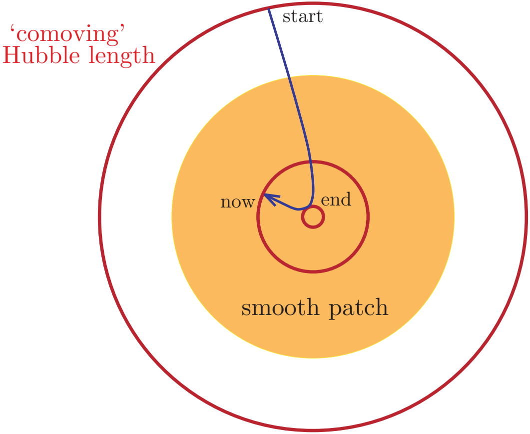
- Very shortly thereafter ☞ as temperature had dropped to $\sim 10^{28}$ K there was another phase transition
  where strong force condensed out
- Now universe was filled with *soup* of quarks and leptons
- About this time ☞ universe underwent exponential expansion increasing in size by a factor of $\gtrsim 10^{26}$
  in a tiny fraction of a second ☞ perhaps $\sim 10^{-34}$ s
- Favored ΛCDM model implicitly includes inflationary hypothesis where scale factor expands exponentially ☞ $a(t) \propto e^{Ht}$
- If interval of exponential expansion satisfies $\Delta t \gtrsim 60/H$
  small casually connected region can grow sufficiently
  to accommodate observed homogeneity and isotropy

- Important to emphasize subtle distinction between comoving horizon $\varrho_{\rm h}$ and comoving Hubble radius $c/(aH)$
- Express comoving horizon as integral of comoving Hubble radius

$$\varrho_{\rm h} \equiv c \int_0^t \frac{dt'}{a(t')} = c \int_0^a \frac{da}{Ha^2} = c \int_0^a \frac{1}{aH}\, d\ln a \qquad (3)$$

- If particles are separated by distances greater than $\varrho_{\rm h}$ they never could have communicated with one another
- If they are separated by distances greater than $c/(aH)$ they cannot talk to each other now
- This distinction is crucial for solution to horizon problem
- It is possible that $\varrho_{\rm h}$ is much larger than $c/(aH)$ now so that particles cannot communicate today but were in causal contact early on
- From (3) we see that this might happen if comoving Hubble radius in early universe $\gg$ that it is now so that $\varrho_{\rm h}$ got most of its contribution from early times
- Hence ☞ we require phase of decreasing Hubble radius

- Evolution of comoving Hubble radius in inflationary universe



- Comoving Hubble sphere shrinks during inflation
  and expands after inflation
- Inflation ☞ mechanism to *zoom-in* on smooth sub-horizon patch

Conditions for inflation

- shrinking Hubble sphere defined by

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \tag{4}$$

- From ☞

$$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{\ddot{a}}{(aH)^2} \tag{5}$$

  shrinking comoving Hubble radius
  implies accelerated expansion $\ddot{a} > 0$

- This explains why
  inflation is often defined as period of accelerated expansion

More conditions for inflation

- Second time derivative of scale factor can be related to first time derivative of Hubble parameter

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon) \Rightarrow \epsilon \equiv -\frac{\dot{H}}{H^2} \tag{6}$$

- Acceleration therefore corresponds to $\epsilon < 1$
- During inflation
  - H is approximately constant during inflation
  - a grows exponentially
  - this implies $c/(aH)$ decreases ☞ just as advertised
- Consulting *acceleration equation*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3P)\,. \tag{7}$$

we infer that $\ddot{a} > 0$ requires a negative pressure ☞ $P < -\rho/3$

- After very brief inflationary period
  universe would have settled back into its more regular expansion
- For $10^{-34}\ \mathrm{s} < t < 10^{5}\ \mathrm{yr}$ ☞ $10^{3}\ \mathrm{K} < T < 10^{27}\ \mathrm{K}$
  universe is thought to have been dominated by radiation
- We have seen equation of state is given by $w = 1/3$
- Neglect contributions to H from Λ
  (this is always a good approximation for small enough a)
  then ☞ $a \sim t^{1/2}$ and $\rho_{\rm rad} \sim a^{-4}$
- Expansion rate as a function of temperature in plasma

$$H = \left(\frac{8\pi G \rho_{\rm rad}}{3}\right)^{1/2} = \left(\frac{8\pi^3}{90} g_\rho(T)\right)^{1/2} T^2/M_{\rm Pl} \sim 1.66 \sqrt{g_\rho(T)}\, T^2/M_{\rm Pl} \qquad (8)$$

(we have adopted natural units $\hbar = c = k = 1$)

- Neglecting T-dependence of g_ρ
  (i.e. away from mass thresholds and phase transitions)
  integration of (8) yields

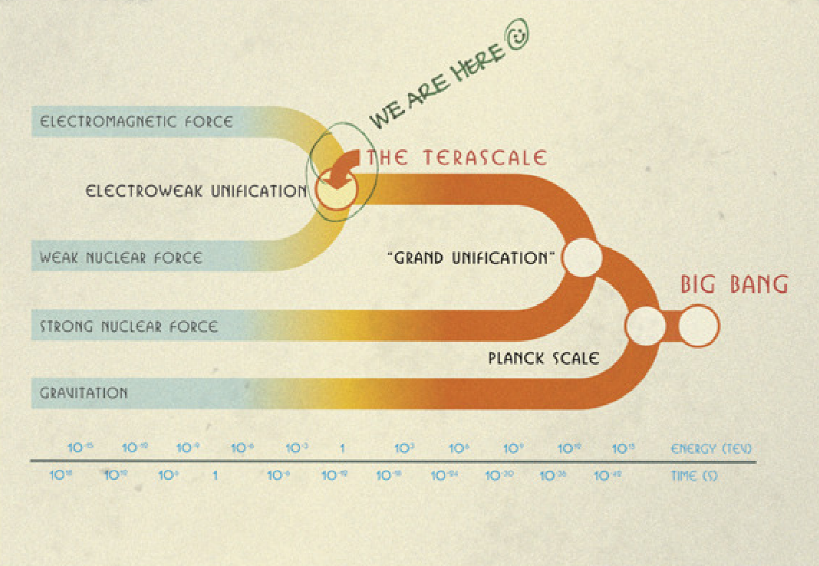
$$a(t) = \left(\frac{t}{t_0}\right)^{1/2} \quad \text{and} \quad \rho(t) = \frac{\rho_0}{a^4} = \frac{\rho_0 t_0^2}{t^2} \tag{9}$$

- (8) leads to useful commonly used approximation

$$t \simeq \left(\frac{3M_{\rm Pl}^2}{32\pi\rho_{\rm rad}}\right)^{1/2} \simeq 2.42 \frac{1}{\sqrt{g_\rho}} \left(\frac{T}{\rm MeV}\right)^{-2} \ {\rm s} \tag{10}$$

Electroweak symmetry breaking

- At about 10^{-10} s ☞ Higgs field spontaneously acquires VEV which breaks electroweak gauge symmetry
- Weak force and electromagnetic force manifest with different ranges
- Through Higgs mechanism quarks and charged leptons become massive
- Fundamental interactions have then taken their present forms but the temperature of the universe ($T \sim 1$ TeV) is still too high to allow quarks to bind together to form hadrons


WE ARE HERE
ELECTROMAGNETIC FORCE
THE TERASCALE
ELECTROWEAK UNIFICATION
WEAK NUCLEAR FORCE
"GRAND UNIFICATION"
BIG BANG
STRONG NUCLEAR FORCE
PLANCK SCALE
GRAVITATION
ENERGY (TEV)
TIME (S)

- @ $t \sim 10^{-6}$ s ☞ $T \sim 1$ GeV

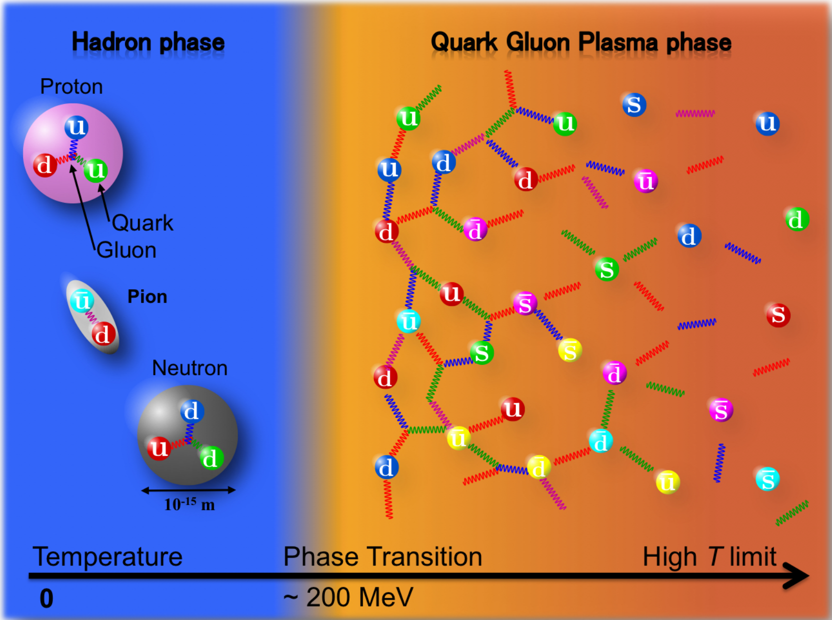
  quarks began to *condense* into mesons and baryons
- If relativistic particles are present that have decoupled from γ's it is necessary to distinguish between two kinds of r.d.o.f.:
  - those associated total energy density g_ρ
  - those associated with total entropy density g_s
- @ energies above deconfinement transition
  quarks and gluons are relevant fields for QCD sector
  effective number of interacting (thermally coupled) r.d.o.f. is

$$g_s(T) = 61.75$$

- As universe cools below confinement scale ☞ $\Lambda_{\rm QCD} \sim 200$ MeV
  SM plasma transitions to regime
  where mesons and baryons are pertinent degrees of freedom
- Precisely ☞ relevant hadrons present in this energy regime

  are pions and charged kaons

$$g_s(T) = 19.25$$


Hadron phase
Quark Gluon Plasma phase
Proton
Quark
Gluon
Pion
Neutron
10^{-15} m
Temperature
Phase Transition
High T limit
0
~ 200 MeV

- Significant reduction in r.d.o.f. is from rapid annihilation or decay of massive hadrons which may have formed during transition
- Quark-hadron crossover transition ☞ has associated large redistribution of entropy into remaining r.d.o.f.
- To connect temperature to effective number of r.d.o.f. use high statistics lattice simulations of QCD plasma in hot phase especially behavior of entropy during changeover
- Concretely ☞ effective number of interacting r.d.o.f. in plasma

$$g_s(T) \simeq r(T) \left(g_B + \frac{7}{8} g_{\rm F} \right) \tag{11}$$

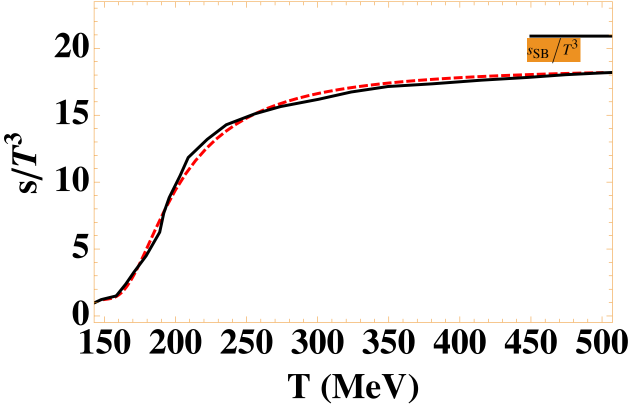
✧ $r = 1$ for leptons
✧ $r = 22$ for photon contributions
✧ $r = s(T)/s_{\rm SB}$ for quark-gluon plasma

  - $s(T)$ ☞ actual entropy
  - $(s_{\rm SB})$ ☞ ideal Stefan-Bolzmann entropy

✧ For $150\ \mathrm{MeV} < T < 500\ \mathrm{MeV}$
entropy rise during confinement-deconfinement changeover

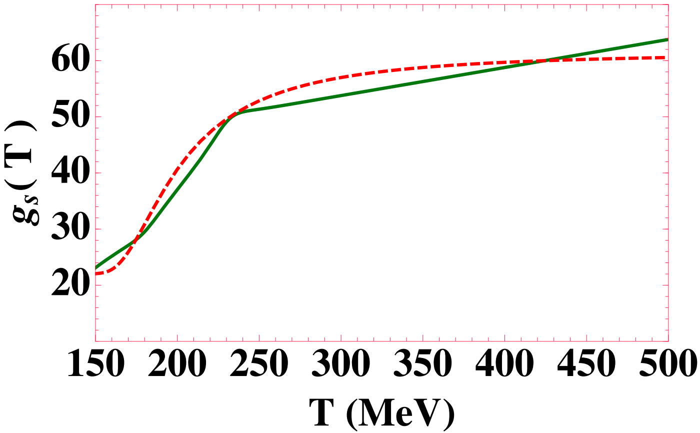
$$\frac{s}{T^3} \simeq \frac{42.82}{\sqrt{392\pi}}\, e^{-C_1} + 18.62\, \frac{C_2^2}{\left[e^{C_2} - 1\right]^2}\, e^{C_2} \qquad (12)$$

$$C_1 = (T_{\rm MeV} - 151)^2/392 \text{ and } C_2 = 195.1/(T_{\rm MeV} - 134)$$



✧ For same energy range

$$g_s(T) \simeq 47.5\, r(T) + 19.25 \tag{13}$$



- Entropy density dominated by contribution of relativistic particles
- To very good approximation

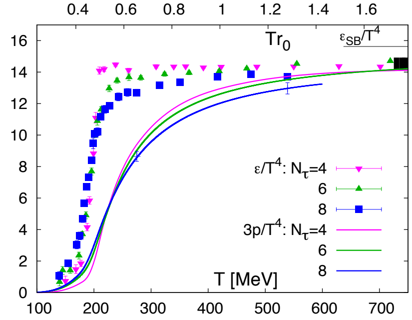
$$s = \frac{2\pi^2}{45} g_s(T)\, T^3 \tag{14}$$

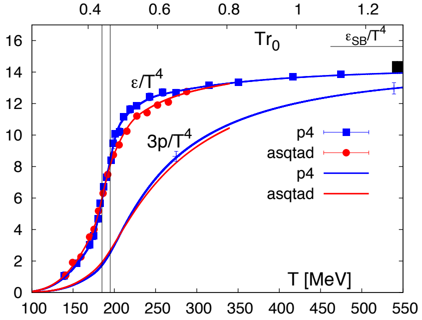
- Conservation of $S = sV$ leads to

$$\frac{d}{dt}(sa^3) = 0 \Rightarrow g_s(T) T^3 a^3 = \text{constant as universe expands} \tag{15}$$

- As one would expect ☞ non-evolving system would stay @ constant entropy density in comoving coordinates even though s is decreasing due to expansion of universe

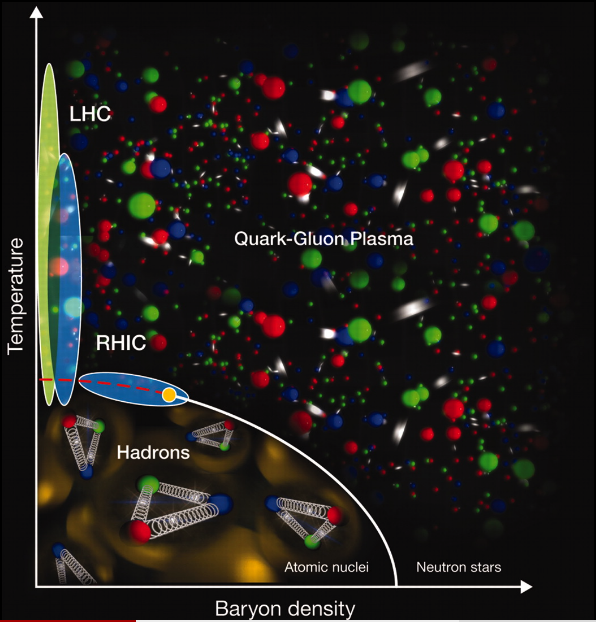
✧ Since quark-gluon energy density in plasma has similar T dependence to that of the entropy





we'll simplify discussion by taking $g = g_\rho = g_s$

Sensitivity of particle colliders



Baryogenesis

- Manned and unmanned exploration of solar system tell us that it is made up of same stuff as Earth: *baryons*
- Observational evidence from radio-astronomy and cosmic rays indicate Milky Way and distant galaxies are made of baryons
- Conclusion: baryon number of the observable universe ☞ $B > 0$
- Requirement: early $q\bar{q}$ plasma contained tiny surplus of quarks
- After all anti-matter annihilated with matter
  only small surplus of matter remained

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 5 \times 10^{-10} \, \frac{\text{excess baryons}}{\text{photons}} \tag{16}$$

- Tiny surplus ☞ explained by interactions in early universe
  that were not completely symmetric
  with respect to an exchange of matter-antimatter