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Problems set # 9 **Physics 307** November 16, 2016

Ordinary Differential Equations V

1. *(i)* Find the Fourier series of $f(x) = |x|$, with $0 \leq x \leq L$.
(ii) Evaluate $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.

2. Show that:

$$\sum_{k=-\infty}^{\infty} \delta(x - a + 2kh) = \frac{1}{h} \left\{ \frac{1}{2} + \sum_{m=1}^{\infty} \cos\left[\frac{m\pi(x-a)}{h}\right] \right\}.$$

3. Using

$$g(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-isx} f(x)\, dx,$$

demonstrate the following Fourier transforms:
(i)

$$f(x) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} e^{-x^2/(4\sigma^2)}, \quad y(s) = \left(\frac{1}{2\pi\sigma'^2}\right)^{1/4} e^{-s^2/(4\sigma'^2)}, \quad \sigma'^2 = (4\sigma^2)^{-1};$$

(ii)

$$f(x) = \frac{1}{a^2 + x^2}, \quad g(s) = \sqrt{\frac{\pi}{2a^2}}\, e^{-|s|a}, \quad a > 0;$$

(iii)

$$f(x) = \sin(bx), \quad g(s) = i\sqrt{\frac{\pi}{2}}\, [\delta(s+b) - \delta(s-b)]\,.$$

(iv) Show that the Fourier transform of $f^{(n)}(x)$ is $(is)^n g(s)$.

4. *(i)* Show that the Laplace transform, $\mathscr{L}[f(x)] = \int_0^\infty e^{-sx}\, f(x)\, dx$, of $f'(x)$ is

$$\mathscr{L}[f'(x) = s\mathscr{L}[f(x)] - f(0).$$

(ii) Generalize the previous result for

$$\mathscr{L}[f^{(n)}(x)] = s^n \mathscr{L}[f(x)] - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0)\,.$$

(iii) Show that

$$\mathscr{L}(x^{n-1} e^{-ax}) = \Gamma(n)\ (s+a)^{-n}\,.$$

(iv) Using Laplace transform solve

$$y'' + 2y' + y = 2\ x\ e^{-x},$$

with $y(0) = 0$ and $y'(0) = 1$.

5. Show that in the vicinity of a simple jump of a function f, the partial sums S_n always overshoot the mark by about 9%. This is the so-called “Gibbs phenomenon.”