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Problems set # 5 **Physics 307** October 19, 2016

Ordinary Differential Equations I

1. Show that a linear equation is homogeneous if and only if 0 is a solution.

2. *(i)* By virtue of Newton's law, the cooling rate of a body in the open air is proportional to the temperature difference between the body and the environment. If the air temperature is 20° and the body is cooled from 100° to 60° in 20 minutes, determine the time it would take for the body temperature to drop to 30°.
(ii) Find the curve that passes through $(0, -2)$ so that the slope of the tangent at each point is equal to the ordinate corresponding to that point plus three units.

3. *(i)* For a given electrostatic field in the plane, show that the equipotential lines are trajectories orthogonal to the lines of force.
(ii) If the electromagnetic potential is of the form $\Phi = xy$, find the equation for the streamlines.

4. Determine which of the following differential equations have singular solutions (*i.e.*, integral curves where uniqueness is violated):
(i) $\frac{dy}{dx} = y^2 + x^2$;
(ii) $\frac{dy}{dx} = (y - x)^{2/3} + 5$;
(iii) $\frac{dy}{dx} = (y - x)^{2/3} + 1$.

5. *(i)* Using Picard's theorem find the general solution of $\frac{dy}{dx} = y^2, \quad \text{with} \quad y(0) = 1$;
(ii) Determine the first and second Picard iterations of $\frac{dy}{dx} = x - y^2, \quad \text{with} \quad y(1) = 0$.